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A
TREATISE ON THE THEORY AND PRACTICE
OF FINDING THE
LONGITUDE AT SEA OR LAND.

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THE
THEORY AND PRACTICE
OF FINDING THE
LONGITUDE
AT SEA OR LAND:

TO WHICH ARE ADDED,
VARIOUS METHODS OF DETERMINING
THE
LATITUDE OF A PLACE,
AND
VARIATION OF THE COMPASS;
WITH
NEW TABLES.

BY
ANDREW MACKAY, A. M. F. R. S. E.

IN TWO VOLUMES.
VOLUME I.

LONDON:
Printed for J. Sewell, Cornhill, P. Elmsly, Strand, and J. Evans,
Paternoſter-row.
MDCCXCIII.

Entered at STATIONERS HALL, according to Act of Parliament.

TO HIS GRACE

ALEXANDER, DUKE OF GORDON,

MARQUIS OF HUNTLY,

EARL OF HUNTLY AND ENZIE,

VISCOUNT INVERNESS,

LORD GORDON OF

BADZENOCH, LOCHABER,

STRATHAVEN, ACHINDOUN,

BALLMORE, KINCARDINE,

GARTLY, &c.

EARL OF NORWICH,

AND

BARON GORDON OF HUNTLEY,

IN ENGLAND.

THIS TREATISE IS,

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ERRATA.

Page 41 line 7 from bottom, for *right or left*, read *left or right*.
2 from bottom, for *exceeds*, read *is leſs than*.

PREFACE.

IN every commercial ſtate, any work that has for its object the improvement of the art of Navigation, will always be favourably received; and ſhould it not in all reſpects anſwer the public expectation, the author may ſtill have ſome claim to indulgence, from his good intentions, and having exerted his beſt abilities for the good of his country. With this view, the author of the following treatiſe has beſtowed much time and labour to render it as complete and as generally uſeful as poſſible; and tho' he has not the vanity to imagine it to be by any means a perfect work, he has yet been flattered, that it may be of ſervice to Navigation, and therefore not altogether unworthy of the notice of the public. He ventures then to ſubmit it, with much diffidence, to their judgment, and if it ſhall be found any ways deſerving of ſuch a character, he will eſteem their approbation his higheſt and beſt reward. Should this be the caſe, he may be encouraged, at a future period, to offer ſome opinions on the mechanical conſtruction of ſhips—an ancient art, but perhaps leſs improved by long experience than any which we now cultivate with ſo much ſucceſs.

It is well known, that altho' the courſe and diſtance between any two places, whoſe latitude and longitude are accurately ſettled, may be aſcertained with the greateſt preciſion; yet a ſhip at ſea is ſo liable to be put out of the proper courſe, by ſtorms, contrary winds, currents, &c. that after all reaſonable allowance is made for the errors ariſing from theſe cauſes, the place of the ſhip by account is very ſeldom found to agree with the real place, or that deduced from obſervation the difference frequently amounting to ſeveral degrees. It is therefore obvious, that the ſhip may be expoſed to the greateſt hazard, when the ſeamen think themſelves moſt ſecure, and from this appears the abſolute neceſſity of taking obſervations, when opportunity offers, for obtaining with accuracy the true place of the ſhip.

At ſea, the latitude of the ſhip is eaſily deduced either from the meridian altitude of the Sun, or from that of a Star or Planet, or from double, treble, &c. altitudes of any of theſe objects; but withregard to the longitude, a more difficult operation becomes neceſſary. At land, indeed, the longitude of any place may be found with very little trouble by ſeveral methods, particularly by the eclipſes of Jupiter's ſatellites; and the longitude at ſea might be found by this method with the ſame facility, provided a teleſcope poſſeſſed of a ſufficient magnifying power could be employed. Attempts to render a teleſcope manageable at ſea have been made by Dr Irwin and others, and an inſtrument for this purpoſe is deſcribed in Rutledge's "Theorie Aſtronomique". A teleſcope, however, with a ſmall magnifying power, may be uſed at ſea in moderate weather, and ſince even with ſuch an inſtrument, ſolar eclipſes and occultations may be often obſerved with tolerable accuracy, and conſequently the longitude aſcertained from ſuch obſervations, they ought never to be neglected, when the ſhip is not in ſight of land, or upon an unknown coaſt. The calculations, indeed, which neceſſarily attend theſe methods, ſeem to be conſidered as too laborious and difficult for the generality of ſeamen, and therefore the method by obſerving the diſtance between the Moon and Sun or a fixed Star, is now commonly preferred, both on rccount of its ſuperior facility, and that it can be practiſed at ſea almoſt as often as neceſſary. Yet, as even this method is attended with a calculation by many thought troubleſome, various ways have been propoſed, to ſhorten as much as poſſible the operation for reducing the apparent to the true diſtance, and it is now accordingly reduced to a tolerably ſimple computation. By this method, the longitude may in moſt caſes be determined within half a degree, (which at ſea is eſteemed no very great error,) and if the calculations are accurately performed, from the mean of ſeveral ſets of good obſervations, taken at ſhort intervals of time, the error will perhaps be greatly diminiſhed.

If a chronometer or timekeeper could be conſtructed, ſo as to go uniformly when placed in every different poſition, and under different degrees of heat, then would this method of finding the longitude at ſea be a moſt valuable acquiſition to the navigator. Indeed ſome of our ingenious countrymen have brought this art to a degree of perfection formerly unknown; and every perſon acquainted with the principles of watchmaking muſt highly admire the accuracy of Harriſon's timekeeper. Thoſe made by Arnold and Kendal are alſo excellent; but it is to be hoped, that inſtruments of this kind may be ſtill farther improved, and may be afforded much cheaper than at preſent, for the high price alone is a very great objection to them, and very much prevents their being more generally uſeful. It is well known, that every chronometer hitherto contrived is ſubject to irregularities; the ſmalleſt ſhock is found to affect them, and the rate of going is found to be altered by changes in the atmoſphere, even tho' provided with thermometer pieces. Upon this and other accounts therefore, their accuracy is very much to be ſuſpected, ſo that at preſent they are chiefly uſed for experiment, or to connect obſervations, for which purpoſe they certainly make a valuable appendage to a ſet of nautical inſtruments.

The following work, which contains the methods of finding the longitude already mentioned, and alſo thoſe from which any advantage can be derived for the ſolution of this important problem, is divided into two volumes, and the firſt volume is divided into ſix books.

The firſt book contains the general principles neceſſary for a proper knowledge of the ſubject.

Book ſecond contains the deſcription, rectification, and uſe of the Quadrant, Sextant, and Circular Inſtrument, in their preſent improved ſtate—alſo, an account of the corrections to be applied to the obſerved altitude of any celeſtial object, in order to reduce it to the true altitude.

In book third is contained a complete ſyſtem of lunar obſervations, with an introductory account of this method of finding the longitude at ſea. It alſo contains a new method of finding the longitude of a ſhip at ſea, together with the apparent time from the ſame ſet of obſervations—for which the author had the honour of receiving the thanks of the Board of Longitude.

Book fourth contains various methods of finding the longitude of a place, ſome of which, though ſcarcely practicable at ſea, are yet perhaps the very beſt for determining the longitude of any place at land. Theſe are, by the Moon's tranſit over the meridian, by lunar eclipſes, ſolar eclipſes, occultations of fixed ſtars by the Moon, eclipſes of the ſatellites of Jupiter, by a Chronometer, and by the Variation Chart.

Every treatiſe muſt be conſidered as defective, wherein rules are given without demonſtrations, and this is particularly the caſe with moſt books on the ſubject of navigation, probably becauſe ſeafaring people in general are very much indifferent as to the *rationale* of any rule given for obſervation, provided it holds good in practice. On the other hand, it not unfrequently happens, that to perſons not very fond of mathematical inveſtigations, the rule and demonſtration, following each other in regular order, may be diſagreeable. For theſe reaſons, and to gratify the ſcientific ſeaman in the preſent highly improved ſtate of nautical knowledge, the demonſtrations of the rules and formulæ are ſeparately inſerted at the end of the work in the fifth book. And, in order to render this work ſtill more generally uſeful, there is farther added to what was originally propoſed, book ſixth, containing various methods of finding a ſhip's latitude, and the variation of the compaſs.

Volume ſecond contains the tables, along with the neceſſary explanations.

This work, therefore, it is hoped, will be found to comprehend a collection of the beſt methods of making and reducing all ſorts of obſervations neceſſary to be made at ſea, for aſcertaining a ſhip's place, or for reducing the true courſe between any two places to the courſe per compaſs, or for correcting the courſe ſteered. The author however flatters himſelf, that it will not be conſidered merely as a compilation from the works of others, but that the intelligent reader will diſcover in various parts of it, ſome things that are at leaſt new and of his own invention (whatever other merit they may poſſeſs) and in every part of it, a ſuitable plainneſs and perſpicuity of ſtile, and a warm zeal for the improvement of one of the nobleſt and moſt uſeful of human arts. In a word, he has beſtowed his beſt endeavours to render his work worthy the notice of the navigator and aſtronomer, as well as in ſome reſpects of the practical geographer and ſurveyor, and worthy of the patronage of thoſe reſpectable individuals who have honoured the author with their ſubſcriptions.

EXPLANATION OF CHARACTERS.

The ſign	+ (plus)	ſignifies	Addition.
	− (minus)	—	Subtraction.
	× or (.)	—	Multiplication.
	÷	—	Diviſion.
	√	—	The Square Root.
	☌	—	Conjunction.
	☍	—	Oppoſition.

The planetary characters are alſo uſed to repreſent the days of the week—thus,

☉ denotes Sunday, ☽ Monday, ♂ Tueſday,
☿ — Wedneſday, ♃ Thurſday, ♀ Friday.
♄ — Saturday,

Owing to the Author's great diſtance from the preſs, and other circumſtances, during the time this work was printing, a few errors, principally typographical, have eſcaped notice. Thoſe already obſerved are marked in the following table of errata—which, it is intreated, the reader will be pleaſed to correct.

ERRATA.

Page	7	line	31	*for*	as are	*read*	are as
	14	—	23	—	(fig. 4)	—	(fig. 3)
	56	—	28	—	$\frac{s^2}{15^\circ\ 36'\ 45'' - s}$	—	$\frac{s^2}{15^\circ\ 46'\ 45'' - s}$
	92	—	13	—	ſine	—	ſecant
	109	—	2	—	the obſervation	—	an obſervation
	108	—	23		cancel the word *true*		
	108	—	27	—	arc firſt	—	arc ſecond
	108	—	28	—	arc ſecond	—	arc firſt
	217	—	26	—	+ b. 2	—	+ L. 2.
	232	—	22	—	ſame the	—	ſame time, the

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THE THEORY AND PRACTICE OF FINDING THE LONGITUDE AT SEA OR LAND.

BOOK I.

CONTAINING,

The Principles of the Aſtronomical Methods of finding the Longitude at Sea or Land.

CHAP. I.

Of the FIGURE and MAGNITUDE of the EARTH.

WITHOUT a previous knowledge of the figure and magnitude of the Earth, the places of the Heavenly Bodies could not be accurately ſettled, from obſervations made on its ſurface; and therefore computations, made from obſervations of theſe bodies, could not be depended on for aſcertaining the ſituation of places on the Earth; hence the neceſſity of knowing both the figure and magnitude of the Earth is apparently obvious.

The opinions of the antients concerning the figure of the Earth were various—the moſt prevailing one being, that it was an extended plane; ſome imagined it to be of a cylindric form, others, that it reſembled a canoe, and a few ſuppoſed it ſpherical; but the diſcovery of its real figure was left to the illuſtrious Sir Iſaac Newton. The following are a few of the arguments commonly uſed to prove, that the figure of the Earth is either ſpherical, or nearly ſo.

I.

The firſt who attempted to circumnavigate the earth was Ferdinand Magellan, a Portugueſe. He ſailed from Seville, Auguſt 10, 1519, was killed on the iſland of Sebu; and one of his veſſels arrived at St Lucar, near Seville, Sept. 7, 1522. Since that time, the circumnavigation of the Earth has been performed, at different times, by Sir Francis Drake, Lord Anſon, Captain Cook, and others—

who, by ſailing in a weſterly direction, allowance being made for promontories, &c. arrived at the country they ſailed from. Hence the Earth muſt either be of a cylindric or globular figure; but it cannot be in the form of a cylinder, becauſe then the difference of longitude and meridian diſtance between any two places would be equal, which is contrary to obſervation; the figure of the Earth therefore is ſpherical.

II.

Lemmata.—1ſt. The diſtance of the neareſt fixed ſtar, when compared with the magnitude of the Earth, is ſo immenſe, that rays flowing therefrom to any two points on the ſurface of the earth, are phyſically parallel.

2d. If in a curve, the arches are proportional to the correſpondent angles, it is a circle.

Now, if the Earth was an extended plane, the meridian zenith diſtance of any given fixed ſtar would be the ſame in all places of the Earth, by lemma 1ſt; but it is found to be variable, and in ſuch a manner, that the difference of the meridian altitudes of the ſame ſtar is proportional to the intercepted arch of a meridian; that meridian muſt therefore be circular—and ſince this is found to be the caſe in every part of the Earth, therefore, by lemma 2d, its figure muſt be ſpherical.

III.

Lemma. If the ſhadow of any body, when turned in every poſition with reſpect to the luminous body, be circular, when projected on a plane perpendicular to the line joining the centers of both, the body itſelf is a ſphere.

Now ſince a lunar eclipſe ariſes from the paſſage of the moon thro' the ſhadow of the Earth, and as the portion of the Earth's ſhadow projected on the lunar diſc is always circular, in every different poſition of the earth, therefore the figure of the earth muſt be that of a ſphere.

IV.

When diſtant ſhips are viewed with a teleſcope, the lower parts are found to be hid by the interpoſed water, and more or leſs is viſible, according to the diſtance. The ſun is obſerved ſooner at riſing, and later at ſetting, by a perſon at the maſt-head, than by one on deck. In making the land, the moſt elevated parts are firſt ſeen; and the lower parts become viſible, as the land is approached.—Theſe phenomena evidently ariſe from the ſpherical figure of the Earth, and therefore ſerve to prove the Earth to be of that figure.

V.

The appearance of the ſun for a continuance of ſeveral months above the horizon, in the neighbourhood of one pole, and that in proportion to the diſtance of the place therefrom, while at a place equally diſtant from the other pole, the ſun is as long abſent—is another proof that the earth is ſpherical.

VI.

The planets that have been hitherto diſcovered are obſerved to be globular; the Earth is alſo a planet, ſubject to the ſame laws, and revolving round the ſun in the ſame manner as the other planets—therefore, by analogy, the Earth muſt alſo be globular.

Although it appears from the foregoing proofs, that the earth is of a ſpherical figure, yet it is not a perfect ſphere, but an OBLATE SPHEROID,* which is a ſolid generated by the rotation of a ſemi-ellipſe about its ſhorteſt axis. The circumſtance that gave riſe to the knowledge of its ſpheroidal figure, was Richer's celebrated experiment, made by order of the Royal Academy of Sciences of Paris, at the iſland of Cayenne, in the year 1672, " That a pendulum of " the ſame length vibrates ſlower at the equator, than in France." This naturally reſults from the rotation of the Earth; but the difference being greater than what ſhould have ariſen from the exceſs of the centrifugal force at the equator above that in France, excited the curioſity of Newton and Huygens, who enquiring minutely into the cauſe of this phenomenon, ſhewed that it reſulted from the rotation of the earth about its axis, combined with its ſpheroidal figure; and Newton ſuppoſing the Earth to aſſume the ſame figure that a homogeneous fluid would take under like circumſtances, computed the ratio of the equatorial diameter to the axis of the earth to be as 230 : 229. The ratio deduced from actual menſuration ſeems to confirm the above computation.

The determination of the magnitude of the Earth ſeems to have engaged the attention both of ancient and modern aſtronomers and mathematicians, and though the concluſions of the ancients differ very conſiderably, yet the meaſures taken within the laſt 120 years, near the ſame place, agree very well.

The only perſon in England who performed this problem with any tolerable degree of accuracy, was Mr Richard Norwood, of which an account is given in his " Seaman's Practice"—the ſubſtance of which is as follows. In the year 1633, he obſerved the apparent meridian altitude of the ſun, at the time of the ſummer ſolſtice, near the Tower of London; and June 11, O. S. 1635, he obſerved the ſun's apparent meridian altitude, near the middle of the city of York; from whence he found the difference of latitude between thoſe places to be 2° 28′. Now this being compared with the difference between the parallels of latitude of thoſe places, deduced from actual menſuration, which was 9149 chains of 99 feet each, he

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* From obſervations made in different parts of the Earth, it would appear, that its figure is not that of a regular ſpheroid.

found that one degree contained 367196 Engliſh feet. However, as Norwood paced ſome of his diſtances, the above meaſure is therefore only an approximation to the truth. In order to obtain a more accurate ſolution to this problem, menſurations have been taken by ſeveral French aſtronomers and mathematicians, particularly by Meſſrs Picard, Caſſini, Maupertuis, de la Caille, Bouguer, &c.— The near agreement of their meaſures taken at the ſame place, is a confirmation of the accuracy of their operations; and the compariſon of their meaſures taken at different places confirms Newton's theory of the ſpheroidal figure of the earth.

The method uſually practiſed to find the length of a degree is, to meaſure a baſe, as accurately as poſſible, of about one tenth of the portion of the meridian intended to be meaſured, and nearly in the middle of that portion, from which a ſeries of triangles is to be extended, ending at the two extreme ſtations, from which the arch of the meridian intercepted between the parallels of theſe ſtations is to be computed. At each extreme ſtation, the zenith diſtance of one or more ſtars is to be obſerved, and corrected for the preceſſion, aberration, nutation, &c.—then the difference between the zenith diſtances of the ſame ſtar gives the correſpondent celeſtial arc. Now,

As the celeſtial arc
Is to one degree,
So is the interval between the two parallels deduced from menſuration
To the length of a degree in the ſame meaſure.

In the year 1756, the Royal Academy of Sciences of Paris appointed eight aſtronomers to meaſure the length of a degree between Paris and Amiens. Theſe gentlemen divided themſelves into two companies, and each company meaſured a baſe ſeparately.— The reſult of their menſurations gave 57069 toiſes for the length of a degree; now ſince 107 French feet are equal to 114 Engliſh feet, and 880 fathoms are contained in an Engliſh mile, therefore the circumference of the Earth, expreſſed in Engliſh miles, is equal to $\frac{57069 \times 114 \times 360^\circ}{107 \times 880} = \frac{57069 \times 57 \times 9}{107 \times 11} = 24873$. Hence alſo one degree contains about 69.09 Engliſh miles, and a nautical or geographical mile contains 6080 feet.

For the purpoſe of aſcertaining the diſtance run by a ſhip in any given time, a half minute glaſs is commonly uſed, and a line divided accordingly; and ſince half a minute is the 120th part of an hour, therefore the interval between two adjacent knots on the line ſhould be $= \frac{6080}{120} = 50$ feet 8 inches; but becauſe it is ſafeſt to have the reckoning a-head of the ſhip, 48 feet are therefore commonly allowed to a knot. If this interval was decimally divided, the computation would be rendered more accurate. We ſhall only further obſerve, that the glaſs ſhould run 29½″, inſtead of 30″, half a ſecond being allowed for the time taken in turning the glaſs.

CHAP. II.

Definitions, Principles, &c.

DAY and Night ariſe from the circumrotation of the Earth. That imaginary line about which the rotation is performed is called the Axis, and its extremities are called Poles. That towards the moſt remote parts of Europe is called the North Pole, and its oppoſite the South Pole: the Earth's axis being produced will point out the Celeſtial Poles.

The Equator is a great circle on the Earth, every point of which is equally diſtant from the poles; it divides the Earth into two equal parts, called Hemiſpheres; that having the North Pole in its center is called the Northern Hemiſphere, and the other the Southern Hemiſphere. The plane of this circle being produced to the fixed ſtars, will point out the celeſtial Equator or Equinoctial.

The Meridian of any place, is a ſemi-circle paſſing through that place, and terminating at the poles of the Equator.

The Latitude of any place, is that portion of the Meridian contained between the Equator and given place. It is either North or South, according as the given place is in the Northern or Southern Hemiſphere, and therefore cannot exceed 90°.

The Parallel of Latitude of any place, is a circle paſſing thro' that place parallel to the Equator.

The Difference of Latitude between any two places, is an arch of a meridian intercepted between the correſponding parallels of latitude. Hence if the places ly between the Equator and the ſame pole, their difference of latitude is found by ſubtracting the leſs latitude from the greater; but if they ly on oppoſite ſides of the equator, the difference of latitude is equal to the ſum of the latitudes of both places.

The First Meridian is an imaginary ſemi-circle paſſing through any remarkable place, and is therefore arbitrary. Thus the Britiſh eſteem that to be the firſt Meridian which paſſes through the Royal Obſervatory at Greenwich, and the French reckon their firſt Meridian that which paſſes through the Royal Obſervatory at Paris, tho' formerly it was that which paſſed through the iſland of Ferro. In any other country, the firſt Meridian is generally eſteemed that which paſſes thro' the principal obſervatory, or chief city in that country.

The Longitude of any place, is that portion of the Equator which is contained between the firſt meridian, and the meridian of that place; and is uſually reckoned either eaſt or weſt, according as the given place is on the eaſt or weſt ſide of the firſt meridian, and therefore cannot exceed 180°.

The Difference of Longitude between two places, is the intercepted arch of the equator between the meridians of thoſe places, and cannot exceed 180°.

In order to illuſtrate the preceding definitions, let Pp (fig. 1ſt) repreſent the axis of the earth, EQ the equator, and PAp, PBp, the meridians of any two places, A and B; then the portion AF, of the meridian of the place A, contained between it and the equator, will be the latitude of that place; in like manner the intercepted arch BG of the meridian PBp will be the latitude of the place B; and the portion AH or BI, of the meridian paſſing thro' either of thoſe places, contained between the parallels of latitude KAIL, MHBN, will be the difference of latitude;—hence it is evident, by inſpection of the figure, that the difference between the latitudes of any two places is equal to the difference or ſum of their latitudes, according as the given places are in the ſame, or contrary hemiſpheres. Again, let PRp be the firſt meridian, then the arch of the equator RF contained between the firſt meridian and the meridian of the place A, is the longitude of that place; RG is the longitude of the place G, and FG the difference of longitude of thoſe places; whence it is apparently obvious, that the difference of longitude between any two places is equal either to the difference or ſum of the longitudes of thoſe places, according as they are on the ſame, or on contrary ſides of the firſt meridian.

There are three different Horizons, the apparent, the sensible, and the true.[*] The apparent or viſible Horizon is that which limits the view of an obſerver. The ſenſible is a plane paſſing thro' the eye of an obſerver, perpendicular to a plumb line, ſuſpended at his eye, and hanging freely—and the true or rational Horizon is a plane paſſing through the center of the earth, parallel to the ſenſible horizon.

Altitudes obſerved at ſea are meaſured from the viſible horizon. At land, when an aſtronomical quadrant is uſed, or when obſervations are taken with a Hadley's quadrant by the method of reflection, the altitude is meaſured from the ſenſible Horizon; and in either caſe the altitude muſt be reduced to the true Horizon.

The Zenith of any given place, is the point immediately above that place, and is therefore the elevated pole of the horizon: the **Nadir** is the other pole or point diametrically oppoſite.

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[*] It may be alſo neceſſary to mention another horizon, which may be called the Reduced Horizon. This is a plane paſſing through the Earth's center, perpendicular to the radius anſwering to the reduced place of obſervation.

A **Vertical** is a great circle paſſing through the Zenith and Nadir, and therefore interſecting the Horizon at right angles.

The **Altitude** of any celeſtial body, is that portion of a Vertical, which is contained between its center and the true Horizon; the Meridian Altitude is the diſtance of the object from the true Horizon, when on the meridian of the place of obſervation; when the obſerved altitude is corrected for the depreſſion of the Horizon, and the errors ariſing from the inſtrument, it is called the Apparent Altitude; and when reduced to the true Horizon, it is called the True Altitude; altitudes are expreſſed in degrees, and parts of a degree.

The **Zenith Distance** of any object, is its diſtance from the zenith, or the complement of its altitude.

The **Declination** of any object, is that portion of its meridian, which is contained between the equinoctial and center of the object, and is either north or ſouth, according as the ſtar is between the equinoctial and north or ſouth pole.

The **Right Ascension** of a ſtar, is the arch of the equinoctial intercepted between its meridian and the vernal equinox, and is reckoned eaſtwards.

The **Ecliptic** is that great circle in which the annual revolution of the earth round the ſun is performed. The inclination of the ecliptic and equinoctial is at preſent about 23° 28′; and by comparing ancient with modern obſervations, the obliquity of the ecliptic is found to be diminiſhing, which diminution in the preſent century is about half a ſecond yearly.[*]

The Ecliptic, like all other circles of the ſphere, is divided into 360°, and is further divided into 12 equal parts, called **Signs**; each ſign therefore contains 30°. The names and characters of theſe ſigns as are follow.

Aries,	♈	Cancer,	♋	Libra,	♎	Capricornus,	♑
Taurus,	♉	Leo,	♌	Scorpio,	♏	Aquarius,	♒
Gemini,	♊	Virgo,	♍	Sagittarius,	♐	Piſces,	♓

Since the ecliptic and equator are great circles, they therefore biſect each other in two points, called the equinoctial points. The ſun is in one of theſe points in March, and in the other in September; hence the firſt is called the Vernal, and the other the Autumnal equinox, and that ſign which begins at the vernal equinox is called Aries. Thoſe points of the ecliptic which are equidiſtant from the equinoxes, and therefore moſt diſtant from the equator, are called Solſtitial Points; the firſt, the ſummer, and ſecond the winter ſolſtice. When the ſun enters Aries, it is in the equator, and therefore has no declination—from thence it moves forward in the ecliptic according to the order of the ſigns, and advances towards the north pole by a kind of retarded motion till it enters Cancer, and is then moſt diſtant from the equator; and moving forward in the ecliptic, the ſun apparently recedes from the north pole with an accelerated motion till it enters Libra, and being again in the equinoctial has no declination; the ſun moving through the ſigns, Libra, Scorpio, and Sagittarius, enters Capricorn, and then its ſouth declination is greateſt, and is therefore moſt diſtant from the north pole, and moving forward thro' the ſigns Capricorn, Aquarius, and Piſces, again enters Aries—hence a period of the ſeaſons is completed, and this period is called a Solar Year.

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[*] It is alſo ſubject to two periodical inequalities; the firſt, ariſing from the attraction of the ſun, is completed in ſix months, the maximum of which is only half a ſecond; the other, called the Nutation of the Earth's Axis, is completed in a revolution of the Moon's nodes, or 18 years 224 days, and is uſually repreſented by ſuppoſing the pole of the Earth to deſcribe the periphery of an ellipſe, in a retrograde manner during a period of the Moon's nodes, the greater axis being in the ſolſtitial colure, and equal to 19″.1, and the leaſt axis, in the equinoctial colure, and equal to 14″.2.

The ſigns Aries, Taurus, Gemini, Cancer, Leo, and Virgo, are called Northern Signs, becauſe they are contained in that part of the ecliptic which is between the equinoctial and north pole, and therefore while the ſun is in theſe ſigns, its declination is north; the other ſix ſigns are called Southern Signs. The ſigns in the firſt and fourth quarters of the ecliptic are called Aſcending Signs, becauſe while the ſun is in theſe ſigns, it approaches the north pole, and therefore in the northern temperate and frigid zones, the ſun's meridian altitude daily increaſes, or, which is the ſame, the ſun aſcends to a greater height above the horizon every day. The ſigns in the ſecoud and third quarters of the ecliptic are called Deſcending Signs.

The TROPICS are circles parallel to the equator, whoſe diſtance therefrom is equal to the obliquity of the ecliptic. The northern tropic touches the ecliptic at the beginning of Cancer, and is therefore called the Tropic of Cancer; and the ſouthern tropic touches the ecliptic at the beginning of Capricorn, and is hence called the Tropic of Capricorn.

Circles about the poles of the equator, paſſing thro' the poles of the ecliptic, are called POLAR CIRCLES; therefore the diſtance of each circle from its reſpective pole is equal to the inclination of the ecliptic and equator. That circle which circumſcribes the north pole is called the Artic, or North Polar Circle, and that towards the ſouth pole, the Antartic or South Polar Circle.

That ſemicircle which paſſes through a ſtar, or any given point in the heavens and the poles of the ecliptic, is called a CIRCLE of LATITUDE.

The REDUCED PLACE of a ſtar is that point of the ecliptic, interſected by the circle of latitude paſſing thro' that ſtar.

The LATITUDE of a ſtar is that portion of the circle of latitude contained between the ſtar and its reduced place, and is either north or ſouth, according as the ſtar is between the ecliptic, and the north or ſouth pole thereof.

The LONGITUDE of a ſtar is that portion of the ecliptic contained between the vernal equinox and the reduced place of the ſtar.

Let the circle PEQ (fig. 2.) repreſent the ſolſtitial colure, EQ the equator, Pp its poles, ♋ ♑ the ecliptic, whoſe poles are Rr, and S the poſition of a ſtar with reſpect to theſe circles; then is AS the declination, and ♈A the right aſcenſion of the ſtar; B is its reduced place, hence BS is the latitude, and ♈B the longitude of the ſtar.

C H A P. III.

OF THE FIXED STARS.

THE Fixed Stars are ſo named, becauſe they are obſerved to retain their places with reſpect to each other. Some ſtars appear of a ſenſible magnitude to the naked eye, but when viewed through a teleſcope, ſeem only as lucid points, without any apparent diameter; hence their immenſe diſtance from the ſolar ſyſtem is inferred, and conſequently they emit their own light, otherwiſe they would be inviſible.— It is therefore reaſonable to ſuppoſe them ſo many ſuns, diffuſing light and heat to planets revolving round them.

The number of fixed ſtars appears very conſiderable to a perſon who looks confuſedly at them; but from a due contemplation it will be found, that without the aſſiſtance of a teleſcope there can very ſeldom be more than a thouſand ſeen above the horizon at the ſame time.

The ſtars, with reſpect to their apparent ſplendour, are divided into orders, called MAGNITUDES. The brighteſt are called ſtars of the Firſt Magnitude; the next to theſe in ſplendour, ſtars of the Second Magnitude, and ſo on to thoſe which are juſt perceptible to the naked eye, and which are called ſtars of the Sixth Magnitude.— Thoſe which cannot be diſcerned without the aſſiſtance of a teleſcope, are called Teleſcopic Stars, and are divided into orders of the Seventh, Eighth, Ninth, &c. Magnitudes accordingly. We are not, however, to infer from this, that the ſtars can be exactly reduced to one or other of theſe magnitudes, for the ſtar α Aquilæ is reckoned by ſome to be of the firſt magnitude, and others eſteem it of the ſecond; hence thoſe ſtars, whoſe magnitudes are doubtful, are generally marked in catalogues as partaking of both magnitudes—thus α Aquilæ is marked 1.2, ſignifying, that it is either of the firſt or ſecond, or rather between theſe magnitudes; and τ Scorpionis is marked 3.4, as being between theſe magnitudes. From what has been ſaid of the magnitudes of the ſtars, we are not to ſuppoſe that their ſizes are in the ratio of their apparent magnitudes; they may perhaps be nearly of the ſame bulk, but the apparent magnitude of a ſtar depends on its diſtance.

The ſtars, for the purpoſe of finding any one more readily, are divided into parcels called CONSTELLATIONS. Theſe, in order to aſſiſt the imagination, are ſuppoſed to be circumſcribed with ſome known figure, as that of a man, woman, ſhip, ſextant, &c. and thoſe ſtars which lie between conſtellations, are called UNFORMED STARS. As it would be an endleſs taſk to give a proper name to each ſtar, it has therefore been cuſtomary to mark the ſtars of each conſtellation with the letters of the Greek alphabet, in ſuch a manner, that the firſt letter is prefixed to the brighteſt ſtar, the ſecond letter to the next in brightneſs, and ſo on.

The celeſtial ſphere is uſually divided into three parts, the ZODIAC, and the NORTHERN and SOUTHERN HEMISPHERES.

The Zodiac extends to about 8° on each ſide of the ecliptic, and contains the orbits of all the planets. The number of conſtellations in the Zodiac are 12, and they ſtill retain their ancient names, or thoſe of the ſigns of the ecliptic, altho' conſiderably removed from the places they formerly occupied, in reſpect of the equinoctial points. There are 36 conſtellations in the Northern Hemiſphere, and 32 in the Southern; and conſequently 80 in all. The number of ſtars of the firſt magnitude in the Zodiac and both Hemiſpheres amounts only to twenty.

By comparing the lengths of a ſidereal and ſolar tropical year with each other, the former is found to exceed the latter by $20' 25\frac{3}{4}''$. Hence the interſection of the ecliptic and equator, moves in a retrograde direction, about $50''.338$ of a degree yearly—therefore, a revolution of the equinoctial points will be completed in about 25746 years; this is called the Annus Magnus, or Great Year.

Since the equator changes its poſition with reſpect to the ecliptic, its axis will alſo be changeable; and its poles, during the annus magnus, will complete a period in a circular, or rather ſpiral orbit—therefore on this account, the longitude, right aſcenſion, and declination of every ſtar will be variable, and conſequently the pole of the equator cannot always be directed to the ſame ſtar, or, which is the ſame, no ſtar can remain at or near the pole of the equinoctial, for any conſiderable ſpace of time. The ſtar which at preſent is neareſt the north pole is Alruccabah, a ſtar of the ſecond magnitude, and the laſt in the tail of the conſtellation of Urſa Minor; the latitude of thisſtar is 60° 4′ 50″, and mean longitude, at the beginning of

1792,

1792, is 85° 39′ 23″. Now the neareſt approach of this ſtar to the pole will be, when it is in the firſt of Cancer—which accordingly happens in the year 2102, at which time its diſtance from the pole will be about 29′ 47″. And ſince the fixed ſtars complete a revolution about the axis of the ecliptic in 25746 years, therefore any given fixed ſtar will perform half a revolution in half that time, and conſequently if that ſtar was before at its neareſt appulſe to the pole, it will now be the moſt remote from it poſſible; therefore 12873 years after the year 2102, that is, in the year 14975, the preſent pole ſtar will be 45° 23′ 16″ diſtant from the pole.

By this motion of the fixed ſtars it follows, that the zodiacal conſtellations have not retained their places with reſpect to the vernal equinox, but ſeem to have moved in conſequentia, or towards the eaſt, almoſt a whole ſign, ſince the time of Hipparchus; ſo that the conſtellation Aries is now where Taurus then was, Taurus where Gemini, &c. However, to avoid confuſion, the ſpaces formerly occupied by the zodiacal conſtellations ſtill retain their ancient names, and are called ANASTRA, or without their former ſtars, whereas the ſpaces they now poſſeſs are called STELLATA.

Beſides the motion of the ſtars ariſing from the receſſion of the equinoctial points, it is found by obſervation, that many alter their poſition with reſpect to the adjacent ſtars; and this change of place is called the PROPER MOTION of the ſtars. The proper motion of Sirius, Caſtor, Procyon, Pollux, Regulus, Arcturus, and α Aquilæ, in 100 years, in right aſcenſion, according to Dr Maſkelyne, are reſpectively 1′ 3″, 28″, 1′ 28″, 1′ 33″, 41″, 2′ 20″, and 57″; the proper motion of Sirius in declination, in a century, is 2′, and of Arcturus 3′ 21″, both tending to the ſouth.

The apparent brightneſs of ſome of the fixed ſtars is alſo obſerved to be periodic, as χ Cygni, of which the periodic time between two ſucceſſive greateſt apparent ſplendors, is about 405 d. 8 h.—and the increaſe and decreaſe of the light of Algol, has been obſerved by many, the period of which, according to Mr Goodrick's obſervations, is 2 days, 20 h. 49′ 3″. For an account of variable ſtars, the Philoſophical Tranſactions may be conſulted, particularly a paper by Mr Pigott on that ſubject, in the 76th volume.

NEBULÆ and DOUBLE STARS have of late engaged the attention of aſtronomers, particularly Dr Herſchell. A Nebula is a portion of the heavens brighter than the circumjacent parts, which brightneſs ariſes from the combined light of a multitude of very diſtant ſtars, apparently near each other. The moſt conſiderable of the Nebulæ is the Milky Way; the next are thoſe uſually called the Magellanic Clouds, ſituated near the ſouth pole, and therefore inviſible in Britain; and θ Orionis is certainly a beautiful phenomenon, when viewed with a proper teleſcope. Dr Herſchel has detected about 2000 nebulæ, and nearly as many double ſtars.

The moon's orbit being contained within the zodiac, its motion is very rapid with reſpect to the ſun, and thoſe ſtars ſituated in that zone. On this account, there is founded one of the beſt methods for finding the longitude at ſea hitherto diſcovered. In order to aſcertain the longitude of a ſhip, with as great accuracy as poſſible, by this method; the moon ought to be compared with thoſe ſtars, which are ſituated in or near its orbit; by which means as little as poſſible of the moon's proper motion will be loſt. The ſtars uſed in the Nautical Almanack, for the purpoſe of determining the longitude at ſea by the above method, are, α *Arietis*, *Aldebaran*, *Pollux*, *Regulus*, *Spica Virginis*, *Antares*, α *Aquilæ*, β *Capricorni*, *Fomalhaut*, and α *Pegaſi*. The diſtance between the moon and the ſun, and one or more of thoſe ſtars, according as they are in a proper poſition with reſpect thereto for obſervation, is given in the Nautical Almanack in the viii. ix. x. and xi. pages of the month, at the beginning of every third hour apparent time, by the meridian of Greenwich.

CHAP. IV.

OF THE PLANETS.

A Planetary Syſtem is an aſſemblage of ſeveral bodies revolving about another body as a common center, which gives light and heat to all, and is therefore called a Sun, and the revolving bodies are called Planets and Comets; the path which a planet or comet deſcribes in its motion round the ſun is called its Orbit.

Planets are of two kinds, Primary and Secondary. A Primary planet regards the ſun as its center of revolution; a ſecondary, is a body revolving about a primary planet, and is called a Satellite.

There are ſeven primary planets, of which the names and characters are as follow.

Name.	*Character.*	*Name.*	*Character.*
Mercury,	☿	Jupiter,	♃
Venus,	♀	Saturn,	♄
Earth,	⊖	Georgian planet,	♅
Mars,	♂		

The number of ſatellites belonging to theſe planets amounts to fourteen, one of which belongs to the Earth, called the Moon, four belong to Jupiter, ſeven to Saturn, and two to the Georgian planet. Meſſrs Caſſini, Short, and Montaigne, obſerved a phenomenon, which induced them to believe that there was a ſatellite belonging to Venus.

Since the Sun is the loweſt body in our ſyſtem, and becauſe the orbits of Mercury and Venus are contained within that of the Earth, they are therefore called Inferior Planets; the other planets, being always more diſtant than the earth from the Sun, are called Superior Planets. It may be obſerved, that all the planets hitherto diſcovered are ſuperior with reſpect to Mercury, and inferior to the Georgian planet.

In the time of a planet's revolution round the Sun, a period of its ſeaſons is completed, and this interval is called a Solar Year.—The periods of the ſeveral planets, together with their mean diſtances from the ſun, expreſſed in Engliſh miles, are as follow.

Periodical Revolution of	*d.*	*h.*	*m.*	*s.*	*Mean Diſtance.*[*]
Mercury,	87	23	15	37	35853400
Venus,	224	16	49	12.7	66995700
Earth,	365	6	9	11.2	92621000
Mars,	686	23	30	43.3	141125100
Jupiter,	4332	8	51	25.6	481887700
Saturn,	10761	14	36	42.5	883671000
Georgian planet,	30445	18			1767375000

The planets Venus, the Earth, Mars, and Jupiter, beſides their annual motion round the Sun, have been obſerved to move round their axis from weſt to eaſt, called their Rotation. The time of rotation is meaſured by the return of any meridian to the ſame fixed ſtar, which is called a Siderial Day; and the return of the ſame meridan to the Sun is called a Solar Day. A terreſtrial ſolar day is divided into 24 equal parts called Hours, each hour is ſubdivided into 60 equal parts called Minutes, and the ſexageſimal diviſion is continued. The rotation of the above planets in mean ſolar time is as follows.

Rotation of Venus, 23 h. 22′ Earth, 23 h. 56′ 4″
Mars, 24 h. 39′ Jupiter, 9 h. 56′

The rotation of Saturn, agreeable to Dr Uſher's computations, is 10 h. 12½ min. A different reſult was however obtained, by taking the denſity of Saturn, as ſtated by M. de la Lande. The rotation of Saturn's ring, as ſettled by Dr Herſchell, is 10 h. 32′ 16″. And the rotation of the ſun, according to the author's obſervations in 1787, is performed in about 25 d. 7 h. 52 min.

The time of rotation of a planet is diſcovered by obſervations on the ſpots on its diſc; and though the nearneſs of Mercury to the Sun, and the great diſtance of the Georgian planet from the earth, have prevented the diſcovery of ſpots on their diſcs, and thereby the times of their rotation, yet from analogy it may be preſumed, that each of theſe planets has a rotatory motion.

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[*] Theſe were computed on the aſſumption of the ſun's mean horizontal parallax, being 8″.8128, according to the determination of M. du Sejour, from the late tranſit of Venus. Vide Tracte Analytique, &c. tom. 1, page 486.

The rotation of the moon is performed in 27 d. 12 h. 43 m. and that each of the ſatellites belonging to the other planets revolves about its axis, is ſcarcely to be doubted.

By means of the rotation of a planet, combined with its annual motion round the ſun, in the time that a period of the planet's ſeaſons is completed, every point on its ſurface will have enjoyed the preſence of the ſun, for nearly an equal interval of time.

The figure of the Earth, Mars, Jupiter, and Saturn, is found by obſervation to be that of an oblate ſpheroid, the ratios of the equatorial diameter to the axis of theſe planets being as 230 : 229, 1355 : 1272, 14 : 13,* and 11 : 10, reſpectively, and by analogy, the other planets are alſo of a ſpheroidal figure, the leaſt diameter being that of rotation.

Since the orbits of the planets, Mercury and Venus, are contained within that of the Earth, they can never be ſeen in oppoſition to the ſun; but becauſe the orbit of the Earth is contained within the orbits of the other planets, theſe planets may therefore be ſeen in oppoſition, or in any other poſition, with reſpect to the Sun. The phenomena of an inferior planet, as obſerved from a ſuperior planet, ariſing from their combined motions round the ſun, may be thus illuſttrated.

Let S (fig. 4.) repreſent the ſun, ABCDEFGH the orbit of an inferior planet, IKLM the orbit of the earth, both being ſuppoſed in the ſame plane, and the circle ♈♋♎♑ the ecliptic; now let the earth be in any point I of its orbit, when the inferior planet is at A, in a right line between the ſun and earth, it is then ſaid to be at its inferior conjunction; when at the point E, it is at its ſuperior conjunction; and when at the points C or G, where a ſtraight line from the earth is a tangent to its orbit, it is then at its greateſt eaſtern or weſtern elongation—the elongation at any other time is meaſured by the angle contained between lines drawn from the center of the earth to the centers of the ſun and planet. Now, if the planet be ſuppoſed to move forward according to the order of the ſigns, with the exceſs of its velocity above the earth's motion, the earth may be ſuppoſed to remain ſtationary at the point I; let the planet be at any point C of its orbit, hence its place referred to in the heavens is N, and when the planet is come to D, its apparent place is O; it has therefore apparently moved thro' the ſpace NO; in like manner the planet having moved through the portion DEFG of its orbit, will have apparently deſcribed the ſpace O♋PQ, and when the planet has moved from G to H, it will appear to have returned to P; and during the time it deſcribes the portion HABC of its orbit, it will apparently move thro' the ſpace P♋ON, contrary to the order of the ſigns; hence during the time the planet is deſcribing the weſtern part of its orbit ABCDE, it ſets ſooner, and riſes earlier than the ſun, and may therefore be called a morning ſtar, and during the time it is deſcribing the eaſtern part of its orbit EFGHA, it riſes and ſets later than the ſun—it will therefore be ſeen after ſun ſet, and of courſe called an evening ſtar; during the time the planet is deſcribing the ſuperior part of its orbit, it moves according to the order of the ſigns, and is therefore ſaid to be direct; but its motion, while deſcribing the inferior part of its orbit, is apparently contrary to the order of the ſigns, as ſeen from the earth, and is hence ſaid to be retrograde; when the planet is towards the points of greateſt elongation, it is then either receding directly from, or approaching towards the earth, and its apparent motion for ſome time is ſcarce perceptible; it is then ſaid to be ſtationary.

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* The proportion of the axes of Jupiter, agreeable to M. de la Place's calculations, is nearly $\frac{69}{74}$

While an inferior planet, as ſeen from the earth, is moving according to the order of the ſigns, it is evident, that the motion of the earth will be direct as ſeen from the planet; when the planet is apparently retrograde, the earth will be alſo retrograde; and when the planet is apparently ſtationary, the earth will be alſo ſtationary, as ſeen from the planet. Hence the phenomena of a ſuperior planet is ſufficiently obvious, and requires no further illuſtration.

Since, by obſervation, the orbit of each planet is found to be inclined to the ecliptic, it therefore interſects the ecliptic in two oppoſite points, called Nodes; that half of the planet's orbit, which is between the ecliptic and north pole, is called the northern part of its orbit, and the other half, the ſouthern. That node which the planet enters, when advancing towards the north, is called the Aſcending Node, and the other, the Deſcending Node; and an imaginary ſtraight line joining theſe is called the Line of the Nodes.

Let ABCD (fig. 4.) repreſent the orbit of the earth, and EFGH the orbit of an inferior planet, S the ſun, and EG the line of the nodes, E being the aſcending, and G the deſcending node; alſo let A be the place of the earth in the ecliptic, and F the ſituation of the planet in its proper orbit; then will M be the Heliocentric place of the planet, or its place as ſeen from the ſun, and N its Geocentric place, or point where it would be ſeen from the earth; if FI be drawn perpendicular to the ecliptic, the angle FSI is the Heliocentric Latitude, and FAI the geocentric latitude of the planet; SI is called the Curtate Diſtance, and the difference between SF and SI, the Curtation

The heliocentric and geocentric longitudes and latitudes of the planets, together with their declination, and paſſage over the meridian in apparent time, are given in the Nautical Almanac, page iv. of the month The declination and tranſit of a planet are uſeful in computing the latitude and apparent time at ſea, from its obſerved altitude.

The Periodic Time of a planet may be found, by obſerving the interval of time between two ſucceſſive paſſages of the planet thro' the ſame node, allowance being made for the motion of the node during that interval; and to obtain a greater degree of accuracy, it will be proper to ſelect two very diſtant obſervations, when the planet was in the ſame node; and the interval, after allowing for the motion of the node, being divided by the number of elapſed periods, will give the periodic time; and becauſe the place of a planet is uſually reckoned from the vernal equinox, therefore the periodic time of revolution with reſpect to the fixed ſtars muſt be reduced to the periodic revolution relatively to the equinoxes, by deducting a part proportional to the receſſion of the equinoxes in that interval. The time when an inferior planet is in its node is moſt accurately determined by obſerving its tranſit over the Sun; and that of a ſuperior planet, when in oppoſition, and at the ſame time in or near the node.

A Synodic Period of a planet, is the time elapſed between two ſucceſſive returns of the planet to the ſame point of its orbit, as ſeen from the Earth.

The periodic time of an inferior planet may alſo be found, by dividing the product ariſing from the multiplication of the ſynodic period of the planet into the earth's period, by the ſum of theſe periods; and that of a ſuperior planet is found by dividing the product of its ſynodic period into the earth's, by the difference between theſe periods.

The Comparative Diſtance of an inferior planet may be deduced from the quantity of its greateſt elongation; for, if the diſtance of the Earth from the Sun be ſuppoſed equal to unity, the diſtance of the planet from the Sun will be expreſſed by the ſine of its greateſt elongation, and its diſtance from the earth at that time, by the coſine of elongation;—the diſtance of a ſuperior planet from the Sun will be equal to the coſecant of the greateſt elongation of the earth, as ſeen from that planet, which is called the Parallactic angle; and the diſtance of the planet from the Earth is equal to the cotangent of that angle, the radius being unity. However, theſe being on the aſſumption of a circular orbit, will deviate a little from the truth.

Kepler, by comparing the periodic times and diſtances of the planets from the Sun with each other, found, that the ſquares of the periodic times were directly as the cubes of the mean diſtances; and ſince the periods of the planets may be very accurately found by obſervation, therefore, by the above law, the relative mean diſtance of the planets may be deduced with the greateſt preciſion; and becauſe the mean diſtance of the Earth from the Sun, in known meaſures, is pretty well aſcertained by the late tranſit of Venus, therefore the abſolute mean diſtances of all the planets are alſo determined.

Since the planets receive their light from the Sun, it is evident, that whatever ſide is oppoſed thereto will be enlightened; therefore at the inferior conjunction of Mercury or Venus, neither can be ſeen becauſe the unenlightened ſide is towards the earth, unleſs at the time of a tranſit, and then it will appear like a ſmall black round ſpot upon the ſun's diſc; but at the ſuperior conjunction, the enlightened hemiſphere of the planet is towards the Earth, conſequently the whole of its diſc will be viſible; and when the planet is at its greateſt elongation, one half of it only can be viſible, becauſe the plane of the terminator or circle bounding light and darkneſs is directed towards the earth. Hence, when either Mercury or Venus is in the inferior part of its orbit, leſs than its half appears enlightened; but more, when in the ſuperior part of its orbit. The diſtance of Jupiter, Saturn, and the Georgian planet, being very conſiderable, in reſpect of the Earth's diſtance from the Sun, the enlightened diſcs of theſe planets are to ſenſe always oppoſed to the earth; but becauſe of the proximity of Mars, it is obſerved to be full and gibbous alternately.

CHAP. V.

OF THE MOON.

THE Moon, though only a ſatellite belonging to the earth, is, next to the Sun, the moſt conſpicuous body in the heavens, both on account of its apparent magnitude, and diverſity of aſpect.

By contemplating the Moon during the courſe of a month, it is obſerved to put on a variety of appearances. When the Moon is firſt obſerved, a little to the eaſtward of the Sun, a ſmall portion of its diſc is enlightened, and that portion is next the Sun; the enlightened part encreaſes more and more until it appears perfectly round—the weſtern part then becomes inviſible, and the enlightened part gradually diminiſhes as it approaches the ſun, and entirely diſappears when in conjunction with that object, except at that time, it happens to paſs directly between the obſerver and the Sun. Theſe various appearances are in general called the Phaſes of the Moon.

By comparing the Moon's phaſes and its elongations from the Sun with each other, it is found, that the ſame phaſe correſponds to the ſame elongation. Hence it evidently appears, that the Moon is an opake body, receiving its light from the Sun, and therefore that part only is enlightened, which is oppoſed to the Sun.

Since the Sun is much bigger than the Moon, it is evident, that more than a hemiſphere is continually enlightened; it is alſo manifeſt, that a complete hemiſphere of the Moon cannot be ſeen by a ſpectator on the earth; but the difference in both caſes is ſo very trifling, that it may be neglected in the following illuſtration of the phaſes of that ſatellite.

Let S (fig. 5.) repreſent the ſun, E the earth, and ACFH the orbit of the moon, in which it revolves from weſt to eaſt, according to the order of the letters. When the moon is at A, it is in conjunction with the ſun; and as its dark ſide is oppoſed to the earth, no part therefore of the moon can be ſeen, unleſs at that time it happens to paſs over the ſun's diſc;—this phaſe is called new moon, and it is ſaid to be in ſyzigy. When the moon is at B, or 45° from the ſun, it is ſaid to be in its firſt octant; in this caſe, part of the moon's enlightened diſc is towards the earth, and it appears in form of a creſcent, as repreſented at L; the curve bounding the enlightened and dark parts of the moon, or terminator, being an ellipſe.—When the moon is at the point C, where the angle of elongation is 90°, it is ſaid to be in quadrature, and the terminator is now apparently a ſtraight line, one half therefore of the moon's enlightened diſc is ſeen from the earth, as repreſented at M: when at D, the moon is its ſecond octant, and as much is now viſible as was inviſible when it was at B, and ſince more than half its diſc is viſible, as ſeen from the earth, the moon is therefore ſaid to be gibbous: and when the moon is at F, it is in oppoſition, and appears to be compleatly illuminated, and is now, as well as when in conjunction, ſaid to be in ſyzigy. The moon having paſſed the oppoſition, and come to G the third octant, is again gibbous, as repreſented by the phaſe O: when at H, the moon is again in quadrature, and appears half full, as at P; and having come to I the fourth octant, it appears like a creſcent, as at Q; and laſtly when to A, it is in conjunction, and the dark ſide being oppoſite to the earth, the moon therefore becomes inviſible.

During the time the moon is moving from A to C, it is ſaid to be in the firſt quarter, from C to F in the ſecond, from F to H in the third, and from H to A in the fourth quarter; when the moon is at A, it comes to the meridian at noon, and when at F, at midnight; therefore, while the moon is in the part ACF of its orbit, it comes to the meridian between noon and midnight, and the weſtern part of the moon's diſc is enlightened. During the time the moon is deſcribing the other part of its orbit FHA, it comes to the meridian between midnight and noon, and the eaſtern part of its diſc is viſible; when the moon is in the firſt and fourth quarters, it appears in form of a creſcent, and when in the ſecond and third quarters, it is gibbous.

Some days before and after new moon, beſides the creſcent, the remaining part of its diſc is faintly ſeen; the reaſon of this will evidently appear, by conſidering, that when the moon is in thoſe parts of its orbit, the enlightened ſide of the earth is oppoſed to the moon; and ſince the quantity of light reflected by each body is ſuppoſed proportional to the ſquares of their diameters, therefore as the earth is much greater than the moon, the quantity of light reflected by the earth to the moon is ſo conſiderable, as to render the dark part viſible.

The interval of time between two ſucceſſive new or full Moons is called a ſynodic period, or uſually a lunation, and is completed in 29 d. 12 h. 44′ 2″.9; a revolution of the Moon with reſpect to the fixed ſtars, is completed in 27 d. 7 h. 43′ 11″.5; and with reſpect to the vernal equinox, in 27 d. 7 h. 43′ 4″.6;—and ſince the ſame face of the Moon is always turned towards the Earth, it therefore turns round its axis in the ſame time that it completes a revolution with reſpect to the fixed ſtars.

From the foregoing illuſtration of the Moon's phaſes it will appear, that if its orbit coincided with the ecliptic, it would paſs directly between the Earth and the Sun at every new Moon, and being an opake body would intercept part of the Sun's diſc, and hence cauſe an eclipſe of the Sun; and at the time of oppoſition, the Moon would paſs through the ſhadow of the earth, and therefore the direct rays of the ſun being intercepted by the earth, an eclipſe of the Moon would take place.

But becauſe the Moon's orbit makes an angle with the ecliptic, of about 5° 8′ 49″, theſe circles therefore interſect each other in two oppoſite points called Nodes, and the common interſection of the plane of theſe circles is called the Line of the Nodes; and if this line were always coincident with the ſyzigial line, the Moon would be eclipſed at every oppoſition, and the Sun at every new Moon; but the mean motion of the nodes is about 19° 20′ 31″ yearly, in a retrograde direction.

It hence obviouſly follows, that an eclipſe cannot happen, but when the Moon is within certain limits of the node, which diſtance in a lunar eclipſe is determinable by the inclination of the Moon's orbit to the ecliptic, and the ſum of the ſemidiameters of the Moon, and that ſection of the Earth's ſhadow thro' which the moon paſſes; and if at the time of oppoſition, the moon's latitude is leſs than the difference between its ſemidiameter, and that of the ſhadow, the Moon will be totally eclipſed with continuance; and the leſs the latitude of the Moon, the longer is the duration of the eclipſe. If the Moon's latitude be equal to the above difference, the eclipſe will be total without continuance; if greater than the above difference, but leſs than their ſum, the eclipſe will be partial; and if it exceeds that ſum, no eclipſe can happen at that time. The limit within which a ſolar eclipſe can happen, is inferred from the inclination of the

Moon's orbit, and the ſum of the Moon's horizontal parallax, and the ſemidiameters of the Sun and Moon, and therefore if the Moon's latitude exceeds this laſt ſum, no part of the Earth can be eclipſed, but if leſs, the eclipſe may be total or annular at one place, partial at a ſecond, and at a third no part of the Sun may be obſcured. A total eclipſe of the ſun is when, at the time of the greateſt obſcuration, the Sun is compleatly hid by the Moon; an annular, when the Sun is obſerved to appear like a luminous ring round the Moon; and a partial eclipſe, when part of the Sun is obſcured.

Becauſe the mean motion of the lunar nodes is about 19° 20′ 31″ yearly, a revolution is completed in about 18 years 224 days; and if in that time an exact number of lunations were contained, there would be a regular return of the ſame eclipſe at the end of every period of the nodes; but this is by no means the caſe—however, in 18 y. 11 d. 7 h. 43′ 20″ leap year being four times contained, but if five times, then in 18 y. 10 d. 7 h. 43′ 20″, the node, after being in conjunction with Sun and Moon, returns ſo near to the conjunction as to be only 28′ 12″ diſtant therefrom; hence if to the mean time of any eclipſe, 18 y. 11 d. 7 h. 43′ 20″, or 18 y. 10 d. 7 h. 43′ 20″, be added, according as there are four or five leap years in the period, the ſum will be the mean time of an eclipſe.

The lunar irregularities, previous to the time of Sir Iſaac Newton, exerciſed the ſkill of the moſt eminent aſtronomers; but that celebrated mathematician, having applied his admirable principle of univerſal gravitation to the Moon, has left a theory of the lunar motion, which, in its preſent improved ſtate, is ſufficient to aſcertain the Moon's longitude within leſs than half a minute of a degree. An explanation of theſe irregularities does not come directly within the preſent plan, but perhaps may be treated of in a following work. We ſhall however in this place briefly mention a few of the principal equations.

The orbit of the Moon is a variable ellipſe, having the Earth in one of its foci. Hence the elliptic equation anſwering to any given mean anomaly is variable; this equation is divided into two parts, the one called the Equation of the Center, and the other the Evection. The mean elliptic equation, when greateſt, amounts to 6° 18′ 32″, and the quantity of the greateſt mean evection is 1° 20′ 29″.

The true longitude of the Moon is found to precede the mean, while the Moon is moving from the ſyzigy to the quadrature, and to be behind it, from the quadrature to the following ſyzigy. Hence ariſes an equation, depending upon the Moon's diſtance from the ſyzigy, called the Variation, which when greateſt amounts to 37′ 8″.

When the Earth is in aphelion, the motion of the Moon is found to be accelerated, and retarded when in perihelion. The equation for correcting this irregularity is called the Annual Eqration, which when greateſt amounts to 11′ 9″, and whoſe argument is therefore the Sun's mean anomaly.

Several other equations are alſo neceſſary, in order to compute the Moon's longitude to any tolerable degree of exactneſs, ſome of which ſerve to correct irregularities to which moſt of the former are liable, on account of the change of the Sun's diſtance from the earth.

The lunar nodes move in a very irregular manner, being ſtationary when the Moon is in quadrature, or without latitude; but in other caſes, their motion is retrograde. The Nodes complete a revolution in 18 y. 224 d. $4\frac{1}{3}$ h.

The inclination of the Moon's orbit is alſo variable; it is greateſt when the line of the nodes coincides with that of the quadratures, and leaſt, when that line coincides with the ſyzigies.

The motion of the Moon's apſides is irregular, being direct when the Moon is in the ſyzigies, and retrograde when in the quadratures. The direct motion is quickeſt when the line of the apſides coincides with the ſyzigies, and the retrograde motion is quickeſt, when the above line coincides with the quadratures; however, the direct motion exceeds the retrograde, and a revolution of the line of the apſides according to the order of the ſigns is completed in 8 y. 309 d. $8\frac{3}{4}$h.

The time of the Moon's revolution is alſo found to be ſubject to a periodical inequality, called the Secular Equation, which ariſes from the Sun's action on the Moon, combined with the variation of the excentricity of the Earth's orbit; the Moon's motion is accelerated, as this excentricity diminiſhes.

According to M. de la Place, the ſecular equation of the Moon is $= 11''.135 i^2 + 0''04398 i^3$, wherein *i* repreſents the number of centuries elapſed ſince 1700, and the ſign is to be changed if the given time is before 1700. According to M. de Lambre's compariſon of modern obſervations, the mean ſecular motion of the Moon, as given by M. Mayer, is too great by 25″. In Mayer's laſt tables, the ſecular equation of the Moon is 9″, therefore the mean motion of theſe tables ſhould be corrected to a quantity equal to $-25''i + 2.135 i^2 + 0.04398 i^3$. It hence evidently appears, that the computation of the Moon's place from tables is very tedious; and hence alſo appears the value of the Nautical Almanac, publiſhed annually by order of the Commiſſioners of Longitude, under the inſpection of Dr Maſkelyne, which contains every article relating to the Moon that may be of ſervice either to the practical aſtronomer or navigator. The Moon's longitude and latitude are given at noon and midnight, in page v. of the month; its age and paſſage over the meridian, and right aſcenſion and declination at noon and midnight, are contained in page VI. of the month; its ſemidiameter and horizontal parallax, at noon and midnight, in page VII. and its diſtance from the Sun and proper ſtars, at every third hour by the meridian of Greenwich, are contained in pages VIII. IX. X. XI. of the month.

The right aſcenſion and declination of a planet are deduced from obſervations of its zenith diſtance, and tranſit over the meridian, from which, together with the obliquity of the ecliptic, the longitude and latitude of the planet are found by computation. But the longitude and latitude of a planet are found directly by aſtronomical tables, for any given time; therefore from hence, and the obliquity of the ecliptic, the right aſcenſion and declination of the planet are to be found by computation. The two following problems, communicated to the author by Dr Maſkelyne, without their demonſtrations, will be fonnd very uſeful for the above purpoſe.

PROBLEM I.

Given (AR) right aſcenſion, declination, and (O) the obliquity of the ecliptic, to find the longitude and latitude, true to the neareſt ſecond by Taylor's logarithms, or to the neareſt 10″ by Gardiner's logarithms, or to the neareſt minute by Sherwin's or Hutton's logarithms, without proportioning.

Tan. dec. — S. AR = tan. A, north or ſouth, as declination is.
Call O in firſt ſix ſigns of AR ſouth, in laſt ſix ſigns, north.
A + O = B.

$$\left.\begin{array}{l}\text{A leſs than } 45^\circ\text{. co. ar. coſ. A + coſ. B + tan. AR}\\ \text{A more than } 45^\circ\text{. tan. A + co. ar. ſine A + coſ. B. + tan. AR}\end{array}\right\} =$$

tan. long. of the ſame kind as AR, unleſs B be more than 90°, when the quantity found of ſame kind as AR muſt be taken from twelve ſigns.

$$\left.\begin{array}{l}\text{Long. nearer III. and IX. ſigns, than 0 and VI. ſigns, ſine long. +}\\ \quad\text{tan. B.}\\ \text{Long. nearer 0 and VI. ſigns, than III. and IX. ſigns, tan. long. +}\\ \quad\text{ſine long. + tang. B.}\end{array}\right\} =$$

tan. lat. of ſame name as B.

EXAMPLE.

Let the Moon's right aſcenſion be 40° 22′, declination 11° 37′ N. and obliquity of the ecliptic 23° 28′. Required its longitude and latitude.

Dec. 11° 37′ N. tang. = 9.312967
RA. 40 22 ſine = 9.811358 tang. 9.929452

A = 17 37 N. tang. = 9.501609 co. ar. coſ. 0.020860
O = 23 28 S.

B = 5 51 S. — — — — coſ. 9.997732 tan. 9.010546

Lon. 41 35 — — — — — tan. 9.948044 ſine 9.821977

Lat. 3 53 S. — — — — — — — ſine 8.832523

PRO-

PROBLEM II.

Given the longitude and latitude of a fixed ſtar or planet, and (O) the obliquity of the ecliptic, to find the right aſcenſion and declination.

Tan. lat. — ſine long. = tan. A, north or ſouth, as latitude is.
Call O north in ſix firſt ſigns, and ſouth in ſix laſt ſigns.
A + O = B.
A leſs than 45°, co. ar. coſ. A + coſ. B + tang. long. }
A more than 45, tan. A + co. ar. ſine A + coſ. B + tan. long. } =
tan. right aſcenſion of ſame kind as longitude, unleſs B be more than 90°, when the quantity found of ſame kind as longitude muſt be ſubtracted from twelve ſigns.
AR nearer III and IX ſigns than 0 and VI ſigns, ſine AR + tan. B }
AR nearer 0 & VI ſigns than III & IX ſigns, tan. AR + coſ. AR + tan. B }
= tan. declination of ſame title as B, true to the neareſt ſecond by Taylor's logarithms, to neareſt 10″ by Gardiner's logarithms, or to neareſt minute by Sherwin's or Hutton's logarithms, without proportioning.

EXAMPLE.

Let the Moon's longitude be 7 s. 14° 26′ 21″, and latitude 4° 0′ 34″ N. and the obliquity of the ecliptic 23° 27′ 48″. Required the right aſcenſion and declination?

Lat. ☽ 4° 0′ 34″ tan. 8.8456713
Lon. 224 26 21 ſine 9.8451920 tan. 9.9914974

A = 5 43 0.7 tan. 9.0004793 ar. co. coſ. 0.0021654
O = 23 27 48. S.

B = 17 44 47.3 S. — — — coſ. 9.9788260 tan. 9.5051970

RA. 223 11 11.2 — — — tang. 9.9724888 ſine 9.8352940

Decl. 12 21 14.6 — — — — — — tan. 9.3404910

BOOK

BOOK II.

CONTAINING,

An Account of the Inſtruments for obſerving Altitudes and angular Diſtances at Sea, and of the Corrections to be applied to Obſervations made with thoſe Inſtruments.

CHAP. I.

OF HADLEY'S QUADRANT.

WE are indebted to John Hadley Eſq; for the firſt account of this admirable inſtrument, which he communicated to the Royal Society, May 13th, 1731. This account was publiſhed in the tranſactions of that ſociety, No. 420. In the ſame paper he deſcribes another inſtrument, having a third ſpeculum, and the poſitions of the ſpecula and teleſcope altered, whoſe uſe is to obſerve the Sun's altitude by means of the oppoſite part of the horizon.

The inſtruments in uſe for meaſuring altitudes at ſea, previous to the invention of Hadley's Quadrant, were, the Croſs Staff and Davis' Quadrant. The advantages poſſeſſed by the latter for obſervations of the Sun's altitude brought it into more general uſe, eſpecially among the Britiſh navigators. Obſervations, however, made with that inſtrument, though in the hands of a good obſerver, were found liable to error, when the ſea was agitated; and it was found impoſſible to obſerve the Sun's altitude, when the motion of the ſhip was very conſiderable. Hence appears the ſuperior excellence of Hadley's quadrant, which is deſigned to be of uſe, where the motions of the objects, or any circumſtance occaſioning an unſteadineſs in the common inſtruments, render the obſervations difficult or uncertain.

The form of this inſtrument, according to the preſent mode of conſtruction, is an octagonal ſector of a circle, and therefore the arch contains 45°; but becauſe of the double reflection, the limb is divided into 90°; and therefore this inſtrument is uſually called a Quadrant—of which the following are the principal parts.

1. An Octant, conſiſting of an arch or limb, connected with two radii.
2. An Index, moveable round a center, and accompanied with a dividing ſcale, to ſhow the obſerved altitude.
3. A Speculum, or Index Glaſs, placed on the Index in ſuch a manner, that its plane is over the center of motion of the Index.
4. Two Horizon Glaſſes, with their adjuſters.

4.

5. A ſet of coloured Glaſſes, to prevent the effect of the ſolar rays on the eye of the obſerver.
6. Two ſight Vanes.

OF THE OCTANT.

The Octant conſiſts of two radii, a limb, and two braces, which laſt are intended to ſtrengthen the inſtrument, and prevent it from warping. The arch or limb, although only an eighth part of a circle, is divided into 90°, on account of the double reflection. Theſe degrees are numbered from right to left, and each is commonly divided into three equal parts. Hence one of theſe intervals contains twenty minutes; and by means of a ſcale at the end of the index divided into twenty equal parts, an obſervation may be eaſily read off to the neareſt minute. The graduation on the limb is continued a few degrees to the right of O; this portion is uſually called the *arch of exceſs*, and is found very convenient for ſeveral purpoſes.

OF THE INDEX.

The Index is a flat bar, commonly made of braſs, moveable round the center of the inſtrument, and broader towards the axis of motion; at the other end of the index, a piece of braſs turns up, behind the limb, having a ſpring to make the dividing ſcale lie cloſe to the limb, and a ſcrew to faſten it in any poſition. Some quadrants have an adjuſting ſcrew affixed to the lower part of the index, by which means the index is made to move gently along the limb, without thoſe ſudden ſtartings, to which it is liable when moved by hand, and which may render the obſervation uncertain to two or three minutes. It may be obſerved, that when the index is moved by hand, it ſhould be taken hold of by the lower part, and not by the middle, as is ſometimes the practice.

That part of the index, which moves along the limb, has a ſmall ſcale attached to one ſide of a rectangular aperture, called a Dividing Scale,[*] but moſt commonly a Vernier or Nonius, from the ſuppoſed inventors. It uſually contains a ſpace equal to 21 or 19

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[*] This method of diviſion was firſt clearly explained by Clavius, in Lemma 1ſt of his treatiſe on Aſtrolabes, printed at Mayence in 1611; it was afterwards publiſhed by Pierre Vernier, who claimed it as his own, in a ſmall tract, intitled, "La conſtruction, l'uſage, & les proprietes du Quadrant Nouveau de Mathematique," &c. printed at Bruſſels, 1631. This method of diviſion may be thus illuſtrated.

Let two equal lines or circular arcs be ſo divided, that the number of equal diviſions in the one, is one more than the number of equal diviſions in the other; that is, let the one be divided into n, and the other into $n+1$ parts, and let a part of the former be called unity.

Then $n : 1 :: n+1 : \frac{n+1}{n}$ = a part of the other.

And $\frac{n+1}{n} - 1 = \frac{n+1-n}{n} = \frac{1}{n}$ = difference of the parts.

diviſions of the limb, and is divided into 20 equal parts; hence the difference between a diviſion on the limb, and a diviſion on the dividing ſcale, is one twentieth of a diviſion of the limb, or one minute; and therefore if any diviſion on the dividing ſcale is in the ſame ſtraight line with a diviſion on the limb, then no other diviſion on the dividing ſcale can coincide with a diviſion on the limb, the extreme diviſions excepted.

Sometime ago, it was uſual to reckon the diviſions on the vernier from its middle towards the right, and from the left towards the middle; and ſince this ſcale uſually contains 20 minutes, therefore the firſt ten minutes were contained on the right of the middle line, and numbered 5.10, and the other ten on the left, and marked 10.15. However, this method of diviſion being found inconvenient, a much more commodious method has been lately introduced, wherein the diviſions are numbered from right to left; hence the degree and

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Or let the one be divided into n, and the other into $n-1$ parts;
Then $n : 1 :: n-1 : \frac{n-1}{n}$ = a part of the other.

And $1 - \frac{n-1}{n} = \frac{n-n+1}{n} = \frac{1}{n}$ = the difference of the parts.

Now ſince, in common quadrants, the Vernier is divided into 20 equal parts, whereof the limb contains either 19 or 21, the abſolute length of both being the ſame, it is hence evident, that each diviſion on the dividing ſcale muſt anſwer to 1 minute, for in the preſent caſe n repreſents 20, and therefore $\frac{1}{n} = \frac{1}{20}$ of a diviſion on the limb: but each diviſion on the limb is 20 minutes, and $\frac{1}{20}$ of 20 minutes is 1 minute.

Let a degree on the limb be divided into four equal parts, and let the dividing ſcale contain a ſpace equal to $7\frac{1}{4}$ degrees or 29 of theſe parts, but divided into 30 equal parts, then will the dividing ſcale ſhow half minutes, for $\frac{1}{30}$ of $\frac{60}{4} = 30''$. And laſtly, if a degree is divided into ſix equal parts, and the dividing ſcale contains 39 (or 41) of theſe parts, but is divided into 40 equal diviſions, then will each diviſion ſhown by the dividing ſcale be $15''$ for $\frac{1}{40}$ of $\frac{60}{6} = 15''$.

This method of diviſion has been unjuſtly aſcribed to Petrus Nonius; for Nonius' method is very different from Vernier's, as may be ſeen in his treatiſe, "De Crepuſculis" printed at Liſbon, 1522, and alſo in his treatiſe, " De arte atque ratione Navigandi."

Nonius' method conſiſts in deſcribing within the ſame quadrant 45 concentric arches, dividing the outermoſt into 90 equal parts, the next within into 89, the next into 88, and ſo on till the innermoſt was divided into 46 equal parts only; by which means in moſt obſervations, the plumb line or index muſt croſs one or other of theſe circles very near a point of diviſion, whence by computation the degrees and minutes of the intercepted arch might be eaſily inferred. However this method of diviſion gave way to that of diagonals, publiſhed by Thomas Diggs in his treatiſe, " Alæ ſeu ſcalæ mathematicæ, "printed at London in 1573, who ſays it was invented by a Richard Chanſeler, a famous mathematical inſtrument maker. This method of diagonals was applied to Davis' Quadrants, and to the firſt of Hadley's, but now ſince the dividing ſcale or Vernier has been introduced, the diagonal method is as little eſteemed as that of Nonius after the invention of diagonals.

and minute, pointed out by the dividing ſcale, may be eaſily found thus.

Obſerve what minute on the dividing ſcale coincides with a diviſion on the limb; then this minute being added to the degree and part of a degree on the limb, immediately preceding the firſt diviſion, on the dividing ſcale, will be the degree and minute required.

Thus ſuppoſe the fourteenth minute on the dividing ſcale coincided with a diviſion on the limb, and that the preceding diviſion on the limb to the firſt on the dividing ſcale, was 56° 40′, hence the diviſion pointed out by the vernier will be 56° 54′. If a magnifying glaſs be uſed, the coincidence of the diviſions may be more accurately obſerved.

OF THE SPECULUM.

The Index Glaſs or Speculum is a rectangular piece of plane glaſs, one of whoſe planes is for the moſt part completely ſilvered. The intention of this glaſs is to reflect the image of the obſerved object, either to the fore or back horizon glaſs, and from thence to the eye of the obſerver.

This glaſs is ſet in a braſs frame, and is placed perpendicular to the plane of the inſtrument, immediately over the centre of motion of the index, by means of two ſcrews in a ſmall plate of braſs, connected with the under part of the braſs frame, and projecting at right angles from the back of that frame: one of theſe ſcrews takes into, and the other preſſes againſt the index.

The Index Glaſs, according to the common method of ſetting it in its frame, is often liable to be bent; and hence the difference between the true and obſerved altitude might be conſiderable. This ſource of error, however, is now removed by Mr Dollond, who contrived the frame ſo, that the glaſs lies on three points, which, by means of a ſpring and ſcrew, preſs the glaſs againſt other three points immediately oppoſite to the former; and ſince the poſition of a plane can be adapted to any three points, therefore the plane of the glaſs cannot be bent, though the preſſure be conſiderable.

As every ſilvered glaſs, although its planes be perfectly parallel to each other, gives two reflections (one from the fore ſurface of the glaſs, being the fainteſt, and the other from the ſilvered ſurface, which is the brighteſt) it is therefore evident, that errors may ariſe from theſe double reflections; in order to prevent which, Dr Maſkelyne propoſed, that the back of the upper half of the index glaſs, or that fartheſt from the plane of the inſtrument, inſtead of being ſilvered as uſual, ſhould be ground rough, and painted black, the line joining the ſilvered and blacked parts being parallel to the plane of the inſtrument. Hence when the object to be obſerved is bright, its reflection is to be taken from this half of the glaſs, the reflection from the ſilvered part being prevented by a ſcreen ſet before it; but if the object is not ſufficiently bright, the ſcreen may be turned down, and its reflection taken from the ſilvered part as uſual.

Of the Horizon Glasses.

There are two Horizon Glaſſes, one for the fore, and the other for the back obſervation, which are therefore named the fore and back horizon glaſſes. Each glaſs is ſet in a braſs frame, having an axis paſſing through the frame of the inſtrument, provided with an apparatus, by which it is capable of receiving a ſmall motion, ſo as to be ſet in a proper poſition with reſpect to the index glaſs; and in the pedeſtal of each glaſs are two ſunk ſcrews, one on each ſide of the glaſs, ſo that by ſcrewing up the one, and unſcrewing the other, the glaſs may be ſet perpendicular to the plane of the inſtrument. The lower part of the fore horizon glaſs is ſilvered, and the upper part is left tranſparent, through which one of the objects is to be ſeen directly, the other being ſeen by reflection from the ſilvered part of the glaſs. Both upper and lower parts of the back horizon glaſs are ſilvered, and in the middle is a tranſparent ſlit parallel to the plane of the inſtrument, through which the direct object is to be viewed.

Of the Coloured Glasses.

There are commonly three coloured glaſſes, two of which are tinged red, and the other green, by which the eye of the obſerver is preſerved from the effects of the ſolar rays, and the glare of the Moon in night obſervations is taken away. Each of theſe glaſſes is ſet in a ſeparate cell, and ſo connected, that either one or more may be uſed at the time of obſervation. When the back obſervation is to be uſed, the ſtem of the glaſſes is put into another perforation, between the horizon glaſſes, made on purpoſe to receive it. The two red glaſſes are adapted to obſervations of the Sun; and therefore in order to render them uſeful on all occaſions, the one is tinged darker than the other, ſo that either, or both, may be uſed according to the brightneſs of the Sun; other degrees of ſhade may be alſo obtained, by uſing the green glaſs with either of the former. The green glaſs is particularly uſeful in obſervations of the Moon; it may be alſo uſed in obſerving the Sun's altitude, when that object is very much obſcured.

Of the Sight Vanes.

The Sight Vanes are pieces of braſs, ſtanding perpendicular to the plane of the inſtrument. That Vane which is oppoſite to the fore-horizon glaſs, is called the Fore ſight Vane, and the other, the Back ſight Vane. There are two holes in the fore ſight vane, the lower of which, and the upper edge of the ſilvered part of the fore-horizon glaſs, are equidiſtant from the plane of the inſtrument, and the other is oppoſite to the middle of the tranſparent part of that glaſs. The back ſight vane has only one perforation, which is exactly oppoſite to the middle of the tranſparent ſlit in the horizon glaſs to which it belongs.

Fig. 6. repreſents the quadrant, wherein DB, CD, are the two radii or ſides; BC the limb; P, Q, the braces; DV the index moveable about the center D, upon which is the index glaſs A; and at the other extremity, the dividing ſcale or vernier V; H is the horizon glaſs for the fore obſervation, and G that for the back obſervation; E and F are the correſpondent ſight vanes; and I the coloured glaſſes, whoſe ſtem is to be put into the hole K, when the back obſervation is to be uſed.

ADJUSTMENTS OF HADLEY'S QUADRANT.

Previous to obſervation, the quadrant ſhould be carefully adjuſted,* otherwiſe it will be impoſſible to deduce accurate concluſions from the obſervations. For this purpoſe, the mirrors muſt be ſet perpendicular to the plane of the inſtrument, the fore horizon glaſs muſt be ſet parallel to the ſpeculum, and the planes of the ſpeculum and back horizon glaſs produced, muſt be perpendicular to each other, when the index is at O. Hence the number of adjuſtments is five; however, three of theſe being once made, are not ſo liable to alteration as the other two, which laſt ſhould be always examined before every obſervation.

ADJUSTMENT I.

To ſet the index glaſs perpendicular to the plane of the inſtrument.

Set the index to about 45°, and hold the quadrant ſo, that its plane may be nearly parallel to the horizon, the limb being from the obſerver; then look into the index glaſs, and obſerve if the portion of the limb ſeen by reflection appears in the ſame plane with that ſeen directly; if they do, the ſpeculum is perpendicular to the plane of the quadrant; if otherwiſe, the error is to be taken away, by turning the ſcrews behind the ſpeculum in contrary directions, and both ſcrews muſt be left equally tight.

This adjuſtment may be performed in a more accurate manner, by means of two adjuſting tools, fig. 20. on each of which is a line drawn parallel to its baſe; theſe lines are exactly at the ſame height.

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* We may obſerve, that ſince Davis' Quadrant is not capable of adjuſtment, it therefore becomes neceſſary to find its error, which was uſually determined by obſervations made at a place whoſe latitude had been previouſly well aſcertained, or by compariſon with another inſtrument whoſe error was known: this error was to be allowed in all ſucceeding obſervations made with the ſame quadrant; and according as the error tended to increaſe or diminiſh the ſhip's northern latitude, the Quadrant was therefore ſaid to be *northerly* or *ſoutherly*, by a quantity equal thereto. But Hadley's Quadrant is conſtructed, ſo as to be capable of being perfectly adjuſted; hence when a perſon ſays, his quadrant is northerly or ſoutherly, it may be inferred, that he neither underſtands the principles of the inſtrument, nor for what purpoſe it is adjuſted.

Let the quadrant be laid on a table, with its face upwards, and the index placed towards the middle of the limb; then place the tools on the limb, one at each end, direct the ſight to the index glaſs, and let the poſition of the index be altered, till the half of the direct tool is intercepted by the farther edge of the ſpeculum, and half of the other tool reflected from the ſame part of the ſpeculum, the other half being off the glaſs. Now, if the index glaſs is perpendicular to the plane of the quadrant, the direct and reflected lines on the tools will be in the ſame ſtraight line; if, not, the lines will appear broken, and the coincidence is to be made perfect by means of the ſcrews behind the ſpeculum.

ADJUSTMENT II.

To ſet the fore horizon-glaſs perpendicular to the plane of the quadrant.

Set the index to O; hold the plane of the quadrant parallel to the horizon, and apply the eye to the lower hole, in the ſight vane; then direct the ſight to the horizon, (or any other well defined object;) and if the horizons ſeen directly, and in the ſilvered part of the glaſs, are apparently in the ſame ſtraight line, the fore horizon-glaſs is perpendicular to the plane of the inſtrument; if not, one of the horizons will appear higher than the other: Now, if the horizon ſeen by reflection is higher than the ſame ſeen directly; unſcrew the neareſt ſcrew in the pedeſtal of the glaſs, and ſcrew up that on the further ſide of the glaſs, till the direct and reflected horizons appear to be in the ſame ſtraight line. But if the reflected is lower, than that ſeen directly, unſcrew the fartheſt, and ſcrew up the neareſt ſcrew, till the coincidence of the horizons is perfect; obſerving to leave both ſcrews equally tight; and the fore horizon-glaſs will be perpendicular to the plane of the qnadrant.

ADJUSTMENT III.

To ſet the fore horizon glaſs parallel to the index glaſs, the index being at Zero.

Hold the plane of the quadrant in a vertical poſition, with the arch lowermoſt, the index being previouſly ſet exactly to O: now the eye being applied to the hole in the vane next the quadrant, and the ſight directed to a well defined part of the horizon, or any well defined diſtant object,* then if the horizon ſeen in the ſilvered part of the glaſs, coincides with that ſeen through the tranſparent part, the horizon glaſs is adjuſted; but if the horizons do not coincide, unſcrew the milled ſcrew in the middle of the lever, on the other ſide of the quadrant, and turn the nut at the end of the lever, till both horizons coincide, and then fix the lever in this poſition, by tightning the milled ſcrew.

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* The Sun or a bright ſtar are the beſt objects for this purpoſe.

If this adjuſtment be again examined, it will perhaps be found imperfect; in this caſe, it therefore remains, either to repeat the adjuſtment, or rather, to find the error of adjuſtment, or as it is uſually called, the index error, which may be done thus:

Let the ſight be directed through the vane to the horizon, move the index till the reflected horizon coincides with that ſeen directly, and the difference between O on the limb, and O on the vernier, is the index error, which is additive, when the beginning of the vernier is to the right of O on the limb, otherwiſe ſubtractive.

ADJUSTMENT IV.

To ſet the back horizon glaſs perpendicular to the plane of the inſtrument.

Put the index to O, hold the plane of the quadrant parallel to the horizon, and direct the ſight towards the horizon, through the back ſight vane. Now if the horizon reflected from the ſilvered part of the glaſs, is in the ſame ſtraight line with that ſeen through the tranſparent part, the glaſs is perpendicular to the plane of the inſtrument. If the horizons do not unite, turn the ſunk ſcrews in the pedeſtal of the glaſs, till both appear to make one continued ſtraight line.

ADJUSTMENT V.

To ſet the back horizon glaſs perpendicular to the plane of the index glaſs produced, the index being at Zero.

Let the index be put as much to the right of O as twice the dip of the horizon amounts to, hold the quadrant in a vertical poſition, and apply the eye to the back horizon vane; then, if the reflected horizon coincides with that ſeen directly, the glaſs is adjuſted; otherwiſe, the ſcrew in the middle of the lever on the other ſide of the quadrant muſt be unturned, and the nut at its extremity turned till both horizons coincide. We may obſerve, that the reflected horizon will be inverted, that is, the ſea will be apparently uppermoſt, and the ſky lowermoſt.

The back horizon glaſs may be eaſily adjuſted at land, as follows:

Place three rods perpendicular to the horizon, in a ſandy beach, or level field, ſo as to be exactly in the ſame ſtraight line, and nearly equidiſtant from each other, the diſtance between two adjacent rods being not much leſs than a quarter of a mile; if the diſtance be greater, ſo much the better; then put the index of the quadrant exactly at O, and let the obſerver be ſtationed at the middle rod, with the plane of the quadrant parallel to the horizon, and the index glaſs above the rod. Now direct the ſight to one of the extreme rods, and if the reflected image of the other rod coincides with that ſeen directly, the glaſs is adjuſted; if the coincidence is not perfect, unturn the milled ſcrew, in the middle of the lever belonging to this glaſs, and turn the nut at the end of the lever, till both rods appear in one, and the glaſs will be adjuſted; then faſten the milled ſcrew.

This adjuſtment, as well as that of the fore horizon glaſs, is ſubject to a ſmall error, which is chiefly produced in faſtening the milled ſcrew, in the middle of the lever; it therefore becomes neceſſary to find the error of adjuſtment, which may be done by again examining the coincidence of the objects as before, after the milled ſcrew is faſtened, and making it perfect by moving the index, and the interval between the beginning of the index ſcale, and Zero on the limb, will be the error of adjuſtment, or the index error for the back obſervation, to be applied as formerly. In the above determination, we have ſuppoſed the rods to be in the ſame ſtraight line, which can be very eaſily effected; however, if theſe rods are not exactly in a right line, the error of adjuſtment may be thus found.

Let one of the extreme rods be called the *firſt*, and the other the *ſecond rod;* now the obſerver being placed at the middle rod as before, let the ſight be directed to the firſt rod, make the reflected image of the ſecond rod coincide with it, by moving the index, and mark the diviſion pointed out by the ſcale. Then let the obſerver turn half round, direct the ſight to the ſecond rod, and move the index till the firſt ſeen by reflection coincides with it, and again mark the angle ſhown by the index ſcale.[*] Now if one of theſe angles be meaſured on the limb, and the other on the arch of exceſs, half their difference will be the error of adjuſtment, or index error; but if both angles be meaſured either on the limb, or on the arch of exceſs, half their ſum is the index error; ſubtractive from altitudes given by the quadrant, when the angle, ſhown on the limb, exceeds that on the arch of exceſs, or if both angles be meaſured on the limb: in other caſes, this error is to be added. If the obſervation is that of the angular diſtance between any two objects, then the ſupplement of that given by the quadrant corrected as above will be the angle required.

Various other methods have been propoſed, for effecting the adjuſtment of the back horizon glaſs, ſome of which we ſhall briefly mention. 1ſt, In Mr Dollond's method, an index is applied to the back horizon glaſs, by which it may be moved, ſo as to be parallel to the index glaſs, when O on the index ſcale coincides with O on the limb. When this is effected, the index of the back horizon glaſs is to be moved exactly 90° from its former poſition, which is known by means of a divided arc for that purpoſe, and then the plane of the back horizon glaſs will be perpendicular to the plane of the index glaſs produced. 2d, In Mr Blair's method, the under edge of the index glaſs is ground and poliſhed, ſo as to be at right angles to the plane of the glaſs, and hence the back horizon glaſs is adjuſted, by making the direct and reflected horizons agree, the index being at O, exactly in the ſame manner as the fore horizon glaſs is rendered parallel to the ſpeculum.

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[*] Or invert the poſition of the quadrant, by turning its face downwards; hold its plane level as before, and direct the ſight again to the firſt rod; move the index till the ſecond ſeen by reflection coincides with it, and read off the angle ſhown by the dividing ſcale.

Uſe of Hadley's Quadrant in obſerving Altitudes at Sea.

The altitude of any object is determined by the poſition of the index on the limb, when by reflection that object appears to be in the horizon. In obſerving altitudes, it will be found moſt convenient tohold the quadrant with the left hand, and move the index with the right. Hence the left hand is to be applied to the further radius, towards the end of the curved brace, and the right hand to the lower part of the index.

If the object whoſe altitude is to be obſerved be the Sun, and if ſo bright that its image may be ſeen in the tranſparent part of the fore horizon glaſs, the eye is then to be applied to the upper hole in the fore ſight vane, otherwiſe to the lower hole; and in this caſe, the quadrant is to be ſo held, that the Sun may be biſected by the line joining the ſilvered and tranſparent parts of the glaſs. Hence one half of the Sun will be reflected from the ſilvered part of the glaſs. The Moon is to be kept as nearly as poſſible in the ſame poſition, and the image of a ſtar is to be reflected from that part of the ſilvered part of the glaſs, which is adjacent to the line of ſeparation of the two parts.

In obſerving the altitude of any object, the quadrant muſt be held perpendicular to the horizon, and ſo, that its plane produced would paſs through that object. In obſervations of the Sun, this latter poſition is eaſily obtained, by holding the quadrant ſo, that the ſhadow of the further radius may fall on that next the obſerver; in this caſe, the ſight is to be directed to the horizon. If the obſerved object is not the Sun, it will be proper to direct the ſight to the object; the quadrant being held ſo, that the image of the object may appear in the ſilvered part of the glaſs, as before mentioned, the index is to be moved till either limb of the object appears in contact with the horizon; but becauſe that part of the horizon, which is immediately under the object, is not exactly known, it therefore becomes neceſſary to give the quadrant a ſlow vibratory motion, the axis of which being that of ſight, the obſerver at the ſame time turning himſelf about upon his heel,[*] ſo as to keep the object always in that part of the horizon glaſs, which is at the ſame diſtance as the eye from the plane of the quadrant. By this means the reflected object will deſcribe an arc of a parallel circle round the true Sun, whoſe convex ſide will be downwards in the fore obſervation, and upwards in the back; and therefore when, by moving the index, the loweſt

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[*] Dr Maſkelyne's remarks on Hadley's quadrant, Nautical Almanac, 1774.

point of the arc in the fore obſervation, or higheſt in the back, is made to touch the horizon, the quadrant will ſtand in a vertical plane, and the altitude above the viſible horizon will be properly obſerved.

It is often found, in meaſuring the altitude of any object, or the diſtance between two fixed objects, that a different angle* will be obtained, by finiſhing the obſervation with a motion of the index in contrary directions. Thus the angle, obtained by moving the index according to the order of the diviſions on the limb, is always found greater, than when the index is moved in a retrograde direction, or contrary to the order of the diviſions. As this error ariſes from the bending of the index, it may be obviated by taking care to move the index the ſame way, both in the adjuſting and in taking the obſervations.

It frequently happens, that, when the object is nearly in contact with the horizon, the obſerver, by endeavouring to make this contact better, often puſhes the index too far, and thereby renders the contact worſe than before. In order to remove this inconvenience, the beſt quadrants are provided with a ſcrew at the lower end of the index, called an *adjuſting* or *tangent ſcrew*, by which a ſlow motion may be given to the index. Hence thoſe ſudden ſtarts, to which it is liable when moved by hand, are prevented.

A ſmall magnifying glaſs will be found of great ſervice, in reading off the obſerved angle; in doing which, the limb may be held towards the obſerver, and the eye ſhould be in a plane perpendicular to the quadrant, paſſing thro' the coinciding diviſions.

To take Altitudes by the Fore Obſervation.

I. Of the Sun.

Turn down either of the coloured glaſſes before the horizon glaſs, according to the brightneſs of the Sun; and if the Sun is bright enough to be ſeen in the tranſparent part of the glaſs, let the eye be placed at the upper hole in the ſight vane, otherwiſe at the hole next the plane of the quadrant; direct the ſight to that part of the horizon which is under the Sun, and move the index till the coloured image of the Sun appears in the horizon glaſs; then give the quadrant a ſlow vibratory motion, about the axis of viſion, move the index till the Sun's lower or upper limb, at the loweſt part of the arch deſcribed by this motion, is in contact with the horizon, and the degrees and minutes, ſhown by the index on the limb, will be the obſerved altitude of the Sun.

II. Of the Moon.

Put the index to O, turn down the green tinged glaſs, place the eye at the lower hole in the ſight vane, and look at the Moon thro the ſight; which being found in the ſilvered part of the horizon glaſs, move the index gradually, and follow the Moon's reflected image, till the enlightened limb is in contact with the horizon, at the lower part of the arch deſcribed by the vibratory motion, as in the laſt article; and the index will ſhow the obſerved altitude of that limb of the Moon, which was brought in contact with the horizon. If the obſervation is made in the day time, the coloured glaſs is unneceſſary.

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* An excellent account of this error is given in the Nautical Almanac for 1788.

III. Of a Star or Planet.

Put the index to O, direct the ſight to the ſtar, thro' the lower hole in the ſight vane, and tranſparent part of the horizon glaſs; move the plane of the quadrant a very little to the left, and the image of the ſtar will be ſeen in the ſilvered part of the glaſs; then move the index, and the image of the ſtar will deſcend; continue this motion of the index till the ſtar is in contact with the horizon, at the loweſt part of the arch deſcribed; and the degrees and minutes ſhown by the index on the limb will be the obſerved altitude of the ſtar.

When the horizon, under the object whoſe altitude is intended to be obſerved, is obſcured or rendered imperfect, by fog or any other impediment, the back obſervation becomes neceſſary; we would, however, adviſe the obſerver, before he depends entirely on the altitude taken by the back obſervation, to accuſtom himſelf to this mode of obſerving, and to aſcertain what accuracy he is capable of attaining, by comparing altitudes taken by this method with thoſe of the ſame object, taken at the ſame time by the fore obſervation, allowance being previouſly made for the dip and index error.

To take Altitudes by the Back Obſervation.

I. Of the Sun.

Put the ſtem of the coloured glaſſes into the hole between the horizon glaſſes; turn down either, according to the brightneſs of the Sun, and hold the inſtrument vertically; then direct the ſight thro' the hole in the back ſight vane, and tranſparent ſlit in the horizon glaſs, to that part of the horizon which is oppoſite to the Sun; move the index till the Sun is in the ſilvered part of the glaſs, and by giving the quadrant a vibratory motion, whoſe axis is that of viſion, the image of the Sun will deſcribe an arch whoſe convex ſide is upwards; now bring the limb of the Sun, when in the upper part of the arch, in contact with the horizon; and the degrees and minutes ſhown by the index on the limb will be the altitude of the other limb of the Sun.

II. Of the Moon.

The altitude of the Moon is obſerved in the ſame manner as that of the Sun, with this difference only, that the uſe of the coloured glaſs is unneceſſary, unleſs the Moon is very bright, and that the enlightened limb, whether it be the upper or lower, is to be brought in contact with the horizon. In the day time, the Moon's altitude may be obſerved, as directed in the next article.

III. Of a Star or Planet.

Look directly to the ſtar, through the vane and tranſparent ſlit in the horizon glaſs, move the index till the oppoſite horizon, with reſpect to the ſtar, is ſeen in the ſilvered part of the glaſs, and make the contact perfect as formerly. If the altitude of the ſtar is known nearly, the index may be ſet to that altitude, the ſight directed to the oppoſite horizon, and the obſervation made as formerly.

Unleſs obſervations are made accurately, the concluſions to be made therefrom cannot be depended on; and ſince a ſingle altitude is not to be ſo much relied on, as the mean of ſeveral, taken at ſhort intervals of time, therefore four or five altitudes ſhould be taken at nearly equal intervals of time, and their ſum, divided by their number, will be the mean altitude which the object would probably have had, at the mean interval, or the inſtant ariſing from the diviſion of the ſum of the times of obſervation by their number. If the intervals between the ſucceſſive obſervations are ſhort, and nearly equal, the correſponding differences of altitude will alſo be either nearly equal, or gradually increaſing or decreaſing Hence an erroneous altitude may eaſily be diſcovered; thus, if the altitudes of an object were 19° 58′, 20° 4′, 20° 11′, and 20° 15′, and the correſponding times of obſervation, 9h. 15′ 1″, 9h 15′ 45″, 9h. 16′ 36″, and 9h. 17′ 30″, reſpectively, it may hence be preſumed, that the three firſt altitudes are tolerably accurate, but that the laſt ſhould have been about 20° 18′, and therefore ſuch altitude ſhould be abſolutely rejected.

This method of deducing the mean altitude of the ſame limb of the Sun is, however, attended with an error ariſing from the irradiation, or apparent ſpreading of its rays, beyond the true limb; and therefore in order to diminiſh, if not wholly to deſtroy it, the mean of an equal number of altitudes of each limb, obſerved at nearly equal intervals of time, ſhould be taken.

Examination of a Quadrant.

If a quadrant be carefully made by a good artiſt, it may be very ſafely depended on; but as many quadrants are made by workmen careleſs of their reputation, it therefore becomes neceſſary to examine any inſtrument very particularly, before it is purchaſed—or, if it be already purchaſed, to be able to make allowance in ſubſequent obſervations for any error to which it may be liable. For this purpoſe, it is hoped, the following remarks will be found of ſervice.

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The frame of the quadrant ought to be made of well ſeaſoned wood—otherwiſe, the action of the Sun, at the time of obſervation, will be ready to alter its plane; from this cauſe, an error may be produced in the obſerved angle. This alteration may be obſerved, by looking along the plane of the inſtrument. The frame ſhould alſo be entirely free of rents, and every joint ſhould be as cloſe as poſſible.

The length of the ſcale on the limb ſhould be adapted to the radius of the inſtrument; this may be examined in the following manner. Conſtruct a ſcale of equal parts, as accurately as poſſible, upon which meaſure the length of the radius of the inſtrument, or diſtance between the center of motion and the exterior circular line bounding the diviſions; then will the extent between 0° and 90° be equal to the diameter of the inſtrument ×,38628. If, therefore, the correſpondent number be taken from the ſcale of equal parts, and applied to that on the limb, it ſhould reach from 0° to 90°.

If the diameter of the inſtrument be divided decimally, the extent between any two diviſions on the limb will be equal to the ſine of one fourth of the intercepted angle.

The diviſions on the limb may be alſo verified, as follows. In a ſandy beach or level piece of ground, let two rods be placed in a vertical poſition, at a convenient and nearly equal diſtance from the obſerver; adjuſt the quadrant by the left hand rod, and obſerve, as accurately as poſſible, the angular diſtance between theſe rods; let this obſervation be repeated, and the mean of theſe meaſures will be the correct angle between the rods; now meaſure the diſtance between the center of the index glaſs and each rod, and alſo the diſtance between the rods, as accurately as poſſible. Hence the angle at the center of the index glaſs may be found by computation, the difference between which and that by obſervation, will be the error of the ſcale at that particular diviſion. By increaſing and diminiſhing the diſtance between the rods, and proceeding as above, it will be known if the error is proportionable to the arc; and in any caſe a table of errors may be drawn out.

The intermediate diviſions may alſo be examined, by means of the index ſcale, provided it be of the proper length; which may be eaſily known, by multiplying the radius of the inſtrument by ,05526, if that ſcale takes in 6° 20′, but by ,06108 if it is equal to 7°. Now let one of the extreme diviſions on the index ſcale be put to any degree, and the other extreme diviſion will coincide with a diviſion on the limb; and in this manner may the whole ſcale be verified. To aſſiſt the eye, while examining the coincidence of the diviſions, a magnifying glaſs will be abſolutely neceſſary; and it ſhould be ſo held, that its axis may be perpendicular to the plane of the inſtrument, and over the diviſion to be examined.

The accuracy of the diviſions of the index ſcale may be eaſily examined, by obſerving if the differences of coincidence between theſe diviſions and thoſe on the limb be regular, and that no two diviſions on the index coincide with two, at the ſame time, on the limb, the extreme diviſions excepted.

The index ſhould be of a ſufficient ſtrength to prevent it, as much as poſſible, from bending, when moved along the limb; it ſhould be entirely free of any ſhake or play at the center, nor ſhould its motion there be ſtiff. Both ſurfaces of each mirror ſhould be perfect planes, and ſtrictly parallel to each other; the firſt of theſe requiſites may be verified by means of two diſtant objects, as follows.

Move the index, till both objects are exactly in contact at the upper edge of the ſilvered part of the horizon glaſs; now the plane of the quadrant being ſtill directed to the ſame objects, move it, in its own plane, ſo as to make the united images move along the line of ſeparation of the horizon glaſs, and if in this motion the images continue united, the reflecting ſurface is a good plane—otherwiſe the plane is imperfect. The paralleliſm of the planes of any of the reflecting ſurfaces may be examined, by viewing the image of the flame of a candle reflected obliquely from the glaſs; for if that image appears ſingle and well defined about the edge, the ſurfaces of the glaſſes are parallel to each other—but if the reflected image is repeated, the planes of the glaſs are inclined, and if produced would meet. If this glaſs was conſtructed ſo as to be capable of reverſion, the inclination of its planes might be found, and hence a table drawn up, to exhibit the error anſwering to any given angle.

The coloured glaſſes ſhould be free of veins, and have their planes true and parallel. The veins in a coloured glaſs may be diſcovered, by viewing the Sun through it, and the want of paralleliſm of the planes may be found in the following manner.

Obſerve the Sun's altitude when very near the meridian, and faſten the index by the ſcrew for that purpoſe; then turn the coloured glaſs ſo, that the plane which was next the index glaſs may now be next the horizon glaſs, and if the altitude is the ſame, the planes of the glaſs are parallel to each other; if not, make the contact of the Sun's limb and horizon perfect, and half the difference of altitude will be the error of the coloured glaſs. The leſs the meridian altitude of the Sun, the more accurately this verification will be made; and it ſhould not be attempted, when the Sun is near the zenith.

CHAP.

C H A P. II.

OF THE SEXTANT.

THE Sextant, as its name implies, is the ſixth part of a circle, and therefore contains 60°, but becauſe of the double reflection it is divided into 120°. It is conſtructed on the ſame principles as the quadrant, and may be ſaid to be that inſtrument extended. As the ſextant is particularly intended to meaſure the diſtance of the Moon from the Sun, or a fixed ſtar, it is therefore conſtructed with more care, and provided with ſome additional appendages, that are wanting in the quadrant.

This inſtrument is commonly made of metal; the index and limb are of braſs, but the frame is of a harder compoſition; the only wooden part belonging to it is a handle attached to the back, by which, when obſerving the diſtance between two objects, it is to be held with one hand, while the other is regulating the motion of the index.

A degree on the limb of this inſtrument is commonly divided into three equal parts, and therefore each of theſe parts contains 20 minutes. The index ſcale is divided ſo as to ſhow half minutes, and ſometimes every tenth ſecond.

That end of the index next the limb is furniſhed with an adjuſting, or, as it is ſometimes called, a tangent ſcrew, by which the index may be moved ſlowly and regularly, and therefore the contact of the limbs of any two objects may be made as perfect as the eye is able to diſtinguiſh, when aſſiſted with one of the teleſcopes, which accompany this inſtrument.

In moſt ſextants, the lower half of the index glaſs, or that next the plane of the inſtrument, is ſilvered as uſual, and the back ſurface of the other half is ground and painted black. A thin plate of braſs, equal in length and breadth to the ſilvered part of the ſpeculum, and moving on an axis, is to be raiſed before that part of the index glaſs, when the Sun is very bright; in which caſe, the reflection is taken from the poliſhed ſurface of the upper half of that glaſs, ſo that any error which might probably ariſe from the want of paralleliſm in the planes of the glaſs, is by this means avoided. This plate is alſo painted black.

A ſextant is not fitted up for the back obſervation; and in ſome, the horizon glaſs is ſo fixed to the inſtrument, as to be incapable of adjuſtment. In inſtruments of this conſtruction, it becomes abſolutely neceſſary to find the index error.

The coloured glaſſes are ſimilar to thoſe applied to a common Hadley's quadrant, and are uſually four in number; a ſextant is, however, generally provided with three more, to be placed occaſionſionally on the farther ſide of the horizon glaſs. Theſe are particularly uſeful in obſerving the ſun's altitude by reflection at land; or in finding the index error, by means of the ſun.

There are two teleſcopes belonging to the ſextant, one of which ſhows objects in their natural poſition; and the other is of the common aſtronomical conſtruction, and therefore ſhowing objects inverted: A tube, without glaſſes, alſo accompanies this inſtrument. By means of theſe, the line of ſight may be rendered parallel to the plane of the ſextant; and the contact of the limbs of any two objects more accurately obſerved. The tube, or either teleſcope, is to be ſcrewed into a braſs ring, which is connected with another braſs ring, by means of two ſcrews, in ſuch a manner, as to raiſe or lower the teleſcope, in order that the line of collimation may be directed to a proper part of the horizon glaſs.

A circular head, containing a plate, in which are three coloured glaſſes, and an aperture without glaſs, ſometimes accompanies the ſextant: This head is to be ſcrewed on the eye end of the tube, or on that of either teleſcope; the edge of the plate projects a little beyond the head on one ſide, and is moveable by the finger, ſo as the perforation, or either coloured glaſs, may be brought oppoſite to the circular aperture in the head.

Fig. 7th is a plan of the ſextant, as deſcribed above; fig. 8th is one of the teleſcopes; fig. 9th is the field of view of the teleſcope, with the two parallel wires; fig. 10th is the tube; and fig. 11th is a ſection of the inſtrument at the line AB, in which D is a ſection of the handle, at its proper diſtance from the plane of the inſtrument.

ADJUSTMENTS OF THE SEXTANT.

The adjuſtments of a ſextant, are to ſet the mirrors perpendicular to its plane, and parallel to each other, when the index is at zero, and to rectify the poſition of the line of collimation. The deviation of each of theſe from its true poſition might indeed be found, and the reſulting error of obſervation, from thence computed: But this by no means ought to be admitted, unleſs in caſes of abſolute neceſſity.

ADJUSTMENT I.

To ſet the index glaſs perpendicular to the plane of the ſextant.

Put the index to about 60°, and hold the plane of the ſextant nearly parallel to the horizon, the limb being from the obſerver: then direct the ſight to the ſpeculum, and if the reflected limb of the inſtrument, appears to be exactly in the ſame plane with that ſeen directly; the glaſs is perpendicular to the plane of the inſtrument. But if the limbs are not in the ſame plane, turn the ſcrews in the projecting plate behind the ſpeculum, till they are apparently ſo, and the glaſs will be adjuſted. This adjuſtment might be effected by means of the adjuſting tool.

ADJUSTMENT II.

To ſet the horizon glaſs perpendicular to the plane of the ſextant.

The index glaſs being previouſly adjuſted, ſet the beginning of the diviſions on the index to zero on the limb, and hold the plane of the inſtrument in a horizontal poſition; then direct the ſight to the horizon glaſs, and if the reflected horizon is apparently in the ſame ſtraight line with that ſeen directly, the glaſs is perpendicular to the plane of the ſextant; otherwiſe, turn the adjuſting ſcrew of the horizon glaſs, at the back of the inſtrument, till the coincidence of the reflected and direct horizons is perfect.

ADJUSTMENT III.

To ſet the horizon glaſs parallel to the ſpeculum.

Set the firſt diviſion on the index to zero on the limb, faſten the index in this poſition, and make the coincidence of theſe diviſions perfect, by means of the adjuſting ſcrew at the end of the index, the eye being aſſiſted with the magnifying lens; ſcrew the teleſcope in its ſupport, and turn the ſcrew belonging thereto, at the back of the inſtrument, till the field of the teleſcope is biſected by the line which ſeparates the ſilvered and tranſparent parts of the horizon glaſs; now hold the ſextant vertically, direct the ſight to the horizon, and if the reflected and direct horizons do not coincide, releaſe the ſcrew which lies nearly behind the ſupport of the teleſcope, and turn the nut at the extremity of the lever, till the coincidence of the horizons is perfect; and the horizon glaſs will be adjuſted.

After the ſcrew which retains the lever in its place is faſtened, it will be proper to examine this adjuſtment; if the coincidence of the horizons is not perfect, the adjuſtment is to be repeated till it is ſo; but as it is difficult to obtain an exact coincidence by this means, the horizons may be brought to coincide, by turning the adjuſting ſcrew of the index—and the difference between the two zeros is the index error.

The index error may alſo be found, by meaſuring the diameter of the Sun or Moon twice, with a motion of the index in contrary directions. If both meaſures are taken either to the right or left of O on the limb, half their ſum will be the index error, and is ſubtractive or additive accordingly; but if one of the meaſures is taken to the right, and the other to the left of O, half their difference is the index error, being additive when the diameter meaſured to the left of O exceeds that meaſured to the right—otherwiſe, ſubtractive. Since in ſome ſextants the horizon glaſs cannot be adjuſted, the index error muſt therefore be found, and in this caſe it may be conſidered as a conſtant quantity, to be applied to all angles meaſured with the ſame inſtrument.

In altitudes obſerved at land by the method of reflection, the double altitude affected by the index error is given by the inſtrument; in this caſe, it will be found convenient to call half the index error, the correction, which being applied to half the angle given by this inſtrument, will give the apparent altitude of the object.

ADJUSTMENT IV.

To make the line of collimation parallel to the plane of the Sextant.*

Turn the eye end of the teleſcope, containing the two parallel wires, till the wires are parallel to the plane of the inſtrument, and let two diſtant objects be ſelected, as two ſtars of the firſt magnitude, or the Sun and Moon, whoſe diſtance may not be leſs than 90° or 100°; make the contact of the limbs of theſe objects as perfect as poſſible, at the wire neareſt the plane of the inſtrument; fix the index in this poſition; move the ſextant till the objects are at the other wire, and if the ſame limbs are in contact, the axis is adjuſted; but if the limbs are either apparently ſeparated, or partly cover each other, correct half the error by the ſcrews in the circular part of the ſupporter, one of which is above, and the other between the teleſcope and ſextant; turn the adjuſting ſcrew at the end of the index, till the limbs are in contact; then bring the objects to the wire next the inſtrument, and if the limbs are in contact, the axis of viſion of the teleſcope is parallel to the plane of the inſtrument—if not, proceed as at the other wire, and continue till no error remains.

If the Moon is one of the objects, this adjuſtment muſt be performed as quick as poſſible; otherwiſe, allowance muſt be made for the Moon's apparent motion in the interval.

This adjuſtment may be alſo performed in the manner afterwards deſcribed, for adjuſting the line of collimation of the circular inſtrument. The error of the line of collimation being given, the reſulting error of obſervation may be found by entering table xxii. with the above error at the top, and the obſerved diſtance in the ſide column.

It is ſometimes neceſſary to know the angular diſtance between the wires of the teleſcope; to find which, let theſe wires be placed perpendicular to the inſtrument, hold the ſextant vertical, direct the ſight to the horizon, and move the ſextant in its own plane, till the horizon and upper wire coincide; keep the ſextant in this poſition, and move the index till the reflected horizon is covered by the lower wire, and the diviſion ſhown by the index on the limb, corrected by the index error, will be the angular diſtance between the wires.

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* The line of collimation is an imaginary ſtraight line, joining the center of refractions of the object glaſs of a teleſcope, and either the interſection of the croſs wires, or middle between the parallel wires.

USE OF THE SEXTANT.

The ſextant is particularly adapted to meaſure the angular diſtance between the Moon and the Sun, or a fixed ſtar. When the diſtance between the Moon and either of theſe objects is to be obſerved, the ſextant muſt be held ſo, that its plane produced may paſs through the eye of the obſerver and both objects; and the reflected image of the moſt luminous of the two, is to be brought in contact with the other ſeen directly. To effect this, therefore, it is evident, that, when the brighteſt object is to the right of the other, the face of the ſextant muſt be held upwards; but if to the left, downwards. When the face of the ſextant is held upwards, the inſtrument ſhould be ſupported with the right hand, and the index moved with the left; but when the face of the ſextant is from the obſerver, it ſhould be held with the left hand, and the motion of the index regulated by the right hand.

Sometimes a ſitting poſture will be found very convenient for the obſerver, particularly when the reflected object is to the right of the direct one; in this caſe, the inſtrument is ſupported by the right hand, the elbow may reſt on the right knee, the right leg at the ſame time reſting on the left knee.

If the ſextant is provided with a ball and ſocket, and a ſtaff, one of whoſe ends is attached thereto, and the other reſts in a belt faſtened round the waiſt of the obſerver; the greater part of the weight of the inſtrument may by this means be ſupported by his body.

To obſerve the diſtance between the Moon and any celeſtial object.

I. Between the Sun and Moon.

Put the teleſcope in its place, and the wires parallel to the plane of the inſtrument, and if the Sun is very bright, raiſe the plate before the ſilvered part of the ſpeculum; direct the teleſcope either to the tranſparent part of the horizon glaſs, or to the line which ſeparates the ſilvered and tranſparent parts of that glaſs, according to the brightneſs of the Sun, and turn down one of the coloured glaſſes; then hold the ſextant ſo, that its plane produced may paſs through the Sun and Moon, having its face either upwards or downwards, according as the Sun is to the right or left of the Moon; direct the ſight through the teleſcope to the Moon, and move the index till the limb of the Sun is nearly in contact with the enlightened limb of the Moon; now faſten the index, and by a gentle motion of the inſtrument make the image of the Sun move alternately paſt the Moon, and, when in that poſition where the limbs are neareſt each other, make the coincidence of the limbs perfect, by means of the adjuſting ſcrew. This being effected, read off the degrees and parts of a degree pointed out on the limb by the index, uſing the magnifying glaſs; and thus the angular diſtance between the limbs of the Sun and Moon is obtained.

II. Between the Moon and Star.

Direct the middle of the field of the teleſcope to the line of ſeparation of the ſilvered and tranſparent parts of the horizon glaſs; if the Moon is very bright, turn down the lighteſt coloured glaſs, and hold the ſextant ſo, that its plane may be parallel to that paſſing through the eye of the obſerver and both objects; with its face upwards, if the Moon is to the right of the ſtar—but if to the left, the face is to be held from the obſerver. Now direct the ſight thro' the teleſcope to the ſtar, and move the index till the Moon appears by reflection to be nearly in contact with the ſtar; faſten the index, and turn the adjuſting ſcrew, till the coincidence of the ſtar and enlightened limb of the Moon is perfect—and the degrees and parts of a degree, ſhown by the index, will be the obſerved diſtance between the Moon's enlightened limb and the ſtar.

The contact of the limbs muſt always be obſerved in the middle between the parallel wires.

It is ſometimes difficult, for thoſe not much accuſtomed to obſervations of this kind, to find the reflected image in the horizon glaſs; it will perhaps be found more convenient to look directly to the object, and by moving the index, to make its image coincide with that ſeen directly; or if the diſtance between the objects be known nearly, the index may be ſet to this diſtance; the ſight being directed to one of the objects, and the ſextant held as formerly deſcribed, the other object will be ſeen in the field of the teleſcope.

CHAP. III.

OF THE CIRCULAR INSTRUMENT OF REFLECTION.

THIS inſtrument was propoſed, with a view to correct the errors to which a ſextant is liable, and in particular the error ariſing from the inaccuracy of the diviſions on the limb. It was invented by the late celebrated aſtronomer Profeſſor Mayer, and ſince has been greatly improved by the Chevalier de Borda, and M. Jean Hyacinth de Magellan.

This inſtrument conſiſts of the following parts, a circular ring or limb, two moveable indices, two mirrors, a teleſcope, coloured glaſſes, &c.

The limb of the inſtrument is a complete circle of metal, and is connected with a perforated central plate by ſix radii; it is divided into 720 degrees, becauſe of the double reflection; each degree is divided into three equal parts, and the diviſion is carried to minutes by means of the index ſcale, as uſual.

The two indices are moveable round the ſame axis, which paſſes exactly thro' the center of the inſtrument; the firſt index carries the central mirror, and the other, the teleſcope and horizon glaſs, each being provided with an adjuſting ſcrew for regulating its motion, and a ſcale for ſhewing the diviſions on the limb.

The central mirror is placed on the firſt index, immediately above the center of the inſtrument, and its plane is inclined to the middle line of the index about 30°. The four ſcrews in its pedeſtal, for making its plane perpendicular to that of the inſtrument, have ſquare heads, and are therefore eaſily turned either way by a key for that purpoſe.

The horizon glaſs is placed on the ſecond index near the limb, ſo that as few as poſſible may be intercepted, of the rays proceeding from the reflected object, when to the left. The perpendicular poſition of this glaſs is rectified in the ſame manner, as that of the horizon glaſs of a ſextant, to which it is ſimilar. It has alſo another ſmall motion, whereby its plane may be diſpoſed ſo, as to make proper angles with the axis of the teleſcope and a line joining its center, and that of the central mirror.

The teleſcope is attached to the other end of the index. It is an achromatic aſtronomical one, and therefore inverts the obſerved objects; it has two parallel wires in the common focus of the glaſs, diſtant from each other between two and three degrees, and which, at the time of obſervation, muſt be placed parallel to the plane of the inſtrument. This is eaſily done, by making the mark on the eye piece coincide with that on the tube. The teleſcope is moveable by two ſcrews in a vertical direction with regard to the plane of the inſtrument, but is not capable of receiving a lateral motion.

To this inſtrument belong two ſets of coloured glaſſes, each containing four, and differing in ſhade from each other. The glaſſes of the larger ſet, which belongs to the central mirror, ſhould have each about half the degree of ſhade with which the correſpondent glaſs, of the ſet belonging to the horizon mirror, is tinged—becauſe the ray from the luminous object paſſes twice thro' the coloured glaſs placed before the central mirror, and only once through the other coloured glaſs. Theſe glaſſes are kept tight in their places, by ſmall preſſing ſcrews, and make an angle of about 85° with the plane of the inſtrument; by which means, the image from the coloured glaſs is not reflected to the teleſcope. When the angle to be meaſured is between 5° and 34°, one of the glaſſes of the largeſt ſet is to be uſed; in other caſes, one of the leſs ſet is to be placed before the horizon glaſs.

The handle is of wood, and is ſcrewed to the back of the inſtrument, immediately under the center, with which it is to be held at the time of obſervation.

Fig. 12. is a plan of the inſtrument, wherein the limb is repreſented by the divided circular plate; A is the central mirror, *aa* the places which receive the parts *aa* of the coloured glaſs, fig. 15. EF the firſt or central index, with its ſcale and adjuſting ſcrew—MN the ſecond or horizon index—GH the teleſcope—IK the ſcrews for moving it to and from the plane of the inſtrument—C the place of the coloured glaſs, fig. 14.—and D its place in particular obſervations.

Fig. 13. is a ſection of the inſtrument, wherein the ſeveral parts are referred to by the ſame letters as in fig. 12.—fig. 14. repreſents one of the horizon coloured glaſſes—and fig. 15. one of the central coloured glaſſes—fig. 16. is the key for turning the adjuſting ſcrews of the mirrors—fig. 17. is the handle—fig. 18. a ſection of one of the radii, towards its middle—fig. 19. the *ventelle*, which is only uſed in terreſtrial obſervations, for diminiſhing the light of the direct object, and whoſe place at the time of obſervation is D.—fig. 20. is the tool for adjuſting the central mirror vertically, and for rectifying the poſition of the teleſcope with regard to the plane of the inſtrument. There is another tool exactly of the ſame ſize of the former; the height of theſe is nearly equal to that of the middle of the central mirror.

ADJUSTMENTS OF THE CIRCULAR INSTRUMENT.

The tendency of theſe adjuſtments is, to make the mirrors perpendicular to the plane of the inſtrument, to make the line of collimation parallel thereto, to prevent any falſe light from entering the teleſcope, &c. Some of theſe adjuſtments being once made, will continue perfect, for a conſiderable ſpace of time, provided the inſtrument receives no injury; but it will be neceſſary to examine others from time to time.

ADJUSTMENT I.

To ſet the horizon glaſs ſo, that none of the rays from the central mirror ſhall be reflected to the teleſcope from the horizon mirror, without paſſing thro' the coloured glaſs belonging to this laſt mirror.

Place the coloured glaſs before the horizon mirror, direct the teleſcope to the ſilvered part of that mirror, and make it nearly parallel to the plane of the inſtrument; move the firſt index, and if the rays from the central mirror to the horizon glaſs, and from thence to the teleſcope, have all the ſame degree of ſhade with that of the coloured glaſs uſed, the horizon glaſs is in its proper poſition; otherwiſe, the pedeſtal of this glaſs muſt be turned, till the uncoloured images diſappear.

ADJUSTMENT II.

To ſet the central mirror perpendicular to the plane of the inſtrument.

Place the two adjuſting tools on the limb, about 350° of the inſtrument diſtant, one on each ſide of the diviſion on the left, anſwering to the plane of the central mirror produced; then, the eye being placed at the upper edge of the neareſt tool, move the central index, till only half of the reflected image of this tool is ſeen in the central mirror towards the left, and move the other, till its half to the right is hid by the ſame edge of the mirror; then if the upper edges of both tools are apparently in the ſame ſtraight line, the central mirror is perpendicular to the plane of the inſtrument; otherwiſe, bring them into this poſition, by the ſcrews in the pedeſtal of the mirror.

This adjuſtment may be performed in the ſame manner as uſed in ſetting the ſpeculum of a ſextant or quadrant perpendicular.

Hence, by examining this adjuſtment in different parts of the limb, it will be known if the limb be in the ſame plane.

ADJUSTMENT III.

To ſet the horizon mirror perpendicular to the plane of the inſtrument.

The central mirror being previouſly adjuſted, direct the ſight through the teleſcope to any well defined object; then if, by moving the central index, the reflected image paſſes exactly over the object ſeen directly, the mirror is perpendicular; if not, its poſition muſt be rectified by means of the ſcrews in the pedeſtal of the glaſs.

A planet, or ſtar of the firſt magnitude, will be a good object for this purpoſe; the Sun will alſo be found very convenient for making this adjuſtment, in which caſe, one of the coloured glaſſes muſt be put on the farther ſide of the horizon mirror.

This adjuſtment may be alſo performed by obſerving, if the horizon of the ſea, ſeen in the ſilvered part of the horizon glaſs, and by the edge of that glaſs appears to make one continued ſtraight line.

ADJUSTMENT IV.

To make the line of collimation parallel to the plane of the inſtrument.

Lay the inſtrument horizontally on a table; place the two adjuſting tools on the limb, towards the extremities of one of the diameters of the inſtrument, and, at about 15 or 20 feet diſtant, let a well defined mark be placed, ſo as to be in the ſame ſtraight line with the tops of the tools; then raiſe or lower the teleſcope, till the plane paſſing through its axis and the tops of the tools is parallel to the plane of the inſtrument, and direct it to the fixed object; turn either or both of the ſcrews of the teleſcope, till the mark is apparently in the middle between the wires; then is the axis of the teleſcope parallel to the plane of the inſtrument, and the difference, if any, between the diviſions pointed out by the indices of the ſcrews will be the error of the indices. Hence this adjuſtment may, in future, be eaſily rectified.

In this proceſs, the eye tube muſt be placed ſo, as to obtain diſtinct viſion.

ADJUSTMENT V.

To find that diviſion, to which the ſecond index being placed, the mirrors will be parallel, the central index being at zero.

Having placed the firſt index exactly to O, direct the teleſcope to the horizon mirror ſo, that its field may be biſected by the line joining the ſilvered and tranſparent parts of that mirror; hold the inſtrument vertically, and move the ſecond index, till the direct and reflected horizons agree—and the diviſion ſhown by the index will be that required.

This adjuſtment may alſo be performed by meaſuring the diameter of the Sun or Moon in contrary directions. The middle between the diviſions on the limb will be that required. Or it may be effected, by making the reflected and direct images of a ſtar or planet coincide.

Since it very often happens, that the contact of the objects cannot be obſerved exactly in the middle of the field of the teleſcope, it therefore becomes neceſſary to know the interval between the point where the contact was obſerved, and the middle of the field, expreſſed in minutes. This may be eſtimated, by knowing the angular diſtance between the wires, which may be thus determined:

Turn round the eye piece of the teleſcope, till the wires are perpendicular to the plane of the inſtrument, and put the firſt index to O; direct the teleſcope to any well defined object, and move the horizon index, till the reflected and direct objects are coincident; then make one of the wires coincide with the object, and turn the central index till the reflected image of the object coincides with the other wire—and the arch paſſed over by that index will be the angular diſtance between the wires.

The interval between the point where the contact of the limbs was obſerved, and the middle of the field, being eſtimated, the correſponding error of obſervation is found directly from table xxii.

Vari-

Verification of the Paralleliſm of the Surfaces of the Glaſſes.

I. Of the Central Mirror.

Select two well defined objects, whoſe angular diſtance exceeds 90° or 100°; now the inſtrument being accurately adjuſted, let ſeveral obſervations of the angular diſtance between the objects be taken, in the manner hereafter to be deſcribed, and from thence find the mean angular diſtance. Then take out the central mirror, and turn it ſo, that the edge which was formerly uppermoſt may now be next the plane of the inſtrument; rectify its poſition, and take an equal number of obſervations of the angular diſtance of the ſame two objects; now half the difference between the mean of theſe, and that of the former, will be the error of the mirror anſwering to the obſerved angle. If the firſt mean exceeds the ſecond, the error is ſubtractive; otherwiſe ſubtractive, the mirror being in its firſt poſition.

The error correſponding to any other angle may be found thus. Add together the proportional logarithm of the error found as above, the proportional logarithm of the error anſwering to the given diſtance from col. 4. tab. xxi. and the arithmetical complement of the proportional logarithm of the error correſponding to the obſerved diſtance from the ſame table; the ſum, rejecting 10 in the index, will be the proportional logarithm of the required error. Hence a particular table of errors for a given mirror, whoſe planes are inclined, may be conſtructed.

Since the angle between the plane of the horizon glaſs and axis of the teleſcope produced is conſtant in all obſervations, therefore no error ariſes from the want of paralleliſm in its ſurfaces.

II. Of the Coloured Glaſſes.

Place one of the coloured glaſſes on this, and another on the farther ſide of the horizon mirror with reſpect to the teleſcope; ſet the firſt index to O, direct the teleſcope to the Sun, and move the ſecond index till the limbs of the direct and reflected images coincide; obſerve at leaſt five or ſix contacts, and from thence find the mean point of diviſion anſwering thereto; then turn the neareſt coloured glaſs, ſo that the ſurface which was from the horizon mirror may now be towards it, obſerve the contact of the ſame two limbs as often as before, and find the mean diviſion; then half the difference between theſe two means will be the error of the neareſt coloured glaſs, which is additive or ſubtractive, according as the firſt mean is leſs or greater than the ſecond, the coloured glaſs being in its firſt poſition. In like manner, the paralleliſm of the ſurfaces of the other coloured glaſſes may be examined, and that of the green glaſs may be verified by means of the Moon.

As in croſs obſervations, the errors ariſing from the coloured glaſs are nearly counterbalanced, in this caſe the above error may therefore be neglected. This error will alſo vaniſh in obſervations to the right or to the left, by taking an equal number, and changing the poſition of the coloured glaſs at each obſervation.

USE OF THE CIRCULAR INSTRUMENT.

There are three different methods of obſerving the angular diſtance between two objects with this inſtrument. The obſervation taken by the firſt method is named, an Obſervation to the Right; that by the ſecond, an Obſervation to the Left, and the laſt is called a Croſs Obſervation.

An obſervation to the right is that, wherein the object, whoſe image is to be reflected, and the central mirror, are on the ſame ſide of the teleſcope. An obſervation to the left, when the object to be reflected and the central mirror are on oppoſite ſides of the teleſcope—which, in both caſes, is ſuppoſed to be directed to the other object; and a Croſs Obſervation is the combination of two ſucceſſive obſervations, the one being to the right, and the other to the left.

GENERAL PRECEPTS,

For obſerving the Angular Diſtance between any two Objects.

Adjuſt the inſtrument, as before directed; then put the firſt index to O, and the other index ſo, that both mirrors may be parallel; hold the inſtrument in ſuch a poſition, that its plane produced may paſs through both objects, having its face upwards, if the reflected object is to the left of the other, but downwards, if to the right; direct the teleſcope to the object which is apparently the leaſt luminous, and move the ſecond index according to the order of the diviſions, till the limb of the reflected object is in contact with that of the object ſeen directly; faſten the ſecond index, and make the coincidence of the limbs perfect, by means of the adjuſting ſcrew—and the arch paſſed over by that index, will be nearly equal to the diſtance between the limbs of the objects. Now move the firſt index towards the ſecond, by a quantity equal to twice the meaſured diſtance; invert the poſition of the inſtrument, by directing its face to the oppoſite point; direct the teleſcope to the fainteſt object, and both will be ſeen in the field; make the contact of the ſame two limbs perfect, and half the angle ſhown by the central index on the limb will be the angular diſtance between the limbs of the objects.

If the Sun is one of the objects, a coloured glaſs muſt be placed before the central mirror, when the diſtance is between 5° and 35°. In other diſtances, one of the coloured glaſſes belonging to the horizon mirror is to be uſed.

In order to render the obſervations as accurate as poſſible, it will be neceſſary to continue meaſuring the angular diſtance as above, till the central index has paſſed once or twice over the limb; and the degrees and parts of a degree, contained in the revolutions and exceſs of a revolution, being divided by twice the number of croſs obſervations, will be the angular diſtance between the objects. On this the principal advantage of the circular inſtrument depends.

It might ſeem ſuperfluous to enlarge upon the uſe of this inſtrument; however, as ſome may wiſh to have inſtructions for every particular operation, it is therefore thought proper to ſubjoin the following

PARTICULAR PRECEPTS.

I.

To obſerve the Diſtance between the Sun and Moon.

1ſt. The Sun being to the right of the Moon.

Prepare the inſtrument for obſervation, as directed above; ſet a proper coloured glaſs before the central mirror, if the diſtance between the objects is leſs than 35°, but if above that quantity, place a coloured glaſs before the horizon mirror; make the mirrors parallel or nearly ſo, the firſt index being at O, and hold the inſtrument ſo, that its plane may be directed to the objects, with its face downwards, or from the obſerver; direct the ſight thro' the teleſcope to the Moon; move the ſecond index, according to the order of the diviſions on the limb, till the neareſt limbs of the Sun and Moon are almoſt in contact; faſten that index, and make the coincidence of the limbs perfect, by the adjuſting ſcrew belonging thereto; then invert the inſtrument, and move the central index towards the ſecond, by a quantity equal to twice the arch paſſed over by that index; direct the plane of the inſtrument to the objects, look directly to the Moon, and the Sun will be ſeen in the field of the teleſcope; faſten the central index, and make the contact of the ſame two limbs exact, by means of the adjuſting ſcrew; then half the arch paſſed over by the central index will be the diſtance between the neareſt limbs of the Sun and Moon.

2d. The Sun being to the left of the Moon.

The inſtrument being previouſly prepared; hold it with its face upwards, ſo that its plane may paſs through both objects, direct the teleſcope to the Moon, and make its limb coincide with the neareſt limb of the Sun's reflected image, by moving the ſecond index; now put the inſtrument in an oppoſite poſition, direct its plane to the objects, and the ſight to the Moon, the central index being previouſly moved towards the ſecond, by a quantity equal to twice the meaſured diſtance; and make the ſame two limbs that were before obſerved coincide exactly, by turning the adjuſting ſcrew of the firſt index; then half the angle ſhown by the firſt index will be the angular diſtance between the obſerved limbs of the Sun and Moon.

II.

To obſerve the angular diſtance between the Moon and a fixed ſtar or planet.

1ſt. The ſtar being to the right of the Moon.

In this caſe, the ſtar is to be conſidered as the direct object, and the enlightened limb of the Moon's reflected image is to be brought in contact with the ſtar or planet, both by a direct and inverted poſition of the inſtrument, exactly in the ſame manner as deſcribed in the laſt article. If the Moon's reflected image is very bright, the lighteſt tinged glaſs is to be uſed.

2d. The ſtar being to the left of the Moon.

Proceed in the ſame manner as directed for obſerving the diſtance between the Sun and Moon, the Sun being to the right of the Moon, and uſing the lighteſt tinged glaſs, if neceſſary.

Beſides the inſtruments already deſcribed, for meaſuring the angular diſtance between the Moon, and the Sun or a fixed ſtar, ſeveral others have been propoſed for the ſame purpoſe, particularly the *Megameter*, by M. de Charnieres, which is conſtructed on the ſame principle as the object glaſs micrometer; but ſince this inſtrument does not meaſure angles above 10°, and becauſe the diſtances given in the Nautical Almanac always exceed that quantity, we therefore will not enter upon a deſcription of it, but refer to his treatiſe, entitled, *Theorie et Pratique des Longitudes en Mer*, printed at Paris in 1772. A deſcription of Mr Garrard's Antimeter may be ſeen in the pamphlet, which accompanies that inſtrument.

CHAP. IV.

Of the manner of taking a complete ſet of Lunar Obſervations.

IN order to take a ſet of lunar obſervations, in a regular and accurate manner, three aſſiſtants will be neceſſary, whereof two are to obſerve the altitudes of the Moon, and the Sun or ſtar, at the ſame time that the diſtance is taken by the principal obſerver; the other aſſiſtant, having a watch ſhowing ſeconds, is to mark the time when theſe obſervations are made. If the Sun or ſtar is at a proper diſtance from the meridian, the time may be inferred from the altitude obſerved at the ſame time with the diſtance, and therefore the watch is not neceſſary; but if the object with which the Moon is compared be two near the meridian, the watch is abſolutely neceſſary, in order to connect the obſervations for aſcertaining the apparent time at the ſhip, and the longitude, with each other. This laſt aſſiſtant muſt be provided with a pencil and paper, to write down the obſervations, as they are taken—which may be as follows:

The ſextant and quadrants being accurately adjuſted, or the error of adjuſtment found, ſet their indices to the eſtimate diſtance and altitudes reſpectively. Now let the principal obſerver place his aſſiſtants in the moſt convenient ſituation poſſible, and deſire them to be prepared when he is ready to obſerve the diſtance; then all are to begin to obſerve at the ſame time, and when the principal obſerver has obtained a perfect coincidence of the limbs of the objects, he is to make it known to his aſſiſtants. The perſon having the watch is immediately to write down, firſt the ſecond, and then the minute of obſervation; the hour being previouſly marked, the principal obſerver is to read off the diſtance, uſing the magnifying glaſs, which he is to communicate to the time aſſiſtant, that it may be wrote after the time of obſervation. The other aſſiſtants are in like manner to report the obſerved altitudes of the limbs of the objects ſeparately. Hence a complete ſet of obſervations is obtained. In this manner, let theſe obſervations be repeated, till at leaſt four or five different ſets are obſerved, which may be eaſily accompliſhed in the ſpace of ſix or eight minutes, and theſe ſets ought to be taken at nearly equal intervals of time. The mean of each particular obſervation is to be taken—that is, the ſum of each is ſeparately to be divided by the number of ſets; and the longitude of the ſhip is to be inferred from thence, in the ſame manner as from a ſingle ſet, but much more to be depended on.

To illuſtrate the above, the following obſervations, taken in the manner deſcribed, are ſubjoined.

☽ March 2d, 1789, Height of the eye 18 feet.

	Times p. watch.	Diſt. ⊙ and ☽'s neareſt limbs	Alt. ☽'s up. limb	Alt. ⊙'s low. limb
	23h. 8′ 31″	71° 50′ 10″	19° 5′	24° 56′
	11 18	52 20	19 26	25 1
	13 15	53 0	19 45	25 5
	15 1	53 50	20 2	25 9
	16 36	54 30	20 15	25 13
Sum	14 41	14 50	213	24
Mean	23 12 56	71 52 58	19 42.6	25 4.8

The aſtronomical method of reckoning the time has been followed, in order to make it agree with the Nautical Almanac. The mean of the above times reckoned nautically would be, March 3, 11 h. 12′ 56″, A. M. and if the above mean had been March 3d, 1 h. 13′, then by the nautical account it would be, March 4th, 1 h. 13′, P. M. Hence, in the preſent caſe, the aſtronomical method of reckoning time ſeems preferable to the nautical method.

An expert obſerver may, in caſe of neceſſity, diminiſh the number of his aſſiſtants, and might even be capable of making all the neceſſary obſervations himſelf, with a tolerable degree of accuracy, as follows:

Let two or three altitudes of the limbs of the Sun and Moon be taken, and write down the time of each; then obſerve five or ſix diſtances between the limbs of theſe objects, and laſtly, ſeveral altitudes of each are to be again obſerved. Now find the mean of each particular ſet of obſervations, and reduce the altitudes to the time of the mean diſtance, which may be done by even proportion, the motion in altitude being ſuppoſed uniform during the interval between the obſervations; and hence a complete ſet will be obtained.

The following obſervations being given, it is required to reduce the altitudes to the time of the mean diſtance.

Times p. watch,	o h. 24′ 31″	Alt. ☉'s l. limb =	54° 6′	Means, o h. 25′ 37″—54° 5′
	25 41	— — — —	54 5	
	26 40	— — — —	54 4	
	27 50	Alt. ☽'s u limb	20 2	—o h. 28′ 47″—20° 3.3
	28 44	— — — —	20 3	
	29 46	— — — —	20 5	
	31 10	Diſt. neareſt limbs	73 13	—o h. 33′ 43″—73° 13′ 57″
	32 50	— — — —	73 13½	
	33 30	— — — —	73 14	
	35 0	— — — —	73 14¼	
	36 40	— — — —	73 14½	
	37 15	Alt. ☉'s l. limb	53 54	—o h. 38′ 24″—53° 52′.7
	38 20	— — — —	53 53	
	39 38	— — — —	53 51	
	41 12	Alt. ☽'s up. limb	20 57	—o h. 42′ 14″—21° 4′.3
	42 4	— — — —	21 4	
	42 36	— — — —	21 12	

o h. 25′ 37″ — — — o h. 25′ 37″——54° 5′
o h. 38 24 — — — o h. 33 43——53 52.7

12 47 : 8 6 :: 12.3 : 8′
Sun's altitude at o h. 25′ 37′ — — — 54 5

[*]Reduced altitude of Sun's lower limb — 53 57

o h. 28′ 47″ — o h. 28′ 47″ — 20° 3′.3
o h. 42 14 — o h. 33 43 — 21 4.3

13 27 : 4 56 :: 1 1 : 22′.3
Moon's altitude at oh. 28′ 47″ — — — 20 3.3

Reduced altitude of the Moon's upper limb — 20 25.6

We have now obtained the following ſet of obſervations.

Times p. watch.	Diſt. neareſt limbs of ☉ & ☽	Alt. ☉'s l. limb	Alt. ☽'s up. limb.
o h. 33′ 43″	73° 13′ 57″	53° 57′	20° 25′.6

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[*] When the object is near the meridian, as the Sun in the preſent example, the change of altitude is not proportionable to the time. However, as theſe altitudes are to be uſed only in clearing the apparent diſtance, and becauſe no material error can ariſe in the diſtance, from an error of a few minutes in the altitude—therefore the rule of proportion will be ſtill found ſufficiently accurate for the above purpoſe, when the interval of time is ſhort. The change of altitude may be more accurately computed by the rule given in the text.

One ſet of altitudes of the Sun and Moon would be ſufficient, provided the azimuths of the objects be known,* as then the change of altitude in the interval between the obſervations of the altitude and diſtance may be found by computation as follows:

To the conſtant logarithm 8,8239, add the log. ſecant of the latitude, the log. co-ſecant of the azimuth, and the proportional logarithm of the interval of time between the obſervations of the altitude and diſtance; the ſum, rejecting radius, will be the proportional logarithm of the correſponding change of altitude.

If the Sun is near the meridian, at the time when the diſtance is obſerved, the apparent time cannot be deduced from thence with a ſufficient degree of accuracy; if, therefore, the error of the watch is not known, altitudes of that object, when at a proper diſtance from the meridian, ſhould have been taken before the obſervation of the diſtance, or muſt be taken afterwards, for the expreſs purpoſe of aſcertaining its error. The apparent time might, indeed, be inferred from the Moon's altitude—but for ſeveral reaſons, it is not ſo much to be depended on, as that deduced from the Sun's altitude.

If, in places where the Sun riſes and ſets daily, the obſervations are taken any time between half an hour after ſun riſe and ten o'-clock in the forenoon, or between two in the afternoon, and half an hour before ſun ſet—the apparent time may be inferred from the Sun's altitude. In this caſe, a watch is not abſolutely neceſſary.

A ſet of obſervations may be taken with accuracy during the time of the evening or morning twilight; and the obſerver, though not much acquainted with the ſtars, will not find it difficult to diſtinguiſh the ſtar with which the Moon is to be compared; for the time of obſervation, and the ſhip's longitude by account, being known, the eſtimate time at Greenwich may be found; and by entering the Nautical Almanac with the reduced time, the diſtance between the Moon and given ſtar will be found nearly. Now ſet the index of the ſextant to this diſtance, and hold the plane of the inſtrument ſo, as to be nearly at right angles to the line joining the Moon's cuſps; direct the ſight to the Moon, and, by giving the ſextant a ſlow vibratory motion, the axis of which is that of viſion, the ſtar, which is uſually one of the brighteſt in that part of the heavens, will be ſeen in the ſilvered part of the horizon glaſs.

CHAP.

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* Sine of azimuth = Secant altitude. Coſine latitude. Sine diſt. object from meridian, to radius 1.

C H A P. V.

Of the Corrections to be applied to the Altitude of an Object obſerved at Sea, and to the obſerved Diſtance between two Objects.

THE altitude of the limb of a celeſtial body, obſerved at ſea, requires four ſeparate corrections, independent of thoſe ariſing from the errors of the inſtrument, in order to obtain the true altitude of its center. Theſe are on account of the Semidiameter, Depreſſion of the Horizon, Refraction, and Parallax. The ſemidiameter and parallax however vaniſh, when the obſerved object is a fixed ſtar. The diſtance between the limbs of the Sun and Moon, or between a ſtar and the Moon's limb, is affected by refraction and parallax; but the diſtance between two ſtars is affected by refraction only.

Of the SEMIDIAMETERS of the SUN, MOON, and PLANETS.

The ſemidiameter of the Sun is given in the Nautical Almanac, page III. of the month for the noon of every ſixth day, and that of the Moon in page VII. to the time of noon and midnight, by the meridian of Greenwich. Theſe ſemidiameters may be reduced to any intermediate time, and to any other meridian, by proportion.

The Moon's ſemidiameter, as given in the Ephemeris, is the angle under which it would be ſeen when in the horizon, or from the center of the earth; but the apparent magnitude of an object is inverſely as its diſtance from the obſerver, and ſince the Moon is nearer an obſerver, by a complete ſemidiameter of the earth, when in the zenith, than it is when in the horizon; and becauſe the ſemidiameter of the Earth has a ſenſible ratio to the diſtance of the Moon from its center, the ſemidiameter of the Moon will be apparently increaſed by a quantity equal to $\frac{ſ^2}{15^\circ\ 36'\ 45''-ſ}$, wherein $ſ$ is the horizontal ſemidiameter of the Moon. This quantity is called the greateſt *augmentation;* and the augmentation at any given altitude will be equal to the ſine of that altitude, the radius being the greateſt augmentation. Table XXX. contains the augmentation of the Moon's ſemidiameter.

The diſtance of the Sun from the Earth being immenſe, when compared with the Earth's radius, the augmentation of the Sun's ſemidiameter is therefore inſenſible.

When the altitude of the lower limb of the Sun or Moon is obſerved, its ſemidiameter muſt be added thereto; but if the upper limb be obſerved, the ſemidiameter muſt be ſubtracted, in order to obtain the central altitude. The contrary rule is to be applied to altitudes taken by the back obſervation. In altitudes of the Sun, taken for the purpoſe of aſcertaining the latitude, and when no great preciſion is required, 16 minutes may be allowed. If a greater degree of accuracy is ſought, the ſemidiameter may be taken from the Nautical Almanac to the neareſt tenth of a minute. But when altitudes are obſerved at land with a good inſtrument, the ſemidiameter is to be taken out to the neareſt tenth of a ſecond.

If the diſtance between the neareſt limbs of the Sun and Moon is obſerved, their apparent ſemidiameters muſt be added thereto, in order to obtain the central diſtance. If the neareſt limb of the one of theſe objects is compared with the remote limb of the other, the apparent ſemidiameter of the firſt muſt be added to, and that of the ſecond ſubtracted from the obſerved diſtance. If the diſtance between the Moon and a fixed ſtar be obſerved, the Moon's augmented ſemidiameter muſt be added to, or ſubtracted from the obſerved diſtance, according as the neareſt or remote limb of the Moon is compared with the ſtar.

The apparent ſemidiameter of a planet is variable. This variation depends on the diſtance of the planet from the Earth; and ſince the apparent ſemidiameter of any of the planets never exceeds half a minute, the altitude of the center may therefore be obſerved with ſufficient accuracy. A fixed ſtar has no ſenſible magnitude, when viewed with a very powerful teleſcope.

Of the DEPRESSION of the HORIZON.

The Depreſſion, or as it is commonly called, the *dip of the horizon*, is the angle contained between the ſenſible and apparent horizons, the angular point being the eye of the obſerver.

Since the altitude of any object, obſerved at ſea, is meaſured from the apparent horizon, and as this horizon is below the ſenſible, by a quantity depending on the height of the eye, the altitude of a celeſtial object, as taken by the fore obſervation, is therefore greater, but leſs when the back obſervation is uſed, by a quantity equal to the angle contained between theſe horizons.

ILLUSTRATION.

Let ABD, (fig. 21.) be a ſection of the earth, whoſe plane produced paſſes thro' the obſerver and object, and let BE be the height of the obſerver's eye above the earth's ſurface; hence FEG will be the ſenſible horizon, EHI the apparent horizon, and therefore the angle FEI the depreſſion of the apparent, below the ſenſible horizon. Let O be any object whoſe altitude is to be obſerved, which is done by bringing its image in contact with the ray from the eye to the apparent horizon produced; and therefore the angle OEI is the obſerved altitude, being greater than the angle OEF, the altitude unaffected by dip, by the angle FEI. In the back obſervation, the obſerved

alti-

altitude is OEK, to which the angle HEF = GEL muſt be added, to obtain the altitude above the ſenſible horizon FE.

The dip may be computed by the following rule. To the conſtant logarithm 0,4236, add the proportional logarithm of the height of the eye above the ſea, in feet; half the ſum will be the proportional logarithm of the dip of the horizon.

EXAMPLE.

Required the dip of the horizon, anſwering to an elevation of 100 *feet above the ſurface of the ſea?*

Conſtant logarithm — —	0,4236
Given height, 100 feet, = 1′ 40″ P. L.	2,0334
Sum	2,4570
Dip of the horizon, 10′ 38″ P. L.	1,2285

The dip is affected by Terreſtrial Refraction, which, according to Dr Maſkelyne, amounts to about one tenth of the whole; Bouguer and Lambert ſuppoſe it to be $\frac{1}{9}$ and $\frac{1}{14}$ reſpectively—but, according to General Roy,* it varies from $\frac{1}{3}$ to $\frac{1}{24}$ of the comprehended arc.

Since the figure of the Earth is that of an oblate ſpheroid, the radius of curvature is therefore variable with the latitude; it hence follows, that ſpeaking ſtrictly, no ſingle table of dip can anſwer in all places. Tables of dip ſhould be conſtructed, ſo as to anſwer to the latitude of the place, and azimuth of the obſerved object.—It, however, may be obſerved, that the difference of dip ariſing from the above cauſe is ſo inconſiderable as to have been hitherto neglected.

OF REFRACTION.

The refraction of any celeſtial body is the difference between its apparent place, and that wherein it would be ſeen, if the ſpace between the ſpectator and object was either a void, or of an uniform denſity.

The courſe of a ray of light from any luminous body is rectilineal, while it moves thro' a medium † of an uniform denſity. If a medium of a different denſity be interpoſed, the ray will be bent from its original courſe at the junction of the two media, and aſſume a new rectilineal direction, provided this laſt medium be of an equal denſity throughout; but if the medium be of a different denſity, and ſo, as to become either more denſe or continually rarer, the ray will deſcribe a curve. Now the denſity of the atmoſphere increaſes, as it approaches the Earth's ſurface; hence a ray from any celeſtial body will deſcribe a curve, in paſſing thro' the atmoſphere to the eye of an obſerver; and the angle contained between the tangent to this curve at the eye, and a ſtraight line from the eye to the object is called the Refraction.

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* Philoſophical Tranſactions for 1790, Part I.

† Any tranſparent ſubſtance, as the atmoſphere, water, glaſs, &c. is called a medium.

ILLUSTRATION.

Let the circle ABC (fig. 22.) repreſent a ſection of the earth, DEF the ſurrounding atmoſphere, S the apparent place of a ſtar, and R the point wherein it would be ſeen by an obſerver at A, if the interveening ſpace was a void, or of an uniform denſity. Now let the ray RD be incident on the ſurface of the atmoſphere at D, which, therefore, inſtead of proceeding in a rectilineal direction to E, will be refracted to A in the direction of the curve DA, becauſe the denſity of the atmoſphere increaſes, as it approaches the earth's ſurface; hence an obſerver at A will ſee the object at S, the line AS being a tangent to the curve at A. The arch RS is called the Refraction of the object in altitude, and is evidently to be ſubtracted from the obſerved altitude, to obtain the true.

Phyſical Cauſe of Refraction.

LEMMA 1ſt. A ray of light from any luminous object moves in a rectilineal direction, in a medinm of an uniform denſity.

2d. A ray, paſſing out of one medium into another of a different denſity, will be bent out of its rectilineal direction, at the junction of the media; and if the medium from which the ray proceeds be rarer than that into which it enters, it will be attracted towards the perpendicular at the point of incidence. This is evident, from the courſe of a ray out of air into water, &c. For the proof of theſe, the writers on Optics may be conſulted.

It is demonſtrable, that every body is endowed with an attractive power, which reaches to ſome diſtance beyond its ſurface, as that of Coheſion, Magnetiſm, &c. Now a ray of light from any of the heavenly bodies will, at its entrance into the terreſtrial atmoſphere, be attracted towards the denſer parts; and ſince the denſity of the atmoſphere increaſes, the nearer the Earth's ſurface, therefore the ray, as it approaches the obſerver, will be more and more attracted, its velocity accelerated, and of courſe its rectilineal direction changed. Hence that portion of the ray contained between an obſerver and the extremity of the atmoſphere, will be a curve, except in that caſe, when the ray is perpendicular to the refracting medium.

Several eminent mathematicians, particularly Meſſrs Bernoulli, Bouguer, Euler, Mayer, Sejour, Simpſon, &c. have inveſtigated the law of this curve, and have given rules for computing the refraction correſponding to any given altitude. We are however indebted to the celebrated Dr Bradley for the following rule,* which he

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* Explanation and Uſe of Dr Maſkelyne's Aſtronomical Tables, page v. Requiſite Tables, 1ſt edit. p. 129.

he deduced from his obſervations, namely, *That the refraction at any altitude is to 57 ſeconds, in the direct ratio of the tangent of the apparent zenith diſtance, leſſened by three times the eſtimate refraction, to the radius.* If a greater degree of accuracy is required, the computation muſt be repeated, uſing the refraction thus found, in place of that by eſtimation, or taken from tables.

EXAMPLE.

Required the mean refraction † anſwering to 10° *of apparent altitude, the corresponding refraction by Caſſini's tables being* 5′ 28″*?*

Apparent altitude, — —	10° 0′ 0″		
Thrice given refraction =	16 24	Conſtant log.	2.2775
Sum	10 16 24	Tangent —	9.2583
Approximate refraction — —	5 14	P. Log. —	1.5358
	3		
Thrice approx. refraction —	15 42		
Apparent altitude,	10 0 0	Conſtant log.	2.2775
Sum	10 15 42	Tangent —	9.2578
Mean refraction, — —	5 15	P. Log. —	1.5353

If the denſity of the atmoſphere remained conſtantly the ſame, it might be naturally concluded, that the refraction anſwering to any given altitude would alſo continue invariable—but this is by no means the caſe; for it is found by obſervation, that the denſity of the atmoſphere is variable, and according to the experiments of Mr Haukſbee and others, the refractive power is proportionable to its denſity; but the denſity of the atmoſphere is directly as its compreſſing force, and inverſly as its heat—and ſince the weight and heat of the

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† The mean refraction anſwering to any given apparent altitude, may be computed by the following rule, deduced from a formula given by M. du Sejour, in the firſt vol. of his Traite Analytique des Mouvemens apparens des Corps Celeſtes, p. 655.

To the conſtant log. 9.9992828 add the log coſine of an arch; the ſum, rejecting radius, will be the log. One ſixth of the difference between this arch and the apparent altitude, will be the refraction.

EXAMPLE.—Let the apparent altitude be 10°; required the correſponding refraction?

Conſtant logarithm — — — — — — — —			9.9992828
Apparent altitude	10° 0′ 0″ — — —	Coſine	9.9933515
Arch —	10 31 22 — —— —	Coſine	9.9926343
	31 22		

$$\frac{31\ 22}{6} = 5'\ 13'',\ 7 = \text{Mean Refraction.}$$

the atmoſphere are ſhewn by the barometer and thermometer reſpectively, it hence follows, that the mean refraction may be reduced to the true, by allowing for the difference between the actual and mean heights of the Mercury in thoſe inſtruments.

The refractive power of the atmoſphere is as its weight, while its heat remains the ſame; hence if a table of refraction be adapted to the mean heat of the atmoſphere, as 55°, and to a given altitude of the barometer, as 30 inches, then the mean refraction will be to the true, as 30 inches to the obſerved height of the barometer,* the thermometer being at 55°.

The change of refraction, ariſing from a given change in the heat of the atmoſphere, is determined by obſervation. M. de la Caille makes the change of refraction, anſwering to a change of 10° in Reaumur's thermometer, to be $\frac{5}{27}$ of the whole; M. Mayer makes this change to be $\frac{1}{31}$. According to Dr Bradley, *The true refraction is to the mean, in a direct ratio of the altitude of the barometer to 29.6 inches; and in an inverſe ratio of the altitude of the thermometer increaſed by* 350 *to the number* 400. Agreeable to this rule, table viii. was computed; from which the corrections of refraction, anſwering to a given altitude of the barometer and thermometer are found by inſpection.

Since the altitudes of the barometer and thermometer, are obſerved at one extremity only of the curve deſcribed by the ray in its paſſage through the atmoſphere; it is hence evident, that theſe corrections ſerve only to approximate to the true refraction.

OF PARALLAX.

That part of the heavens in which a planet would appear, if viewed from the ſurface of the earth, is called its *Apparent Place:* and the point wherein it would be ſeen at the ſame inſtant from the earth's center, is called its *True Place*. The difference between the true and apparent places, is called the ***Parallax in Altitude.***

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* The mean refraction is adapted to an altitude of 30 inches of the barometer, and 55° of Fahrenheit's thermometer: If, therefore, the barometer be at the ſtandard, but the thermometer higher than 55°, it is evident, the mercury in the barometer will be expanded, and conſequently, the obſerved height will exceed the true; otherwiſe it will be leſs than the true height. Again, ſince the expanſion of the mercury is as its height; hence, the higher the barometer, the greater is the expanſion anſwering to a given change in the thermometer. It therefore becomes neceſſary to correct the obſerved altitude of the barometer.

It is found, that the expanſion of the mercury in the barometer, anſwering to a change of 1° in the thermometer, is about ,00304 of an inch. Now let the difference between the height of the thermometer and 55° be called a, and the obſerved height of the barometer b.

Then $1° : a° :: ,00304 : ,00304a$
And $30 : b :: ,00304a : ,000101ab$ = correction.

Hence, if a cypher be prefixed to the obſerved height of the barometer, and the whole conſidered as a decimal, then the product of this quantity by the decimal ,101 and by the difference between the height of the thermometer and 55°, will be the correction; which is additive, when the thermometer is above 55°, otherwiſe, ſubtractive.

ILLUSTRATION.

Let C, (fig. 23.) repreſent the center of the earth, A the place of an obſerver on its ſurface, whoſe ſenſible horizon is AE, true horizon CF, and zenith Z, alſo let ZKEF be a great circle in the heavens, and GHD the apparent diurnal path of a planet, ariſing from the rotation of the earth. Now let the planet be in any point H of its diurnal path, then will I be its apparent place referred to in the heavens, and K its true place; the difference between theſe or the arch KI is called the parallax in altitude. If the planet be in the zenith, its true and apparent places coincide, and therefore the parallax vaniſhes; but if the planet be in the horizon at D, then the arch LE, which is the meaſure of the angle LDE = ADC, will be the horizontal parallax. Hence alſo, the horizontal parallax of a planet is the angle which the Earth's ſemidiameter, when viewed directly, ſubtends at that planet; and the parallax in altitude is the apparent angular diſtance of the obſerver from the center of the Earth, as ſeen from the planet's center.

The more elevated a planet is above the horizon, the leſs is the parallax, its diſtance from the Earth's center continuing the ſame. When the planet is in the zenith, it has no parallax; but when in the horizon, its diurnal parallax is greateſt.

Since the apparent place of a planet is more diſtant from the zenith than its true place, it is therefore evident, that the parallax in altitude muſt be added to the obſerved altitude, in order to obtain the place of the planet, as ſeen from the Earth's center.

The horizontal parallax being given, the parallax correſponding to any given altitude may be found by the following rule. To the log. ſecant of the apparent altitude, add the proportional logarithm of the horizontal parallax; the ſum, rejecting radius, will be the proportional logarithm of the parallax in altitude.

EXAMPLE.

Let the apparent altitude of the Moon's center be 23° 32', *and horizontal parallax* 58' 46". *Required the parallax in altitude?*

Apparent altitude,	23° 32' —	Secant,	0,0377
Horizontal parallax,	— 58' 46"	P. Log.	0,4861
Parallax in altitude,	— 53 53	P. Log.	0,5238

Upon account of the revolution of the Earth about its axis, its figure is that of an oblate ſpheroid; the equatorial radius is therefore the greateſt, the polar ſemi-axis is the leaſt, and the other radii are of

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of an intermediate length, depending on the latitude of the place; and ſince the Moon's horizontal parallax is the angle under which the Earth's ſemidiameter appears at the Moon, it hence follows, that the horizontal parallax of the Moon will vary with the latitude, being greateſt at the equator, and leaſt at the poles; but the horizontal parallax of the Moon, as given in the Nautical Almanac, is that anſwering to the equatorial radius, and therefore, in order to reduce it to any given latitude, it muſt be diminiſhed by a certain quantity. Table XXXII. contains this quantity, agreeable to Sir Iſaac Newton's hypotheſis.

Summary of the Corrections.

When the altitude of the lower limb of any object is obſerved. its ſemidiameter is to be added thereto, in order to obtain the central altitude; but if the upper limb be obſerved, the ſemidiameter is to be ſubtracted. If the altitude be taken by the back obſervation, the contrary rule is to be applied.

The dip is to be ſubtracted from, or added to, the obſerved altitude, according as the fore or back obſervation is uſed.

The refraction is always to be ſubtracted from, and the parallax added to, the obſerved altitude.

If the diſtance between the neareſt limbs of any two objects be obſerved, that diſtance is to be increaſed by the ſum of their ſemidiameters; but if the remote limbs be obſerved, the diſtance is to be diminiſhed by the above ſum. If the Moon's enlightened limb be compared with the neareſt and remote limbs of the Sun alternately, half the ſum of theſe diſtances will be the diſtance between the Sun's center and the Moon's enlightened limb. The diſtance between a ſtar and the Moon's neareſt limb is to be increaſed by the Moon's apparent ſemidiameter; and the diſtance between the Moon's remote limb and a ſtar is to be diminiſhed by the Moon's ſemidiameter. If, at the time of full Moon, both limbs of the Moon be compared with a ſtar, half the ſum of theſe diſtances will be the apparent central diſtance between theſe objects. In like manner, if the diſtance between the Moon's enlightened limb, and the neareſt and remote limbs of a planet, apparently full, be obſerved—then half the ſum of theſe diſtances will be the apparent diſtance, between the Moon's enlightened limb and the center of the planet. If the planet be not full, and the diſtance between the enlightened limbs obſerved, allowance is to be made for the planet's ſemidiameter: this may, however, be avoided, by obſerving the diſtance between the Moon's limb and the planet's center.

B O O K III.

CONTAINING,

The Method of finding the Longitude at Sea or Land by Lunar Obſervations.

C H A P. I.

INTRODUCTION.

IF the latitude and longitude of a ſhip at ſea were accurately known, the courſe and diſtance therefrom to any given port, might be eaſily deduced by charts or otherwiſe. The latitude is aſcertained from obſervation; but the common method of deducing, the ſhip's longitude, from the courſe and diſtance made good, is at beſt only an approximation to the truth, and, at the end of the voyage, the accumulated error is ſometimes very conſiderable; having, in a run from Britain to the Weſt Indies, been found to exceed eight or ten degrees. It has therefore been the wiſh of the practical navigator, and of every maritime nation, to have ſome certain method of aſcertaining the longitude of a ſhip at ſea. In order to obtain ſo uſeful a diſcovery, ſeveral very conſiderable rewards have been offered to any perſon, who would give a method, by which the ſhip's longitude might be determined within proper limits, as often as neceſſary.

The firſt who offered a reward for the diſcovery of the longitude at ſea, was Philip III. of Spain in the pear 1598, and ſoon after, the States of Holland followed his example. The reward offered by Philip was 1600 crowns, and that by the States 10,000 florins. In the year 1714, the Britiſh Parliament offered a premium of 20,000l. and in 1716, a reward of 100,000 livres was promiſed by the Duke of Orleans, who was then Regent of France. In conſequence of which, many and various methods have been propoſed to ſolve this important problem; of theſe which have hitherto appeared, that by obſerving the diſtance between the Moon and the Sun or a fixed ſtar ſeems to be the moſt proper for this purpoſe, both on account of the frequency of the obſervations, and of the ſhortneſs of the calculations.

John Werner of Nuremberg appears to be the firſt who propoſed the method of finding the longitude, by obſerving the diſtance between the Moon and a ſtar, in his annotations on the firſt book of Ptolemy's Geography, printed 1514. He recommends the croſs ſtaff as a very proper inſtrument for the purpoſe of obſerving the diſtance between theſe objects.

This method was again recommended by Gemma Friſius, in his treatiſe, intitled, *Structura radii aſtronomici et geometrici*, printed at Antwerp in 1545, which uſually accompanies his edition of Apian's coſmography. At folio 39th, he expreſſes himſelf as follows, "*Sed quotidie fere, ſi quis velit longitudinem loci alicujus perquirere, is diligenter conſideret lunæ diſtantiam ab aliquo ſidere, ſydere firmamento per radium noſtrum. Ita tamen ut illa ſtella fixa ſecundum rectum eclipticæ ductum, lunam, præcedit aut ſequatur.* He then proceeds to ſhow how the Moon's longitude is found from the above obſervation, and directs the apparent time of obſervation to be inferred from the altitude of the ſtar obſerved at the ſame inſtant; the Moon's longitude at this time is alſo to be computed from the beſt aſtronomical tables. Hence the difference between the obſerved and computed longitudes of the Moon will be known; with which, and the Moon's horary motion, the difference of longitude, between the place of obſervation and that to which the tables are adapted, may be found.

The celebrated Kepler was fully perſuaded of the utility of this method of finding the longitude at ſea. In his Rudolphine tables, he gave directions for obſerving the diſtance between the Moon and a ſtar, and for making the neceſſary computations, nearly the ſame as thoſe given by Gemma Friſius. This method was alſo recommended by Longomontanus.

Mr Blundevil deſcribes this method of aſcertaining the longitude at ſea, in his Exerciſes, printed at London, the invention of which he attributes to Apian. It is alſo taken notice of by Carpenter in his Geography, printed at Oxford in the year 1635, wherein he ſays, *This way was taught by Apian, illuſtrated by Gemma Friſius and Blundevil.* He then proceeds to deſcribe the manner of obſerving the diſtance, from whence the longitude of the place of obſervation is to be inferred as formerly. He concludes as follows: *This way, though more difficult, may ſeeme better than all the reſt, for as much as an eclipſe of the Moon ſeldom happens, and a watch, clocke, or hour glaſſe cannot ſo well be preſerved, or at leaſt ſo well obſerved in ſo long a voyage, whereas every night may ſeem to give occaſion to this experiment, if ſo bee the ayre be freed from clouds, and the Moone ſhew her face above the horizon.*

M. Jean Baptiſte Morin, profeſſor of mathematics at Paris, improved the above method; which, by order of Cardinal Richlieu, was examined in 1634. It was, however, judged to be incomplete, upon account of the imperfections of the lunar tables. Morin therefore wrote againſt thoſe appointed to examine his method, particularly M. Pierre Herigone, who had propoſed ſeveral methods of finding the longitude, in the fourth volume of his *Courſe of Mathematics;* which methods Morin endeavoured to confute. An anſwer was given thereto by Herigone, at the end of the fifth volume of the above work, printed at Paris in 1637.— Although Morin did not obtain the reward he claimed for his improvement of the above method, yet, in 1645, Cardinal Mazarin procured him a penſion of 2000 livres.

A method for the above purpoſe was now much deſired in England. The Royal Obſervatory at Greenwich was founded by Charles II. in the year 1675, and Mr Flamſteed appointed Aſtronomer Royal. The words of his commiſſion were, *To apply himſelf with the utmoſt care and diligence to the rectifying the tables of the motions of the heavens, and the places of the fixed ſtars, in order to find out the ſo much deſired longitude at ſea, for perfecting the art of navigation.*

Among all the celeſtial objects, the Moon, upon account of the quickneſs of its motion, appeared to be the beſt adapted for the above purpoſe. Any method, however, depending on obſervations of the Moon, which were to be compared with lunar tables, was very uncertain, as long as thoſe tables remained ſo inaccurate. *The longitude*, ſays Mr Flamſteed*, *might be alſo attained by obſervations of the Moon, if we had tables that would anſwer her motions exactly; but after* 2000 *years, we find the beſt tables extant, erring ſometimes* 12 *minutes or more in her apparent place, which would cauſe a fault of half an hour, or* 7½ *degrees in the longitude deduced, by comparing her place in the heavens with that given by the tables.*

That juſtly celebrated aſtronomer and navigator, Dr. Edmund Halley, recommended obſervations of the Moon, as the moſt certain method of aſcertaining the longitude at ſea, having, by his own experience, found the impractibility of all the other methods propoſed for that purpoſe. He gave an excellent paper on that ſubject, in the Philoſophical Tranſactions, No. 420, wherein he ſhews the defects of the lunar tables extant. By comparing the place of the Moon, as given by the tables, with that deduced from obſervation, he found the errors of the tables recur with great regularity, at the end of 18 years 11 days; ſo that whatever error was found in a former period, the ſame error was again repeated, under the like circumſtances, of the ſame diſtance of the Moon from the Sun and apogeon. Being encouraged by this, he next examined what difference might ariſe, from the period of nine years wanting nine days; in which time there are performed very nearly, one hundred and eleven lunations; but the return of the Sun to the apogee in that time, differing above four times as much from an exact revolution, as in the period of eighteen years, a like agreement was not to be expected. Having, however, entered upon the 10th year of his obſervations of the Moon's tranſit, he compared his late obſervations of 1730 and 1731, with thoſe he had made in 1721 and 1722, and very ſeldom found a difference of more than one ſingle minute of motion, but moſt commonly this difference was wholly inſenſible; ſo that, by the help of what he obſerved in 1722, he preſumed that he was able to compute the true place of the Moon with certainty within two minutes of motion, during the year 1731, and ſo on for the future. He finiſhes this paper, by recommending Hadley's quadrant as a proper inſtrument for taking the neceſſary obſervations at ſea. This inſtrument had been deſcribed in the preceding number of the Tranſactions of the Royal Society.

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* Phil. Tranſactions, Lowthorp's abridgement, vol. 1. page 537.

Dr Halley again treats of this ſubject in his Aſtronomical Tables, in which are given two complete examples of finding the longitude, from the obſerved diſtance between the Moon and the ſtars, ν Leonis and ζ Tauri, and concludes with obſerving, that *by a like method of computation, may the difference of the meridians be found from obſervations of the Moon's diſtance from the Sun, in her firſt and laſt quarter.*

M. Emanuel Swedenborg takes notice of the above method of finding the longitude, in his *Dædalus Hyperboreus*, printed in Swediſh, at Stockholm, in the year 1716—and other tracts. It is alſo briefly explained in a very ſmall pamphlet, intitled, *An introduction to a true method for the diſcovery of longitude at ſea, by Stephen Plank*, printed at London in 1720.

The Abbé de la Caille recommends it as the only practical method at ſea. He was, however, ſenſible, from his own experience, of the errors to which it was then liable. In his edition of Bouguer's Navigation, printed at Paris in 1781, he ſays, "La grande incertitude à laquelle nous avons dit qu'étoit ſujette la méthode d'employer les obſervations de la Lune faites ſur mer, ne doit pas décourager le marin, ni la lui rendre ſuſpecte, puiſque dans les voyages de long cours, où l'on a eſſuyé beaucoup de vents contraires, & de long coups de vents, il arrive ſouvent qu'aux atterrages on ſe trouve en erreur de ſept ou huit degrés ſur la longitude eſtimée ſelon les regles du pilotage." In the ſame work, he gave a new and eaſy method of reducing the apparent to the true diſtance, by means of ſcales conſtructed expreſsly for that purpoſe.

In 1755, Profeſſor Mayer of Gottingen ſent a manuſcript copy of his lunar tables* to the Britiſh Admiralty, claiming at the ſame time ſome one of the rewards, promiſed by Parliament, which he might be thought to merit. In theſe tables, the arguments are inveſtigated on the Newtonian principle of univerſal gravitation, and the maxima of the equations are deduced from his own obſervations, and from thoſe of Dr Bradley and others. The above tables were delivered to Dr Bradley, to be examined; who, having compared them with a great number of his own obſervations, was convinced of their excellence. He then ſent an account of them to Mr Cleveland, ſecretary of the Admiralty, in a letter dated Greenwich, Feb. 10, 1765, from which the following is extracted.

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* Mayer's tables firſt appeared in the Memoirs of Gottingen for 1742.

" In obedience to their Lordſhips commands, I have examined " the ſame, and carefully compared ſeveral obſervations that have " been made (during the laſt five years) at the Royal Obſervatory " at Greenwich, with the places of the Moon computed by the " ſaid tables. In more than 230 compariſons which I have already " made, I did not find any difference ſo great as 1'½ between the " obſerved longitude of the Moon and that which I computed by " the tables; and although the greateſt difference which occurred " is, in fact, but a ſmall quantity, yet as it ought to be conſidered " as partly ariſing from the error of the obſervations, and partly " from the error of the tables, it ſeems probable, that, during this " interval of time, the tables generally gave the Moon's place true " within one minute of a degree.

" A more general compariſon may perhaps diſcover larger errors; " but thoſe which I have hitherto met with being ſo ſmall, " that even the biggeſt could occaſion an error of but little more " than half a degree in longitude, it may be hoped that the tables " of the Moon's motions are exact enough for the purpoſe of finding " at ſea the longitude of a ſhip, provided that the obſervations " that are neceſſary to be made on ſhipboard can be taken with ſufficient " exactneſs."

In another letter to Mr Cleveland, dated Greenwich, April 14, 1760, Dr Bradley ſhows the utility of theſe tables, from the compariſon of obſervations made at ſea by Capt. Campbell, in the years 1757, 1758, and 1759. By the mean of twelve different days or nights obſervations, taken at ſea, and reduced to Uſhant, he found the longitude of the middle of that iſland to be about 5° 23' weſt of Greenwich, and the greateſt difference between the mean reſult, and that of any particular day, amounted in one caſe only to 37 minutes in defect, and the greateſt difference in exceſs was but 23 minutes. He alſo obſerves, that ſince his former account of the near agreement of Profeſſor Mayer's lunar tables with the obſervations that had been made at the Royal Obſervatory, he had compared ſeveral others, which concurred to prove, that the difference between the obſerved and computed places no where amounted to more than one minute and an half; and he found that the difference, ſmall as it is, may yet be diminiſhed by making alterations in ſome of the equations, whoſe true quantity could not be determined without proper obſervations. After making the neceſſary corrections, it appeared, by the compariſon of above eleven hundred obſervations taken ſince the new inſtruments were fixed up, that the difference did no where amount to more than one minute. It may therefore be reaſonably concluded, that ſo far as it depends upon the lunar tables, the true longitude of a ſhip at ſea may in all caſes be found within half a degree, and generally much nearer.

Profeſſor Mayer died in the beginning of the year 1762, and left a ſtill more complete and accurate ſet of lunar tables. Theſe were ſoon after tranſmitted to the Board of Longitude, by his widow, for which ſhe received 3000l. Mr Euler alſo received a reward of 300l. in conſideration of Mayer having availed himſelf of Euler's lunar theory. Theſe tables, together with a ſet of Mayer's ſolar tables, were printed at London in 1770, under the inſpection of Dr Maſkelyne; from which, all the articles in the Nautical Almanac relative to the Sun and Moon are computed. An edition of the lunar tables, improved by Mr Maſon from Dr Bradley's ſeries of lunar obſervations, publiſhed in the Nautical Almanac for 1774, was printed at London in 1787, and reprinted at Paris in the *Connoiſſance des Temps* for 1790. "Theſe new tables," ſays Dr Maſkelyne, "when compared with the abovementioned ſeries of obſervations, ſeem to give always the Moon's longitude in the heavens, correctly within 30 ſeconds of a degree."

Our preſent aſtronomer royal, the Rev. Dr Nevil Maſkelyne, very much deſerves the thanks of every navigator, and of the public at large, for his great induſtry and exertions in bringing this method of aſcertaining the longitude at ſea into common practice. In his *Britiſh Mariners Guide*, printed at London in 1763, he very ſtrongly recommends it, and gives new precepts for making the neceſſary obſervations and computations. The rules for computing the effects of refraction and parallax, which he had formerly communicated to the Royal Society, and which were publiſhed in the Britiſh Mariners Guide, were again communicated to that Society with the demonſtrations, in 1764, and publiſhed in the LIV. vol. of the Phil. Tran. On the 9th of February 1765, he preſented a memorial to the Commiſſioners of Longitude, in which he ſhows, that in his voyage to St Helena, and return thence, he made frequent obſervations of the diſtance of the Moon from the Sun and fixed ſtars, with Hadley's Quadrant, from which, by the help of Mayer's printed tables, he computed the longitude of the ſhip from time to time; and from the near agreement of the obſervations, eſpecially at making the land, when the ſhip's common reckoning was very erroneous, he inferred that the longitude thus determined would never err a whole degree. That the ſame method which he had explained in his Britiſh Mariners Guide, had been practiſed with ſucceſs by the commanders and mates of ſeveral of the Eaſt India ſhips.—He then proceeds to point out the method he uſed to adjuſt his quadrant, in his voyage to Barbadoes, and return to England, which was, by meaſuring the Sun's diameter with the index placed alternately to the right and left of the beginning of the diviſions; by which means, he found his obſervations to agree ſtill nearer than in his former voyage to St. Helena, having made Barbadoes within half a degree of its true ſituation; and at his return home, he made the iſle of Wight, by obſervations taken 24 hours before, within 16 minutes of its true ſituation. He concludes his memorial in the following words, "I am authoriſed by them* to ſay, they humbly apprehend that nothing is wanting to make this method generally practicable at ſea, but a Nautical Ephemeris—an aſſiſtance which they, with many more, hope for from this Board."

The gentlemen mentioned in the memorial were examined ſeparately, as to the utility and practicability of the abovementioned obſervations; and all agreed in teſtifying that they had determined the longitude of their reſpective ſhips from time to time, by obſervations of the moon taken in the manner directed in the Britiſh Mariner's Guide; and that the longitude reſulting always agreed with the making of land, to one degree. In conſequence of which, the Board came to the following reſolution:

"That it is the opinion of this Board, upon the evidence given of the utility of the late Profeſſor Mayer's Lunar Tables, that it is proper the ſaid tables ſhould be printed; and that application ſhould be made to Parliament for power to give a ſum not exceeding 5000 l. to the widow of the ſaid Profeſſor, as a reward for the ſaid tables, part of which have been communicated by her, ſince her huſband's deceaſe; and alſo for power to give a reward to perſons to compile a Nautical Ephemeris, and for authority to print the ſame when compiled, in order to make the ſaid lunar tables of general utility."

This was immediately put into execution; and accordingly, ſince the year 1766, a work entitled *The Nautical Almanack and Aſtronomical Ephemeris* has been publiſhed annually, by order of the Commiſſioners of Longitude, under the inſpection of Dr Maſkelyne.—This is perhaps the beſt and moſt accurate publication of the kind that has hitherto appeared.

At the ſame time with the firſt Nautical Almanac, a ſet of tables to facilitate the calculations, was publiſhed by Dr Maſkelyne; in which are given two excellent methods, with their demonſtrations, for clearing the apparent diſtance between the moon and the ſun or a fixed ſtar, from the effects of refraction and parallax; the firſt by Mr Lyons, and the other by Mr Dunthorne: alſo two examples for aſcertaining the longitude from obſervation.— About twelve years afterwards, this edition being nearly all ſold off, it became neceſſary to reprint it; previous to which it was new-modelled by Dr Maſkelyne, and the ſecond edition appeared in 1781. With which, and the Nautical Almanac, every perſon ought to be provided, who wiſhes to aſcertain a ſhip's place from celeſtial obſervations.

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* The gentlemen mentioned in his memorial—namely, the commanders and mates of the ſhips that had uſed the method he had deſcribed.

The publications on the longitude, by obſervations of the Moon, now became very numerous; almoſt every book on Navigation, publiſhed in Britain ſince the commencement of the Nautical Almanac, contained a few rules for that purpoſe. In Robertſon's Navigation, there is a good tract on this ſubject. Beſides thoſe already mentioned, ſome of the other writers on the lunar obſervations in Britain are Meſſrs Adams, Biſhop, Dunn, Emerſon, Heath, Margetts, Moore, Nicholſon, Vince, Waddington, Wilſon, Witchell, &c.

In 1772, a treatiſe, entitled *Theorie et Pratique des Longitudes en Mer*, was publiſhed at Paris, by M. de Charnieres, wherein he endeavours to bring the Megameter into common practice. This inſtrument is conſtructed on ſimilar principles with the object glaſs Micrometer, and is only applicable to meaſure diſtances leſs than 10°. In page 21. he ſays, "Intimement perſuadé, que la vraie ſolution du problème des longitudes en mer, tenoit à la meſure exact de la diſtance de la lune aux etoiles, je trouvai que l'héliométre pourroit en le modifiant, s'appliquer aux obſervations de mer." He then deſcribes the inſtrument in a very particular manner, and ſhows its uſe in meaſuring the diſtance between the Moon and a ſtar, and how the longitude is deduced from that obſervation.

This method of determining the longitude at ſea is mentioned in in a treatiſe, entitled, *Abrege du Pilotage*, Paris, 1766. In the ſame year, Pere Pezenas publiſhed his *Aſtronomie des Marins* at Avignon, which contains, beſides other articles, various methods of finding the longitude from obſervations of the Moon. This ſubject is alſo very well treated in the ſixth volnme of Bezout's *Cours de Mathematiques*, Paris 1781, and in M. Callet's edition of *Gardiner's Logarithmic Tables*, Paris 1783.

An excellent tract was publiſhed at Paris in 1787, intitled, *Deſcription et Uſage du Cercle de Reflexion, par le Chevalier de Borda*, in which, that inſtrument is very particularly deſcribed, its uſe ſhown in nautical obſervations, and the manner of calculating theſe obſervations.

M. de la Lande treats very perſpicuouſly on this ſubject, both in his *Expoſition du Calcul*, and in his *Aſtronomie*. In the former of theſe works (p. 168) he ſays, "Sans une méthode pour trouver ces longitudes, la navigation eſt toujours incertaine; l'eſtimé à laquelle on a recours, peut être fautive de 4 or 500 lieues après quelques mois de navigation, & jeter ceux qui naviguent dans les plus grands dangers." He then recommends the lunar method of finding the longitude at ſea, and ſhews the manner of conſtructing and uſing M. de la Caille's *Chaſſis de Reduction*, for clearing the apparent diſtance from the effects of refraction and parallax, and the method of deducing the ſhip's longitude from obſervation. In his *Aſtronomie** he writes, "Cette méthode des diſtances a l'avantage de ne dependre eſſentiellement que d'une ſeule obſervation de diſtance: elle ne ſuppoſe pas la hauteur connue avec une extreme préciſion: elle depend tres-peu de la déclinaiſon de la lune, et de la hauteur du pole; elle n'exige pas qu'on ait un horizon *clair-fin*, c'eſt-a-dire, bien dégagé des vapeurs; elle ne ſuppoſe pas des calculs auſſi longs que ceux de l'aſcenſion droite de la lune: enfin, la reduction de la diſtance apparente en diſtance vraie, à raiſon de la réfraction & de la parallax, ſe peut faire avec la regle, & le compas par une operation graphique."

From the mutual concurrence of the opinions of ſo many eminent men, it appears, that the method of finding the longitude at ſea, by obſerving the diſtance between the Moon and the Sun or a fixed ſtar, commonly called the *Lunar Obſervations*, is the beſt in uſe at preſent; and therefore any improvement that may be made therein, either in the lunar tables from which the articles in the Nautical Almanac relative to the Moon are computed, or in the conſtruction of inſtruments for obſerving the diſtance, or in the methods of making and calculating the neceſſary obſervations, muſt certainly be of very great advantage to the practical navigator.

CHAP. II.

PREPARATORY PROBLEMS.

PROBLEM I.

To convert Degrees, or parts of the Equator, into Time.

RULE. Multiply the given motion by 4, and the product will be the correſponding time.

REMARK. Minutes multiplied by 4 produce ſeconds, degrees multiplied by 4 produce minutes, which divided by 60 give hours.

EXAMPLE.

Let 26° 45′ be reduced to time.

$$\begin{array}{r} 26^\circ\ 45' \\ 4 \\ \hline \end{array}$$

$$1\text{h. } 47'\ 0'' = \text{time required.}$$

PROBLEM II.

To convert Time into Motion.

RULE. Multiply the given time by 10, to which add half the product; the ſum will be the correſponding degrees, &c.

EXAM-

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* Prem. edit. tome II. art. 3224, or ſec. edit. tome III. art. 3977.

Example.

Let 3 h. 4′ 28″ be converted into motion.

3 4 28
10

30 44 40
Half = 15 22 20

Correſponding motion = 46 7 0

Prob. III.

Given the time under any known meridian, to find the correſponding time at Greenwich.

Rule. Let the given time be reckoned from the preceding noon, to which the longitude of the place in time is to be applied by addition or ſubtraction, according as it is weſt or eaſt, and the ſum or difference will be the correſponding time at Greenwich.

Examples.

I. What is the time at Greenwich, when it is 6 h. 15′ at a ſhip in longitude 76° 45′ W.?

Time at ſhip, — — 6 h. 15′
Longitude in time, — 5 7 W.

Time at Greenwich, 11 22

II. Required the time at Greenwich, anſwering to 5 h. 46′ 39″ of May 1ſt at Canton, whoſe longitude is 113° 2′ 15″ E.?

Time at Canton, May 1ſt, — 5 h. 46′ 39″
Longitude in time, — — — 7 32 9 E.

Time at Greenwich, April 30, 22 ′ 14 30

Prob. IV.

To reduce the time at Greenwich to that under any given meridian.

Rule. Reckon the given time from the preceding noon, to which add the longitude in time, if eaſt, but ſubtract it if weſt; and the ſum or remainder will be the correſponding time under the given meridian.

Examples.

I. What is the expected time of the beginning of the lunar eclipſe of April 18, 1791, at a ſhip in longitude 109° 48′ E.

Beginning of eclipſe at Greenwich, p. Naut. Alm. 3 h. 15′ 30″
Ship's longitude in time, — — — 7 19 12

Beginning of the eclipſe at the ſhip, — — 10 34 42

L II.

II. When may the immerſion of the firſt ſatellite of Jupiter be obſerved at Port St. Julian, in long. 68° 44′ W. which, by the Nautical Almanac, happens at Greenwich 24th March 1792, at 17h. 53′ 1′?

Apparent time of imm. at Greenwich,	17h.	53′	1′
Longitude of Port St. Julian in time,	4	34	56 W.
Apparent time of imm. at Port St. Julian,	13	18	5

PROB. V.

To reduce the declination of the Sun, as given in the Nautical Almanac, to any other meridian, and to any given time of the day.

RULE. Take the Sun's declination from the Ephemeris for the noon of the given day, and convert the ſeconds into tenths of a minute, by dividing by 6; then enter table XIII. with the declination at the top, and take out the equations anſwering to the given longitude, and time from noon. Theſe will be the change of declination, anſwering to a north increaſing declination; when the Sun is in either of the other quarters of the ecliptic, a correction from one of the three following tables accordingly, is to be applied to each of the above equations. Now, between the 20th of March and 21ſt of June, and from the 22d of September to the 21ſt of December, or while the Sun's declination is increaſing, the equation anſwering to the ſhip's longitude is to be added to, or ſubtracted from the declination, according as the longitude is eaſt or weſt; and the equation anſwering to the time from noon, muſt be added or ſubtracted, according as the given time is after or before midday. But from the 21ſt of June to the 22d of September, and from the 21ſt of December to the 20th of March, or while the Sun's declination is decreaſing, the equation anſwering to the longitude is additive or ſubtractive, according as the longitude is eaſt or weſt; and that anſwering to the time from noon is to be applied to the declination by addition or ſubtraction, according as the given time is before or after noon*.

EXAM-

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* Thoſe acquainted with the firſt principles of Algebra will prefer the following rule.

Let the ſign + be prefixed to the declination if increaſing, to the equation correſponding to the longitude if weſt, and to that anſwering to the time from noon, if the given time be in the afternoon; but if the declination is decreaſing, the longitude eaſt, or the time before noon, prefix the ſign — to theſe quantities, and their ſum, according to the rules of Algebraic addition, will be the reduced declination.

EXAMPLE. Required the Sun's declination April 2, 1791, at 6 h. 35′ A. M. in longitude 38° 40′ W.

Sun's declination at noon, per Naut. Alm.	+ 5°	1′,	1
Equation to longitude 38° 40′ W.	+	2,	5
—— to time from noon, 5h. 25′	—	5,	2
Reduced declination, —— ——	4	58,	4

EXAMPLES.

I. Required the Sun's declination at noon, November 2d, 1792, in longitude 10° 5′ E.?

Sun's declination at Greenwich, —	15° 4′.7
Equation to declination and longitude,	— 5.2
Reduced declination, — — —	14 59.5

II. What is the Sun's declination 24th May 1792, at 10 h. 48′ Greenwich time?

Sun's declination at noon, — — —	20° 57.0
Equation to declination and time from noon, +	4.8
Declination at given time, — — —	21 1.8

III. Required the Sun's declination, 13th Auguſt 1792, at 5 h. 46′ A. M. in longitude 143° 7′ W.?

Sun's declination at noon — — —	14° 24′.8
Equation to declination and longitude, —	— 7.4
— — — and time from noon,	+ 4.8
Reduced declination, — — — —	14 22.2

IV. What is the Sun's declination, 19th February 1792, at 2 h. 15′ P. M. in longitude 164° 56′ E.?

Sun's declination at noon, per Naut. Alm. —	11° 15′.6
Equation to decl. and longitude 164° 56′ E. —	+ 9.8
— — — and time from noon 2h. 15′ P. M. —	2.0
Reduced declination, — — —	11 23.4

PROB. VI.

To reduce the Sun's Right Aſcenſion, as given in the Nautical Almanac, to any given time of the day, under a known meridian.

RULE. Enter table XVIII. with the Sun's right aſcenſion at the top, and take out the equations anſwering to the ſhip's longitude, and the interval between the given time and noon. Then if the longitude be weſt, or the given time, between noon and midnight, add the correſponding equations to the Sun's right aſcenſion; otherwiſe ſubtract them; and the ſum or remainder will be the reduced right aſcenſion.

EXAMPLES.

I. Required the Sun's right aſcenſion at noon, 21ſt April 1792, in longitude 76° 45′ W.

Sun's right aſcenſion at noon, p. Naut. Alm. —	1 h. 59′ 23″
Equation to right aſcenſion and longitude, 76° 45′ W. +	48
Reduced right aſcenſion, — — —	2 0 11

II. What is the Sun's right aſcenſion at Greenwich, October 17th, 1792, at 8 h. 40′ P. M.

Sun's right aſcenſion at noon, per Naut. Alm.		13h. 31′ 55″
Equation to right aſcenſion and time from noon,	+	1 21
Reduced right aſcenſion, — — —		13 33 16

III. What is the Sun's right aſcenſion, June 5, 1792, at 1 h. 48′ A. M. in longitude 63° 10′ E.?

Sun's right aſcenſion at noon, per Naut. Alm.		4h. 56′ 32″
Equation to right aſcen. and long. 63° 10′ E.	—	43
— — — and time from noon 10h. 12′	—	1 45
Reduced right aſcenſion, — — —		4 54 4

PROBLEM VII.

To reduce the Declination of the Moon, as given in the Nautical Almanac, to any given time, under a known meridian.

Rule. Find the variation of the Moon's declination in twelve hours, with which enter table XIX. at the top, and take out the equations anſwering to the ſhip's longitude, and to the time from the preceding noon or midnight. Then if the Moon's declination is increaſing, the equation correſponding to the time is additive; and that anſwering to the longitude is to be added, if the longitude is weſt—but ſubtracted, if eaſt. If the declination is decreaſing, the contrary rule is to be applied.

EXAMPLES.

I. Required the Moon's declination, Sept. 22, 1792, at 5 h. 13′ apparent time under the meridian of Greenwich?

Moon's declination 22d Sept. at noon, — —		17° 31 S.
Equation to variation 29′, and time from noon 5 h. 13′	+	13
Reduced declination, — — —		17 44 S.

II. What is the Moon's declination, 30th December 1792, at 14 h. 9′ apparent time, in longitude 65° 14′ W.?

Moon's declination 30th December, at midnight,		15° 1′ N.
Equation to variation 1° 13′ and longitude 65° 14′ W.	—	27
— — — and time from midnight 2 h. 9′	—	13
Reduced declination — — — —		14 21 N.

PROBLEM VIII.

To reduce the Horizontal Parallax and Semidiameter of the Moon, as given in the Nautical Almanac, to any given time, under a known meridian.

Rule. Reduce the given time to the meridian of Greenwich by Prob. iii. and find the variation of parallax in twelve hours, the product of which, by the interval between the reduced time and the preceding noon or midnight, being divided by 12 hours, will be the proportional part; which being added to or ſubtracted from the Moon's horizontal parallax at the preceding noon or midnight, according as it is increaſing or decreaſing, will give the required parallax. In like manner, ~~may~~ the Moon's ſemidiameter may be found.*

Example.

Required the Moon's horizontal parallax and ſemidiameter, Dec. 7, 1792, at 11h. 15′ in longitude 38° 40′ E.?

Given time, — — — — —			11° 15′
Longitude in time — — — —			2 35 E.
Reduced time, — — — — —			8 40
Moon's horizontal parallax at noon }	= 55′ 44″	Moon's ſemidiameter	15′ 11″
Prop. part } = $\frac{8h.\ 40'\times 22''}{12h.}$	= + 16	P. part = $\frac{8h.\ 40'\times 6''}{12h.}$	= + 4
Reduced parallax,	56 0	Reduced ſemidiam.	15′ 15″

Problem IX.

Given the obſerved altitude of a Fixed Star, to find the true altitude.

Rule. To the obſerved altitude of the ſtar, apply the index error, if any, and the dip of the horizon; from which, ſubtract the refraction anſwering thereto, from table v, and the remainder will be the true altitude of the ſtar.

Examples.

I. The obſerved altitude of Arcturus is 38° 40′, and the height of the obſerver's eye 18 feet above the ſurface of the water. Required the true altitude?

Obſerved altitude of Arcturus,	38°	40′.0
Dip, table iii. — —	—	4.1
Apparent altitude, — —	38	35.9
Correction, table v. —	—	1.2
True altitude of Arcturus, —	38	34.7

II. Let the obſerved altitude of Antares be 49° 52′, the error of the inſtrument 5′.3 ſubtractive, and height of the eye 14 feet. Required its true altitude?

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* Three elevenths, or one fourth nearly, of the proportional part of parallax, will be that of ſemidiameter.

Obſerved altitude of Antares	49 52	
Index error —	— 5.3	
Dip — —	— 3.6	
Apparent altitude, —	49 43.1 = 49° 43′ 6″	
Refraction, table vi. —		— 48
True altitude of Antares, — —		49 42 18

PROBLEM X.

Given the obſerved altitude of the lower or upper limb of the Sun, to find the true altitude of its center.

RULE. Correct the obſerved altitude of the Sun's limb by the index error, if any, the dip of the horizon, and ſemidiameter; hence the apparent altitude of the Sun's center will be obtained; from which, the correction* in table IV. anſwering thereto, is to be ſubtracted, and the remainder will be the true altitude of the Sun's center.

EXAMPLES.

I. The obſerved altitude of the Sun's lower limb is 18° 41′, height of the eye above the ſurface of the ſea 22 feet, and Sun's ſemidiameter 16′.1. Required the true central altitude?

Obſerved altitude of Sun's lower limb	18° 41.0
Sun's ſemidiameter, — —	+ 16.1
Dip — — — —	— 4.5
Apparent altitude Sun's center, —	18 52.6
Correction — — —	— 2.6
True altitude of Sun's center,	18 50.0

II. The obſerved altitude of the Sun's upper limb is 28° 31½′, index error 40″, height of the eye 12 feet, and Sun's ſemidiameter 15′ 54″. Required the true altitude?

Obſerved altitude of Sun's upper limb	28° 31′ 30″
Index error, — — —	+ 40
Sun's ſemidiameter, — —	— 15 54
Dip of the horizon 3,3, or —	— 3 18
Apparent altitude of the Sun's center,	28 12 58
Refraction, table VI. — —	— 1 46
Parallax, table VII . — —	+ 8
True altitude of the Sun's center,	28 11 20

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* In ſome caſes, it will be proper to apply the refraction and parallax from tables VI. and VII. reſpectively, in place of the above correction.

PROBLEM XI.

Given the obſerved altitude of the lower or upper limb of the Moon, to find the correct central altitude.

RULE. Agreeable to the reduced time of obſervation, find the Moon's horizontal parallax and ſemidiameter, by means of the Nautical Almanac, and increaſe the ſemidiameter by the augmentation from table xxx, anſwering to the ſemidiameter and obſerved altitude.

To the obſerved altitnde of the Moon's limb, apply the index error, the augmented ſemidiameter and dip as formerly; the aggregate will be the apparent altitude of the Moon's center; to which the correction from table IX. anſwering to the Moon's horizontal parallax and apparent altitude being added, the ſum will be the correct altitude of the Moon's center.

EXAMPLES.

I. Let the obſerved altitude of the Moon's lower limb be 31° 18′, horizontal parallax 58° 37′, ſemidiameter 15° 58′, and height of the eye 16 feet. Required the true altitude of the Moon's center?

Moon's ſemidiam.	15′ 58″	Obſerved alt. Moon's low. limb		31°	18′.0
Augment. tab. xxx.	+ 8	Dip — — —	—		3.8
Augm. ſemidiam.	16 6	— — — —	+		16.1
		Apparent alt. of Moon's center		31	30.3
		Cor. to alt. 31° 30′ and h. par 58′ 37″ p. tab. IX. = 48′ 26″	= +		48.4
		True alt. of Moon's center,		32	18.7

II. The obſerved altitude of the Moon's upper limb is 41° 25′, horizontal parallax 55′ 40″, ſemidiameter 15′ 10″, and height of the eye 15 feet. Required the true altitude of the Moon's center?

Moon's ſemidiam.	15′ 10″	Obſerved alt. Moon's up. limb		41°	25′	0″
Augment. tab. xxx.	+ 10	Dip of the horizon = 3′, 7	—		3	42
Augm. ſemidiam.	15 20	— — ——	—		15	20
		Apparent alt. of Moon's center		41	5	58
		Cor. to alt. 41° 6′ and h. par 55′ 40″ p. tab. IX.	= +		40	51
		True alt. of Moon's center,		41	46	49

Altitudes, uſed for the computation of the apparent time or the latitude, ought to be expreſſed in degrees, minutes, and tenths of a minute; but in ſome of the methods for correcting the apparent diſtance between the Moon and the Sun or a fixed ſtar, it is neceſſary, that the true altitudes of the objects be expreſſed in degrees, minutes, and ſeconds. In the firſt example to each of the three laſt problems, the correct altitude is, therefore, found in degrees, minutes, and tenths; and in the ſecond, it is expreſſed in degrees, minutes, and ſeconds.

The correction of a ſtar in altitude is the refraction anſwering to that altitude, and the correction of the altitude of the Sun or Moon, is the difference between the refraction and parallax anſwering to the given altitude. As the refraction in altitude is greater than the Sun's parallax at the ſame altitude, this correction is therefore ſubtractive; but the Moon's parallax in altitude being ever greater than the refraction anſwering to the ſame altitude, the Moon's correction is therefore to be added. This laſt correction may be found, independent of table IX, by ſubtracting the refraction from the Moon's parallax in altitude, computed by the rule given in page 62.

It has already been obſerved, that in order to reduce the Moon's horizontal parallax, as given in the Nautical Almanac, to a given place; it muſt be diminiſhed by an equation from table XXXII, depending on the latitude. The horizontal parallax in the Connoiſſance des Temps is adapted to the latitude of Paris; in order, therefore, to adapt it to a place whoſe latitude is leſs than that of Paris, it muſt be increaſed by the difference of the equations anſwering to thoſe latitudes, and diminiſhed by the above difference, when the latitude of the place exceeds that of Paris. Moreover the altitude uſed for computing the parallax in altitude muſt be increaſed or diminiſhed by a certain quantity, from table XXVI.

Hence, in order to compute the altitude of the Moon's center above the true horizon, the obſerved altitude ſhould be corrected by the error of the quadrant and the dip of the horizon, from which the corrected refraction, anſwering thereto, is to be ſubtracted. This laſt quantity being increaſed by the parallax in altitude, (computed with the reduced horizontal parallax and altitude) will be the true altitude of the Moon's limb, to which the Moon's true ſemidiameter being applied, will give the true altitude of the Moon's center.

CHAP. III.

Of the methods of aſcertaining Time, and regulating a Chronometer or Watch at Sea, or Land.

TIME, as inferred from obſervations of the Sun, is denominated *Apparent* and *Mean Solar Time.*

Apparent Time is that which is deduced immediately from altitudes, or other obſervations of the Sun, and Mean Time ariſes from a ſuppoſed uniform motion of the Sun. Hence, a mean ſolar day is always of the ſame determinate length of abſolute time, but the length of an apparent day is ever variable, being longer at one time, and ſhorter at another, than a mean day; the inſtant of apparent noon will, therefore, ſometimes precede, and ſometimes follow that of mean noon. The interval between apparent and mean time is called the Equation of Time.

The equation of time reſults from the inclination of the ecliptic and equator, combined with the unequal motion of the Earth in its orbit. Tables have been conſtructed, to exhibit the parts of the equation of time ariſing from theſe cauſes; which parts being taken out with their proper arguments, and connected according to their reſpective ſigns, will give the compound equation of time; but in order to ſave the trouble of this computation, a table of the abſolute equation of time is given in the Nautical Almanac, page II. of the month, for the inſtant of apparent noon by the meridian of Greenwich.

As the principle upon which a watch or time keeper is conſtructed, is that of an equable or uniform motion; it is therefore evident, that when a ſeries of ſucceſſive and repeated obſervations are to be made, the watch ought to be regulated according to mean ſolar time, or its rate eſtabliſhed on that time. But as it is apparent time that is immediately deduced from obſervations of the Sun, the equation of time becomes, therefore, an eſſential article, and muſt be applied thereto, in order to reduce it to mean ſolar time. Indeed, when a watch is uſed for no other purpoſe than that of connecting a few obſervations, taken within ſhort intervals of time of each other, the reduction of apparent to mean time is not neceſſary.

PROBLEM I.

To find the Equation of equal Altitudes.

RULE.

Enter table XXIII. with the interval of time betwee.. the obſervations at the top, and the latitude of the place of obſervation in the ſide column, and take out the correſpondent number; take out the number from table XXIV. anſwering to the interval of time and the Sun's declination; ſubtract it from the former, if the latitude and declination are of the ſame name, otherwiſe, add them, and find the log. correſponding to the remainder or ſum, which ſubtracted from the P. Log. of the daily variation of the Sun's declination, increaſed by 5, the remainder* will the P. Log. of the equation of equal altitudes.

EXAMPLE.

Let the latitude of the place of obſervation be 57° 9′ N. the interval of time between the obſervations 5 h. 17 m. Sun's declination 17° 48′ S. and change of declination 16′ 19½″. Required the equation of correſponding altitudes?

No. from tab. XXIII. to interval and lat. =	1782		
No. from tab. XXIV. to inter. and decl. =	284		
Sum —— ——	2066	log.	3.3151
Daily variation of declination,	16′ 19.″5	P. log. + 5 =	6.0424
Equation of equal altitudes,	20.″2	P. log.	2.7273

PROBLEM II.

To find the Error of a Watch by equal Altitudes of the Sun.

RULE.

In the morning, when the Sun is more than two hours diſtant from the meridian in theſe latitudes, let a ſet of obſervations be taken, conſiſting, for the ſake of greater accuracy, of at leaſt three altitudes—which, together with the correſponding times per watch, are to be wrote down regularly, the time of each obſervation being previouſly increaſed by 12 hours. In the afternoon, obſerve the inſtants when the Sun comes to the ſame altitudes, and write down each oppoſite to its reſpective altitude. Now half the ſum of any two times, anſwering to the ſame altitude, will be the time of noon per watch uncorrect; find the mean of all the times of noon, thus deduced from each correſponding pair of obſervations, to which the equation of equal altitudes is to be applied, by addition or ſubtraction, according as the Sun is receding from, or approaching to, the elevated pole; the ſum or difference will be the time per watch of apparent noon, or the inſtant when the Sun's center was on the meridian—the difference between which and noon is the error of the watch for apparent time, and the watch will be faſt or ſlow, according as the time of noon thereby is more or leſs than twelve hours.

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* Or, from this remainder ſubtract the conſtant log. 1.7782, and the remainder will be the P. Log. of a certain number of minutes and ſeconds; which, being eſteemed ſeconds and thirds, will be the equation of equal altitudes. The thirds may be reduced to a decimal by dividing by 60.

If the watch be regulated to mean ſolar time, it is obvious, the time of noon found as above ſhould agree with that found, by applying the equation of time to noon, according to its ſign in the Nautical Almanac. If theſe times do not agree, their difference will be the error of the watch or mean ſolar time. Inſtead of applying the equation of time to twelve hours, it perhaps will be found more convenient, to apply it with a contrary ſign to the time per watch, of apparent noon; and the difference between this time and 12 hours will be the error of the watch as formerly.

EXAMPLES.

January 29, 1786, in latitude 57° 9′ N. the following equal altitudes of the Sun were obſerved. Required the error of the watch?

Altitude = 8° 5′	Time, 21 h. 35′ 8″ A. M.	2 55 43 P. M.
8 10	36 8	54 42
8 20	38 9	52 41.2
8 25	39 12½	51 38

	21 h. 35′ 8″	21 36 8	21 h. 38′ 9″	21 h. 39′ 12″.5
	2 55 43	2 54 12	2 52 41.2	2 51 38
Sum	24 30 51	24 30 50	24 30 50.2	24 30 50.5
Mean	12 15 25.5	12 15 25	12 15 25.1	12 15 25.2
				25.1
				25.0
				25.5
Sum	—	—	—	.8

Time of noon per watch uncorrected,		12	15 25.2
Equation of equal altitudes,	—	—	20.2
Time per watch of apparent noon	—	12	15 5.0
Watch faſt for apparent time,	—		15 .5
Time per watch of apparent noon,	—	12	15 5.0
Equation of time,	— —	—	13 29.8
Time per watch of mean noon,	—	12	1 35.2
Watch faſt for mean time,	—		1 35.2

In obſerving equal altitudes, it will be found convenient to put the index of the quadrant to a certain diviſion, and to wait till either

limb

limb of the Sun attains that altitude. If the ſucceſſive altitudes of the ſame ſet are equidiſtant from each other, the mean of the morning obſervations may be compared with the mean of thoſe obſerved in the afternoon, in order to find the time of noon.

April 20th 1786, in latitude 57° 9′ N. the following obſervations were made, in order to aſcertain the error of the clock.

Alt. =	35° 40′	Time p. clock	21 h.	20′	27″.5		2 h.	37′	29″5
	35 45	— —		21	16.0			36	41.0
	35 50	— —		22	4.5			35	52.5
	35 55	— —		22	53.0			35	4.0
	36 0	— —		23	41.3			34	15.6
				10	22.3			29	22.6
Mean — —			21	22	4.46		2	35	52.52
			2	35	52.52				
			23	57	56.98				
Time p. clock of noon uncorr.			11	58	58.49				
Equation of equal altitudes,			—		19.53				
Time p. clock of app. noon,			11	58	38.96		11	58	38.96
Clock ſlow for apparent noon,				1	21.04	eq. time +		1	16.20
Time p. clock of mean noon,				—	—		11	59	55.16
Clock ſlow for mean time				—	—				4.84

Hence, the obſervations of the two preceding examples being ſuppoſed to be made at the times ſpecified, by the ſame watch or clock, its daily rate may be eſtabliſhed, upon the ſuppoſition of an uniform motion, as follows:

January 29th	Clock faſt at noon =	1′ 35″.3
April 20th	Clock ſlow at noon =	4.8
Interval 81 days.	Difference	1 40.1

Now 1′ 40″.1, div. by 81, gives 1″.236 for the daily rate of the clock.

In obſervations made at land, with a good inſtrument, in favourable weather, the time may be eaſily aſcertained within the fifth part of a ſecond, by obſerving the poſition of the Sun's limb with reſpect to the wire in the focus of teleſcope, at the inſtants immediately preceding and following the tranſit of the limb; and though the time of each obſervation at ſea be only aſcertained within a few ſeconds, yet the error may be ſo far diminiſhed, by taking the mean of repeated ſets, as to be rendered almoſt inſenſible.

Obſervations for this purpoſe ſhould not be taken, when the Sun is within leſs than two hours of the meridian, unleſs in caſe of abſolute neceſſity, becauſe the motion of the Sun in altitude, at places diſtant from the equator, would then be too ſlow; neither ſhould the altitude be under 4 or 5°; otherwiſe, the refraction may be conſiderably altered between the times of the morning and afternoon obſervations, and thereby ſenſibly affect thoſe times. If the rate of the watch is not uniform during the interval between theſe obſervations, an error in the time of noon may thence ariſe, which it is more than probable will augment with the obſerved interval. Moreover, if the rate of the watch differs conſiderably from mean ſolar time, the interval between the means of the morning and afternoon ſets of obſervations ſhould be reduced to that time, with which the equation of equal altitudes is to be taken from the tables.

It very often happens, in places diſtant from the equator, that in the afternoon, by the interpoſition of clouds, a correſpondent ſet to that obſerved in the forenoon cannot be obtained; it is therefore proper to take ſeveral ſets in the morning, and tho' the afternoon be a little cloudy, yet a correſpondent ſet to ſome of thoſe obſerved in the morning will perhaps be obtained.

We have hitherto ſuppoſed the obſervations to have been made in the ſame civil day; the interval may however be conſiderably extended, provided the rate of the watch is regular. Thus equal altitndes may be obſerved in the afternoon, and in the following morning, and the mean of the times will be the time of the intermediate midnight, or inſtant per watch uncorrected, when the oppoſite meridian was directed to the Sun, to which the equation of equal altitudes is to be applied with a contrary ſign. In caſe of abſolute neceſſity, the time between the obſervations may be ſtill farther extended, by taking equal altitudes in the forenoon, and in the afternoon of the following day; in this caſe the error of the watch will be found at the intermediate midnight, the equation of equal altitudes being applied as uſual.*

As the method of aſcertaining time by equal altitudes depends neither on the accuracy of the latitude of the place of obſervation, nor on that of the declination, ſince theſe elements are only neceſſary in taking out the equation of equal altitudes, and as any probable error therein will not ſenſibly affect that equation; neither does it depend on the exact quantity of the altitude, provided only it is the ſame at both obſervations—it is therefore univerſally uſed by the prac-

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[*] Theſe methods have been ſucceſsfully practiſed by the Author.

practical aſtronomer, eſpecially when not provided with a tranſit inſtrument, or before a meridian mark is placed for that inſtrument.

Problem III.

To find the Error of the Watch by equal Altitudes of the Sun, the Ship being under way.

Rule.

Let ſeveral ſets of equal altitudes be obſerved in the morning and afternoon, and from thence find the corrected time of noon, as formerly; alſo let the Sun's azimuth be obſerved, to which the variation of the compaſs being applied, the true azimuth at the time of obſervation will be obtained.†

Now to the conſtant log. 9.2219 add the proportional log. of the interval of time between the equal altitudes, the hours and minutes being conſidered as minutes and ſeconds, the prop. log. of the hourly rate of ſailing, the log. co-ſine of the ſhip's latitude, the log. ſecant of the courſe, and the log. tangent of the Sun's azimuth; the ſum, rejecting tens in the index, will be the prop. log. of the correction anſwering to the change of latitude; and to the ſum of firſt four logs. add the log. co-ſecant of the courſe—the ſum, rejecting tens in the index, will be the prop. log. of the change of longitude. The firſt correction is to be added to, or ſubtracted from, the time of noon formerly found, according as the ſhip's latitude is increaſing or diminiſhing, and the ſecond correction is additive or ſubtractive, according as the ſhip's courſe has been in the eaſtern or weſtern hemiſphere. The reſult thus deduced will be the time per watch of apparent noon, under the meridian of the firſt place of obſervation.

If the two laſt corrections be applied with a contrary ſign, the time of apparent noon, under the meridian of the ſecond place of obſervation, will be obtained.

The firſt correction vaniſhes, if the courſe made good between the obſervations is either due eaſt or weſt; and the ſecond, if the ſhip ſails on a meridian.

Example.

Auguſt 6th 1792, equal altitudes of the Sun's lower limb were obſerved, whereof the means were 9h. 14′ 52″ A. M. and 2h. 48′ 18″ P. M. reſpectively, the corrected azimuth of the Sun from the ſouth was 69¾°, the ſhip's courſe during the elapſed time S. W. by W. at the rate of 8.6 knots per hour, and the ſhip's latitude and longitude at noon were 39° 18' N. and 31° 24' W. reſpectively. Required the error of the watch for apparent noon, under the meridian of the place where the firſt obſervation was made?

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† Or if the Sun's magnetic azimuth be taken at the times when the morning and afternoon ſets were obſerved, then half the ſum of the eaſtern and weſtern azimuths, reckoned from the meridian, will be the Sun's true azimuth nearly. Or, if the obſervation of the azimuth had been neglected, it may be computed from the known latitude of the ſhip, the Sun's altitude and declination.

Conſtant logarithm —	9.2219		
Int. time = 5 h. 33' or 5' 33" P. Log. =	1.5110		
Hourly rate of ſailing 8 h. 6' or 8' 36" P. Log. — — =	1.3208		
Latitude 39° 18' coſine	9.8886		
	1.9423		1.9423
Courſe 3 points, — ſecant	0.0801	— co-ſecant,	0.2553
Azimuth 69° 45' tangent —	0.4331	ſec. cor. 1' 9" P.L.	2.1976
Firſt correction 0' 38" P. Log.	2.4555		

Mean of morning ſet — — — —		9h.	14'	52"
afternoon ſet — — — —		2	48	18
		12	3	10
Uncorrected time of noon — — —		12	1	35
Equation of equal altitudes — —	+			7
Equation of latitude — — —	−			38
Equation of longitnde — — —	−		1	9
Time p. watch of apparent noon, under meridian of firſt place of obſervation — — —		11	59	55
Watch ſlow for apparent time — —				5
Equation of time — — —			5	23
Watch ſlow for mean time, — —			5	28

This problem may otherwiſe be performed, by eſtimating how many minutes the Sun is higher or lower, in conſequence of the change of latitude in the elapſed time, at the inſtant it will attain the correſponding altitude in the afternoon, and ſetting the index of the quadrant accordingly. This quantity may be found with ſufficient accuracy from a traverſe table.

Or, with the latitude of the ſhip at the time of the firſt obſervation, the declination and correct altitude of the Sun's center, compute its diſtance from the meridian by prob. VII. which being added to the mean of the firſt ſet, will give the time per watch of apparent noon. In like manner, with the ſhip's latitude at the ſecond obſervation, the Sun's declination and altitude, compute the meridian diſtance of the Sun; which ſubtracted from the mean of the afternoon ſet, the remainder will be the time by the watch of apparent noon, under the meridian of the place where that obſervation was made. Now half the difference of longitude made good between theſe obſervations, being applied as formerly to the mean of the computed noons, will give the time per watch of apparent noon, under the meridian of the firſt place of obſervation, and hence the error of the watch for that meridian will be known. If the interval between the computed times of noon is not the ſame as the difference of longitude expreſſed in time, there is an error, either in the difference of longitude, in the rate of the watch during the obſerved interval, or in the actual obſervation or computation. In this method it is not neceſſary, that the altitudes obſerved be correſpondent.

PROBLEM IV.

To find the Error of a Watch, by equal Altitudes of a Fixed Star.

RULE.

Let ſeveral altitudes, and the correſponding times per watch, of a known ſtar, be obſerved when in the eaſtern hemiſphere, and when the ſtar is in the weſtern hemiſphere, obſerve the inſtants when it comes to each of the former altitudes.

Take the mean of each correſponding pair of times, and the mean of theſe will be the apparent time per watch of the ſtar's tranſit over the meridian.

From the apparent right aſcenſion of the ſtar, ſubtract the Sun's right aſcenſion, and the remainder will be the approximate time of the ſtar's tranſit; from which ſubtract the equation correſponding thereto, and the Sun's right aſcenſion, from table XVIII. and from the ſame table, take the equation anſwering to the ſhip's longitude, which muſt be added, if the longitude is eaſt, but ſubtracted, if weſt. Hence the apparent time of the paſſage of the ſtar over the meridian will be obtained.

Now the difference between the obſerved and computed times of the ſtar's tranſit, will be the error of the watch for apparent time, and which is faſt or ſlow, according as the time by obſervation is later or earlier than the computed time of the ſtar's tranſit.

EXAMPLE.

July 23d 1792, in latitude 35° 48′ S. and longitude 23° 26′ E. the following equal altitudes of Altair were obſerved. Required the error of the watch for apparent time?

	Time per watch.		Altitude.		Time per watch.
	8h. 17′ 0″	—	27° 23′	—	14h. 35′ 57′
	19 16	—	27 40	—	33 42
	20 12	—	27 55	—	32 44
	21 54	—	28 12	—	31 5
	23 16	—	28 30	—	29 41
	8 25 55	—	28 52	—	14 27 1
Sum	7 33				10 10
Mean	8 21 15.5	—	—	—	14 31 41.6
					8 21 15.5
					22 52 57.1

		h	′	″
Obſerv'ed time of tranſit,	—	11	26	28.6
Altair's right aſcenſion,	— —	19	40	40
Sun's right aſcenſion at noon, p. N. Alm.		8	13	41
Approximate time of Altair's tranſit,		11	26	59
Equation to 8h. 14′ and 11h. 27′ tab. xviii.	—		1	53
Equation to 8h. 14′ and 23° 26′ tab. xviii.	+			16
Apparent time of ſtar's tranſit,	—	11	25	12
Apparent time of tranſit per watch,	—	11	26	29
Watch faſt,	— — — —		1	17

PROBLEM V.

To find the Error of a Watch, by equal Altitudes of a Fixed Star, the Ship being under way.

RULE.

Let the time per watch of the ſtar's tranſit be inferred from the equal altitudes as uſual, and let it be reduced to the meridian of either place of obſervation, by applying thereto the equations ariſing from the ſhip's run, as directed in prob. III.

Then compute the apparent time of the tranſit of the obſerved ſtar, as in laſt problem, the difference between which, and the time per watch inferred from equal altitudes, will be the error of the watch for apparent time.

EXAMPLE.

November 26, 1792, the time per watch of the tranſit of Bellatrix, as inferred from the mean of ſeveral equal altitudes, was 12h. 52′ 18″, and the corrected azimuth of its center 59° 40′, from the ſouth; the diſtance run in the elapſed time was 3¼ miles E. N. E.

and the latitude and longitude of the ſhip at the time of the ſtar's tranſit were 41° 33′ N. and 51° 18′ W. reſpectively. Required the error of the watch for apparent time, under the meridian of the weſternmoſt place of obſervation?

Conſtant logarithm 1.4771*
Diſt. run 34′ P. L. = 0.7238
Lat. 41° 33′ coſ. = 9.8741

2.0750 — — — 2.0750
Courſe 6 points, ſec. = 0.4171 — co-ſecant — 0.0344

Azimuth 59° 40′ tan. = 0.2327 Second corr. 1′ 24″ P. L. 2.1094

Firſt cor. 20″ P. L. = 2.7248

Time of tranſit of Bellatrix per equal altitudes		12h.	52′	18″
Firſt correction,	+			20
Second correction,	+		1	24
Time of tranſit reduced to the firſt place of obſervation,		12	54	2
Right aſcenſion of Bellatrix,		5	14	2
Sun's right aſcenſion,		16	12	16
Approximate time of tranſit		13	1	46
Equation to 16h. 12′ and 13h. 2′ p. tab. xviii.	−		2	20
Equation to 16h. 12′ and 51° 18′§ tab. xviii.	−			36
Apparent time of tranſit,		12	58	50
Time per watch of tranſit,		12	54	2
Watch ſlow			4	48

In obſerving equal altitudes of a fixed ſtar, it will be proper to uſe one whoſe declination is a little greater than the latitude of the ſhip, and of the ſame name. Hence, in places diſtant from the equator, the motion of the ſtar in altitude will be a maximum, when near the meridian; and its altitude when in this poſition may be found by the following formula: Sine alt. = ſine lat. coſecant declination, to radius unity. In places conſiderably diſtant from the equator, the interval between the obſervations will be much contracted, and therefore any probable irregularity in the rate of the watch will be diminiſhed. In theſe latitudes, the refraction near the horizon is very variable; but as the altitude of a ſtar, when in a proper poſition for obſervation, increaſes with the latitude, therefore in this caſe no material error can ariſe from the change of refraction between the obſervations.

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* When the courſe and diſtance made good between the obſervations, are given in place of the obſerved interval, and hourly rate of ſailing, as in the above example, the conſtant log. 1.4771 (= 9.2219 + P. L. of one min.) is to be uſed inſtead of the former.

§ In ſtrictneſs, the argument of this equation ſhould be the longitude of that meridian, to which the error of the watch is to be found.

PROBLEM VI.

Given the Latitude of a place, the Altitude and Declination of the Sun, to find the Apparent Time of Obſervation, and the Error of the Watch.

RULE.

Correct the obſerved altitude of the Sun's limb, and reduce the declination to the time and place of obſervation, which ſubtracted from or added to 90°, according as the declination and latitude are of the ſame or of contrary names, the remainder or ſum will be the Sun's *polar diſtance.*

Now add together the Sun's corrected altitude and polar diſtance, and the latitude of the place of obſervation, and call the difference between half the ſum of theſe and the altitude, the *remainder.*

Then to the log. co-ſecant of the polar diſtance, add the log. ſecant of the latitude, the log. co-ſine of the half ſum, and the log. ſine of the remainder; half the ſum of theſe will be the log. ſine of an arch—which, being multiplied by 8, will be the Sun's diſtance from the meridian in apparent time. Hence the apparent time of obſervation and the error of the watch will be known.

REMARK.—In practice, it will be neceſſary to take ſeveral altitudes and the correſponding times per watch, within one or two minutes of each other; and in making the computations, it will be found very convenient, and at the ſame time ſufficiently accurate, to eſtimate the altitude, declination, and latitude in degrees, minutes, and tenths of a minute, inſtead of degrees, minutes, and ſeconds. Moreover, in this and the following computations, 10 is always rejected from the index of the log. ſecant or tangent, when it exceeds that quantity.

EXAMPLES.

March 3, 1792, in latitude 45° 37′ N. and longitude 51° 19′ W. the following altitudes of the Sun's lower limb were obſerved, the height of the eye being 16 feet above the ſurface of the ſea. Required the apparent time of obſervation, and the error of the watch?

Time per watch.		Alt. ☉'s l. limb.		
2h. 53′ 32″	—	24° 59′	☉'s dec. at noon p. N. A. =	6° 25.′4
54 30	—	52	Eq. tab. XIII. to dec. and 2h. 55′ P. M. } =	− 2.8
55 36	—	44	— to dec. and 51° 19′ W. =	− 3.2
56 47	—	35		
20 25		190	Reduced declination,	6 19.4
Mean 2 55 6		24 47.5	Polar diſtance, —	96 19.4
Semidiameter,	—	+ 16.2		
Dip	—	− 3.8		
Correction	—	− 1.9		

Cor. alt. ☉'s center	= 24 58.0			
Sun's polar diſtance	= 96 19.4	—	co-ſecant	0.00265
Ship's latitude	= 45 37.0	—	ſine —	0.15524
Sum —	166 54.4			
Half —	83 27.2	—	co-ſine	9.05695
Remainder —	58 29.2	—	ſine —	9.93071
				19.14555
Arch —	21 57.4	—	ſine —	9.57277
	8			
Apparent time,	2 55 39			
Time per watch,	2 55 6			
Watch ſlow, —	— 33			

At noon, October 9, 1794, the latitude of the ſhip by obſervation was 43° 12′ N. and longitude by account 18° 20′ W. In the afternoon, ſeveral altitudes of the Sun's lower limb, and the correſponding times by the watch, were obſerved, in order to find the error of the watch. The mean of the altitudes was 22° 51′, and that of the times of obſervation 3h. 16′ 47″, and height of the eye 18 feet. Since noon the courſe per compaſs was N. E. by E. 7 knots per hour, and variation 1¾ points weſterly. Required the error of the watch?

Diſtance run = 3h. 17′ × 7k. = 23 miles; and 1¾ points allowed to the left of N. E. by E. gives N. E. ¾ N. the true courſe, to which and the diſtance 23 miles, the difference of latitude is 18.5 m. and departure 13.7 m. Now to latitude 43°, taken as a courſe, in a traverſe table, and departure 13.7 in a lat. column, the diſtance is 19 m. which is therefore the difference of longitude.

La-

Latitude at noon		43°	12′ N.	Long. at noon —	18°	20′ N.
Diff. of latitude			18 N.	Difference of longitude,		19 E.
Preſent latitude		43	30 N.	Long. at time of obſerv.	18	1 W.
Alt. Sun's l. limb		22	51	Sun's declination at noon	6	27 S.
Semidiameter		+	16	Eq. to dec. and 3h. 17′ P.M.	+	3
Dip and refract.		—	6	and 18° 1′ W.	+	1
Corrected alt.		23	1	Reduced declination,	6	31 S.
Polar diſtance		96	31	— co-ſecant —		0.00281
Latitude, —		43	30	— ſecant —		0.13944
Sum —		163	2			
Half —		81	31	— co-ſine, —		9.16886
Remainder,		58	30	— ſine, —		9.93077
						19.24188
Arch, —		24	42	— ſine — —		9.62094
			8			
App. time	3	17	36			
Timep. watch	3	16	47			
Watch ſlow,			49			

Many different methods might be given for computing from the ſame data, the horary diſtance of an object from the meridian. It is however thought proper to ſubjoin only the two following, the firſt of which is nearly ſimilar to that given in the ſecond edition of the *Requiſite Tables*; and the other is performed by means of a ſet of tables calculated expreſsly for that purpoſe—vide tables XXVII. XXVIII. XXIX.

METHOD SECOND.

For computing the Horary Diſtance of an Object from the Meridian.

RULE.

If the latitude and declination are of different names, let their ſum be taken, otherwiſe their difference; from the natural co-ſine of this ſum or difference, ſubtract the natural ſine of the corrected altitude, and find the logarithm of the remainder; to which add the log. ſecants of the latitude and declination, the ſum will be the log. riſing of the horary diſtance of the object from the meridian; and if this object is the Sun, the apparent time is known as before.

EXAMPLE.

September 15th, 1792, in latitude 33° 56′ S. and longitude 18° 22′ E. the mean of the times per watch was 8h. 12′ 10″ A. M. and that of the altitudes of the Sun's lower limb 24° 48′; height of the eye 24° feet. Required the error of the watch?

Obſ. alt. ☉'s l. limb	24° 48′	☉'s dec. at noon p. N. A.		2° 40′.5 S.
Semidiameter	+ 16.0	Equat. to 3h. 48′ A. M.		+ 3.7
Dip —	— 4.7	to 18° 22′ E.		+ 1.2
Correction	— 1.9	Reduced declination,		2 45.4
Cor. alt. ☉'s center	24 57.4			
Latitude —	33 56	— ſecant		0.08109
Declination —	2 45.4	— ſecant		0.00050
Sum — —	36 41.4	N. co-ſine	80188	
Sun's altitude	24 57.4	N. ſine	42193	
			37995 log.	4.57973
Sun's mer. diſt.	3h. 48′ 51	— riſing —		4.66132
Apparent time	8 11 9			
Time p. watch	8 12 10			
Watch faſt —	1 1			

METHOD THIRD.

For computing the Meridian Diſtance of an Object.

RULE.

Enter tab. xxvii. with the declination of the object at the top, and the latitude of the place of obſervation in the ſide column; take out the correſponding number, to which prefix the index 4, and add to it the log. ſine of the corrected altitude; find the natural number anſwering thereto, to which apply the number from table xxviii. by ſubtraction or addition, according as the latitude and declination are of the ſame or of contrary names. Now find the above difference or ſum in table xxix. and the correſponding time will be the diſtance of the object from the meridian.

EXAMPLE.

May 6, 1791, in latitude 56° 4′ N. and longitude 38° 30′ W.

at 4h. 37′ 4″ P. M. per watch, the altitude of the Sun's lower limb was 25° 6′.1, and height of the eye 18 feet. Required the error of the watch for apparent time?

Alt. ⊙'s lower limb		25° 6′.1
Semidiameter	+	15.9
Dip	−	4.1
Correction,	−	1.9
Cor. alt. ⊙'s center		25 16.0

⊙'s dec. p. N. Alm. =		16° 35′.8 N.
Eq. to 4h. 37′P. M.	+	3.4
to 38° 30′ W.	+	1.8
Reduced declination,		16 41.0 N.

To latitude 56° 4′, and declination 16° 41′, the number from

Table xxvii. =	4.2719	Table xxviii. =	4455
Alt. 25° 16′ ſine	9.6303		
Sum —	3.9022	Natural number	7984
Apparent time	4h. 37′ 20″	per table xxix.	3529
Time p. watch	4 37 4		
Watch ſlow,	16		

Problem VII.

Given the Latitude of a Place, the Altitude of a known Fixed Star, and the Sun's Right Aſcenſion, to find the Apparent Time, and Error of the Watch.

Rule.

Correct the obſerved altitude of the ſtar, and let its declination and right aſcenſion be reduced to the time of obſervation.

With the latitude of the place, the true altitude, and apparent declination of the ſtar, compute its horary diſtance from the meridian by any of the methods given in laſt problem; which being added to, or ſubtracted from, its right aſcenſion, according as it was obſerved in the weſtern or eaſtern hemiſphere, the ſum or remainder will be the right aſcenſion of the meridian.

From the right aſcenſion of the meridian, ſubtract the Sun's right aſcenſion, as given in the Nautical Almanac for the noon of the propoſed day; the remainder will be the *approximate time* of obſervation, from which ſubtract the equation anſwering thereto, and the Sun's right aſcenſion, from table xviii. and let the equation from the ſame table, correſponding to the longitude, be added or ſubtracted according as the ſhip is to the eaſt or weſt of Greenwich, and the reſult will be the apparent time of obſervation. Hence the error of the watch will be known.

Exam-

Example.

December 12, 1792, in latitude 37° 46′ N. longitude 21° 15′ E. a certain phenomenon was obſerved, and at the ſame inſtant the altitude of Arcturus, eaſt of the meridian, was obſerved to be 34° 6′.4; the height of the eye 10 feet. Required the apparent time of obſervation?

Obſerved alt. of Arcturus =		34° 6′.4			
Dip	—	− 3.0			
Refraction	—	− 1.4			
True altitude of Arcturus =		34 2.0			
Polar diſtance	—	69 43.6	—	co-ſecant	0.02778
Latitude	—	37 46.0	—	ſecant	0.10209
Sum	—	141 31.6			
Half	—	70 45.8	—	co-ſine	9.51781
Remainder	—	36 43.8	—	ſine —	9.77674
					19.42442
Arch	—	31 2	—	ſine —	9.71221
		8			
Arcturus eaſt of meridian =		4 8 16			
Arcturus' right aſcenſion =		14 6 13			
Right aſcenſion of mer. =		9 57 57			
Sun's right aſcenſion		17 21 59			
Approximate time		16 35 58			
Eq. to long. tab. xviii.		+ 16			
Eq. to approx. time		− 3 3			
App. time of obſervation		16 33 11			

Remark. In order to attain the greateſt accuracy from obſervations of this kind, ſeveral ſtars ſhould be obſerved, and the error of the watch deduced from each ſtar ſeparately. If an equal number of ſtars be obſerved on each ſide of the meridian, and nearly equidiſtant therefrom, thoſe errors which ariſe from the inſtrument, the ſpheroidal figure of the Earth, &c. will by this means be rendered almoſt inſenſible. If the ſhip is under way, during the interval between the obſervations of the different ſtars, and if that interval is conſiderable, it will be neceſſary to reduce the error to the ſame meridian, by allowing for the difference of longitude made good between the obſervations.

Example.

January 29, 1792, in latitude 53° 24 N. and longitude 25° 18′ W. by account, the following altitudes of Procyon to the weſt, and Alphacca to the eaſt of the meridian, were obſerved by two ſeparate perſons at the ſame inſtant. The elevation of the perſon's eye, who obſerved the altitude of Procyon, was 20 feet above the ſurface of the ſea, and the height of his eye, who obſerved the altitude of Alphacca, was 22 feet. Required the error of the watch?

	Time per watch.	Alt. Procyon.	Alt. Alphacca.
	14h. 53′ 0″	20° 46′	41° 23′
	14 57 0	20 12	42 2
	15 0 32	19 42	42 29
	15 4 0	19 13	43 0
	234 32	233	54
Mean	14 58 38	19 58 2	42 13.5
Dip		— 4.3	— 4.5
Refraction		— 2.6	— 1.0
Cor. altitude		19 51.3	42 8.0

Computation of the app. time from the corrected alt. of Procyon.

Procyon's altitude	= 19° 51′.3			
polar diſtance	= 84 14.8	—	co-ſecant	0.00219
Ship's latitude, —	53 24.0	—	ſecant	0.22459
Sum — —	157 30.1			
Half — —	78 45.0	—	co-ſine	9.29024
Difference —	58 53.7	—	ſine	9.93259
				19.44961
Arch — —	32 2.9 8	—	ſine	9.72480

Procyon's mer. diſtance	4 16 23
—— right aſcen.	7 28 24
Right aſc. meridian	11 44 47
Sun's right aſcenſion	20 47 22
Approximate time,	14 57 25
Equation to long.	— 17
Eq. to approx. time	— 2 36
Apparent time,	14 54 32

Computation of the apparent time from the corr. alt. of Alphacca.

Alphacca's altitude	= 42° 8′		
—— polar diſt.	= 62 34.6	—	co-ſecant 0.05177
Ship's latitude —	53 24	—	ſecant — 0.22459
Sum — —	158 6.6		
Half — —	79 3.3	—	co-ſine — 9.27845
Difference —	36 55.3	—	ſine — 9.77867
			19.33348
Arch — —	27 39.6 8	—	ſine — 9.66674

Alphacca's mer. diſt.	3 41 17
—— right aſcen.	15 25 52
Right aſcenſion mer.	11 44 35
Sun's right aſcenſion,	20 47 22
Approximate time —	14 57 13
Equat. to time and lon. —	2 53
Apparent time —	14 54 20
App. time by ob. Proc.	14 54 32
Mean, — —	14 54 26
Time per watch, —	14 58 38
Watch faſt, —	4 12

PROBLEM VIII.

Given the Altitude of a Planet, to find the apparent time of obſervation.

RULE.

Let the mean of ſeveral altitudes of the planet's center be corrected by the dip of the horizon, and the refraction as uſual.*

Reduce the declination of the planet, as given in the Nautical Almanac, to the time and place of obſervation, and from thence find its polar diſtance; with which, its corrected altitude, and the latitude of the ſhip, compute its meridian diſtance by prob. VI.

Find the proportional part anſwering to the interval between the reduced time of obſervation, and the time of the planet's preceding paſſage over the meridian; which added to, or ſubtracted from the time of the preceding tranſit, according as the following tranſit happens at a later or an earlier hour, and the ſum or difference, will be the reduced time of tranſit; to which the planet's meridian diſtance being applied by addition or ſubtraction, according as it was obſerved in the weſtern or eaſtern hemiſphere, and the ſum or difference, will be the apparent time of obſervation: the difference between which, and that ſhown by the watch, will be its error.

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* If great accuracy is required, the parallax of the planet in altitude muſt be computed from aſtronomical tables, and adde thereto.

EXAMPLE.

February 14, 1792, in latitude 43° 26′ N. and longitude 54° 16′ W. the following altitudes of Jupiter were obſerved, the height of the eye being 14 feet. Required the error of the watch?

Time p. watch =	12h.	21′	10″	Alt. ♃ =	15°	56′
		22	50		16	8
		25	15		16	25
		9	15			29
Mean —	12	23	5		16	9.7
Long. in time	3	37	4	Dip —		3.5
Reduced time	16	0	9	Refract. —		3.2

True altitude of Jupiter	16	3		
Polar diſtance —	100	41	co-ſecant	0.00759
Latitude — —	43	26	ſecant —	0.13896
Sum — —	160	10		
Half — —	80	5	co-ſine —	9.23607
Remainder —	64	2	ſine —	9.95378
				19.33640
Arch — —	27	45¾	ſine —	9.66820
		8		

Jupiter's meridian diſtance 3 42 6

Time of preced. tran.	13d.	16h.	9′		13d. 1h.	9′
of follow. tran.	19	15	46	red. time	14 16	0

5 23 37 : 23′ :: 23 51 : 3′ 49″

Preceding tranſit —	16	9	0
Reduced time of tranſit —	16	5	11
Jupiter's meridian diſtance	3	42	6
Apparent time —	12	23	5
Time per watch —	12	23	5
Watch exact — —			0

PRO-

PROBLEM IX.

Given the Altitude of the Moon, the Latitude and Longitude of a Place, and the eſtimate Time of Obſervation, to find the apparent time.

RULE.

Correct the altitude of the Moon's limb, by prob. XI. page 79. and reduce its declination to the time and place of obſervation, by prob. VII. page 76. Now with the latitude of the place, the altitude and declination of the Moon, compute its diſtance from the meridian.

To the time of the Moon's paſſage over the meridian of Greenwich, as given in the Nautical Almanac, apply the equation from table XX. anſwering to the daily variation of the Moon's paſſage and meridian diſtance, and alſo that correſponding to the daily variation of tranſit, and the longitude of the place of obſervation.— The firſt of theſe equations is additive, if the Moon be in the weſtern hemiſphere at the time of obſervation, but ſubtractive, if in the eaſtern; and the ſecond equation is to be added, if the ſhip's longitude is weſt, otherwiſe ſubtracted. Hence the poſition of the Moon with reſpect to the meridian of the place of obſervation will be known; to which the Moon's meridian diſtance being applied by addition or ſubtraction, according as the Moon was obſerved in the weſtern or eaſtern hemiſphere, and the ſum or remainder will be the apparent time of obſervation.

EXAMPLE.

March 3, 1792, the following altitudes of the Moon's lower limb were obſerved, the height of the eye being 10 feet, and the ſhip's latitude and longitude 51° 38′ N. and 32° 18′ W. reſpectively. Required the apparent time of obſervation, and the error of the watch?

Time per watch = 11h.	24′ 16″	Alt. ☽'s lower l. =	38 12
	27 0	———	37 55
	29 10	——— ———	37 31
	31 0	— ———	37 13
	34 9	——— ———	36 46
	45 35		157
Mean — ——— 11	29 7	——— ———	37 31.4
☽'s decl. at midnight	17 6 N.	augm. ſemidiam.	+ 15.1
Eq. to var. dec. and oh. 31′	+ 2	Dip ———	− 3.0
and long. 32° 18′ W.	− 8	App. alt. ☽'s center	37 43.5
		Correction, tab. IX.	+ 42.0
Reduced declination ———	17 0 N.		
		Cor. alt. ☽'s center	38 25.5

To latitude 51° 38′ N. and declination 17° 0′ N. the number
from table XXVII. = 4.2265 from table XXVIII. = 3862
☽'s alt. 38° 25′.5 ſine 9.7934

4.0199 — Natural number — 10470

Meridian diſtance of moon, 3h. 14′ 33″ Table XXIX. 6608

☽'s paſſage over mer. Gr. 8h. 10′
Eq. to daily var. and 3h. 14′ + 6
and long. 32° 18′ W. + 4

Reduced time — 8 20 0

Apparent time, — — 11 34 33
Time per watch, — 11 29 7

Watch ſlow, — — 5 26

By this method of operation, the time can ſcarcely be found nearer than one minute. If, however, a greater degree of preciſion be required; let the right aſcenſions of the ſun and moon be reduced to the place of obſervation, and to the apparent time before found. Then to the moon's reduced right aſcenſion, apply its meridian diſtance by addition or ſubtraction, according as the moon is obſerved in the weſtern or eaſtern hemiſphere, and the right aſcenſion of the meridian will be obtained: from which ſubtract the ſun's right aſcenſion, and the remainder will be the apparent time of obſervation more correct than that found by the former method. Thus in the preceding example.

⊙'s R. A. at noon is = 22h. 59′ 52″ ☽'s R. A. at noon is = 103° 36′
Eq. to 11h. 34′ t. xviii. + 1 48 — — + 6 0½
to long. 32° 18′ W. + 20 — — + 1 7

Reduced right aſcenſ. 23 2 0 — — 107 43½
4

Moon's right aſcenſion in time — 7 22 54
Moon's meridian diſtance — — 3 14 33

Right aſcenſion of the meridian — 10 37 27
Sun's right aſcenſion — — 23 2 0

Apparent time — — — 11 35 27
Time per watch — — 11 29 7

Watch ſlow, — — — 6 20

If the error of the watch, and the change of the moon's declination be conſiderable, the computation of the moon's diſtance from the meridian muſt be repeated.

If the apparent time at Greenwich be known, the time at the ſhip may be very accurately found, by taking out the right aſcenſions of the ſun and moon, and the moon's declination agreeable to that time. Then with the ſhip's latitude, the moon's correct altitude and declination, its meridian diſtance is to be computed; which being compared with its right aſcenſion, will give the right aſcenſion of the meridian, from which, the ſun's right aſcenſion being taken, the remainder will be the apparent time of obſervation.

Since the right aſcenſion and declination of the moon, cannot be accurately reduced to any given time and meridian by even proportion; this method of aſcertaining the time is, therefore, ſtill liable to ſome uncertainty, unleſs the above data be previouſly corrected by the equation of ſecond difference from table XXXVII. or computed from aſtronomical tables, for the given time reduced to the meridian of Greenwich.

The poſition of a celeſtial object, moſt favourable for determining the apparent time with accuracy is, when it is in the prime vertical; or, if it does not ſet, when in that part of its diurnal path which is in contact with an azimuth circle; as then, the change of altitude is quickeſt, and the latitude is not an eſſential element. The nearer an object is to either of theſe poſitions, the leſs will be the error in the time, ariſing from given unavoidable errors in the obſervations and latitude. Table XXV. contains the altitude moſt proper for this purpoſe.

When the declination and latitude are of contrary names; the object is paſt the prime vertical before it riſes; and is ſet before it comes to that circle: in this caſe, and upon account of the irregularity to which the refraction near the horizon is liable; the altitude of the object ſhould not be under three degrees, nor its meridian diſtance leſs than two hours, at the time of obſervation. Otherwiſe, it is not proper to uſe that object for the purpoſe of aſcertaining the apparent time.

It may be alſo obſerved, that, the leſs the latitude and declination, and the greater the altitude of the object, the leſs is the error in the apparent time, anſwering to a given error in the altitude, declination or latitude.

The error in the apparent time anſwering to a given error in any of theſe quantities may be found by computation; for which purpoſe the three following problems are inſerted.

PROBLEM X.

Given the Latitude, Azimuth, and Error in Altitude; to find the Error in the apparent Time anſwering thereto.

RULE.

To the conſtant log. 1.1761 add the log. co-ſine of the latitude, the log. ſine of the azimuth, and the proportional log. of the error in altitude; the ſum[*] will be the prop. log. of the error of apparent time, which is additive to the meridian diſtance, if the error of altitude be in exceſs, otherwiſe ſubtractive.

EXAMPLE.

Let the latitude of the place of obſervation be 51° 31′ N. azimuth of the obſerved object S. 48° 10′ E. and error of altitude 10′. Required the correſpondent error of the apparent time?

Conſtant log.			1.1761
Latitude	51° 31′	Co-ſine	9.7940
Azimuth	48 10	Sine	9.8722
Error in altitude	10	P. Log.	1.2553
Error in the apparent time	1′ 26″	P. Log.	2.0976

PROBLEM XI.

Given the Error in Declination, to find the correſponding Error in the apparent time.

RULE.

To the conſtant log. 1.1761 add the log. co-ſine of the declination, the log. tangent of the angle of poſition[†], and the prop. log. of the error of declination; the ſum will be the prop. log. of the error of the apparent time, which is additive to the meridian diſtance, if the correction of declination is to be ſubtracted from the elevated polar diſtance; otherwiſe, this correction is ſubtractive.

EXAMPLE.

Let the declination be 18° 37′, angle of poſition 37° 29′, and error of declination 10′. Required the reſulting error of the apparent time?

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[*] It is always to obſerved, that in order to ſimplify the operation, tens are conſtantly to be rejected from the indices, and the ſums of indices.

[†] The angle of poſition may be computed by the following formula. Sine angle of poſition = Coſ. lat. ſec. alt. ſine mer. diſt. = coſ. lat. ſec. decl. ſine azimuth to radius unity.

Conſtant log.	— —	—	1.1761
Declination —	18° 37′	— co-ſine -	9.9767
Angle of poſition —	37 29	— tangent -	9.8847
Error in declination —	10	— P. Log. -	1.2553
Error in the app. time	0.55″	P. Log.	2.2928

PROBLEM XII.

Given the Error in Latitude, to find the Error in the Apparent Time anſwering thereto.

RULE.

To the conſtant log. 1.1761, add the log. coſine of the latitude, the log. tangent of the azimuth, and the prop. log. of the error in latitude; the ſum will be the prop. log. of the error of the apparent time, which is additive, if the eſtimate latitude is greater than the true latitude, otherwiſe ſubtractive, the azimuth being leſs than 90°; if greater, the contrary rule is to be applied.

EXAMPLE.

Let the latitude be 54° 42′, the azimuth 26° 17′, and the error in latitude 10′. Required the error of the apparent time?

Conſtant log.	— —	—	1.1761
Latitude —	54° 42′	— co-ſine —	9.7618
Azimuth —	26 17	— tangent —	9.6936
Error in latitude —	10	— P. Log. —	1.2553
Error in apparent time	2′ 20″	P. Log.	2.8868

In the preceding methods of computing the apparent time at ſea, the altitude and declination of the obſerved object, and the latitude of the ſhip, were the neceſſary data. It is however almoſt impoſſible to obtain theſe elements ſufficiently correct at ſea; we have therefore ſhown how to make an allowance for any probable error that may be in either of theſe quantities.— But if thoſe errors are not known, a correction cannot be applied. It is therefore thought proper to ſubjoin the three following methods of finding the apparent time, in the firſt of which the latitude is not required as a neceſſary element; in the ſecond, neither the latitude nor altitude of the obſerved object are neceſſary; and in the third method, neither the latitude nor declination of the object are requiſite.

PROBLEM XIII.

Given the Latitude of the Place of Obſervation, to find the Apparent Time when two known Fixed Stars were obſerved in the ſame Vertical.

RULE.

To the log. co-tangent of the declination of the ſtar neareſt the elevated pole, add the log. tangent of the other ſtar's declination, and the log. co-ſecant of the difference of their right aſcenſions; the ſum will be the log. of a natural number, the difference between which, and the natural co-tangent of the difference of right aſcenſion,* will be the natural tangent of arc *firſt*, if the declinations of the ſtars are of the ſame name; but if of contrary names, the ſum of thoſe quantities will be arc *firſt*.

Now to the log. co-ſine of arc firſt, add the log. tangent of the latitude, and the log. co-tangent of the declination of the ſtar neareſt the meridian; the ſum will be the log. coſine of arc *ſecond.* The difference between arcs firſt and ſecond, when the common azimuth is leſs than a quadrant, otherwiſe their ſum, will be the meridian diſtance of the ſtar neareſt the meridian. Hence the apparent time is found, as in prob. VII.

EXAMPLE.

July 1, 1792, in latitude 57° 9′ N. and longitude 2° 9′ W. the ſtars Vega and Altair were obſerved in the ſame vertical at 10h. 9′ per watch. Required the apparent time of obſervation, and the error of the watch?

App. R. A. of Vega =	18h. 29′ 56″				
of Altair =	19 40 40				
Diff. of app. right aſ. =	1 10 44 = 17° 41′			Co-ſecant —	0.517475
App. dec. of Vega =	38° 36′ 0″			—— Co-tangent -	0.097840
of Altair =	8 19 50			—— Tangent —	9.165627
			60387	-	9.780942
Difference of right aſc.	17 41 0	N. co-tan.	313656		
Arc firſt ——	68 27 20	N. tang.	253269	Co-ſ.	9.564930
Latitude ——	57 9 0			—— Tangent	0.189975
Appar. decl. of Vega	38 36 0			—— Co-tangent	0.097840
Arc ſecond ——	44 34 0			—— Co-ſine —	9.852745
Arc firſt ——	68 27 20				
Mer. diſtance of Vega	23 53 20 = 1h. 35′ 33″				

P

Mer.

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[*] The difference of the right aſcenſions of the two ſtars is ſuppoſed to be leſs than ſix hours, ſince in practice it will ſcarcely ever exceed that quantity.

Mer. diſtance of Vega 23 53 20 =	1h.	35′	33″
Apparent right aſcenſion of Vega ——	18	29	56
Right aſcenſion of the meridian	16	54	23
Sun's right aſcenſion ——	6	44	23
Approximate time —— ——	10	10	0
Equation to approx. time ——	—	1	45
to longitude ——	—		1
Apparent time —— ——	10	8	14
Time per watch, —— ——	10	9	0
Watch faſt, —— ——			46

Remark. If the right aſcenſions are equal, the vertical will be the meridian; and hence the apparent time is eaſily found, by ſubtracting the Sun's right aſcenſion from that of the ſtar, and applying the equations as formerly.

If the ſtars be obſerved on the ſame vertical at different times, the difference between the obſerved interval reduced to ſiderial time, and the difference of right aſcenſion, is to be uſed in place of this laſt quantity.

Problem XIV.

Given the Interval of Time between the Riſing of two known Fixed Stars, to find the Apparent Time of either Obſervation.

Rule.

Reduce the obſerved interval to ſiderial time, which may be done with ſufficient accuracy, by adding to the interval in mean ſolar time, the proportional part anſwering to 24 hours, the above interval, and 3′ 56″, which is the mean daily increaſe of the Sun's right aſcenſion; and let the difference between the reduced interval, and the difference of the right aſcenſions of the ſtars, be called arc firſt.

Now, to the log. co-ſecant of arc firſt, add the log. co-tangent of the declination of the firſt obſerved ſtar, and the log. tangent of the declination of the other ſtar; the ſum will be the log. of a natural number, the ſum or difference of which, and the natural co-tangent of arc firſt, according as the declination of the ſtars are of different or of the ſame affection, will be the natural tangent of the meridian diſtance of the firſt obſerved ſtar, if the declination of that ſtar, and the latitude are of contrary names; otherwiſe, the ſupplement of the above quantity will be the meridian diſtance of that ſtar, and hence the apparent time is found as formerly.

Exam-

Example.

October 25th, 1792, in longitude 21° E. by account, the interval in mean time between the riſing of Aldebaran and Rigel was 3h. 17′ 30″. Required the apparent time of the riſing of Aldebaran: and ſuppoſe the watch to have pointed out 6h. 39′ at that time, its error is required?

App. R. A. of Rigel = 5h. 4′ 36″ Obſerved interval 3h. 17′ 30″
of Aldebaran = 4 24 3 Proportional part + 32

Difference of R. A. — 0 40 33 Reduced interval, 3 18 2
Reduced interval — 3 18 2

Arc firſt — 2 37 29 = 39° 22′ 15″ co-ſecant 0.197680
App. decl. Aldebaran 16 4 55 —— co-tangent 0.540165
App. decl. Vega — 8 27 9 —— tangent — 9.172029

81260 — 9.909874
Arc firſt — 39 22 15 N. co-tang. 121868

63 47 16 N. tang. 203128

Mer. diſt. Aldebaran 116 12 44 = 7h. 44′ 51″
R. A. Aldebaran — — — 4 24 3

R. A. of the meridian — — 20 39 12
Sun's R. A. — — — 14 2 18

Approximate time — — — 6 36 54
Equation to approximate time — − 1 3
to longitude E. — + 13

Apparent time of riſing of Aldebaran, 6 36 4
Time per watch, — — — 6 39 0

Watch faſt, — — — — 2 56

Problem XV.

Given three Altitudes of the Sun, or a fixed Star, with the Intervals of Time between the Obſervations, to find the apparent time of either Obſervation.

Rule.

Let half the interval between the obſervations of the two greateſt altitudes be reduced to degrees, and called arc *firſt;* call half the interval between the inſtants when the two leaſt altitudes were obſerved, arc *ſecond;* and let the ſum of arcs firſt and ſecond be arc *third.*

From the natural ſine of the greateſt altitude, ſubtract the natural ſines of the other two altitudes reſpectively; call the greateſt remainder arc *fourth*, and the leaſt arc *fifth*.

Now to the ar-co-log. of arc fourth, add the log. of arc fifth, and the log. ſine of arc third; the ſum, rejecting radius, will be the log. of arc ſixth. To the log. ſine of arc firſt, add the log. coſine of arc ſecond; the ſum, rejecting radius, will be the log. of arc ſeventh. To the ar-co-log. of the difference of arcs ſixth and ſeventh, add the log. ſines of arcs firſt and ſecond; the ſum, rejecting radius, will be the log. tangent of arc eight, the difference between which and arc third, is the apparent time from noon, when the greateſt altitude was obſerved.

REMARK. If the object is a ſtar, the obſerved intervals muſt be reduced to ſiderial time.

Call the index of the log. of the difference of the natural ſines 9, when that difference is 1 leſs than the radius of the table, 8 when it is 2 leſs, 7 when 3 leſs, and ſo on.

EXAMPLE.

At 9h. 51′ 58″ A. M. per watch, the correct altitude of the Sun's center was 21° 11′, at 10h. 48′ 54″ the altitude was 24° 40′, and at 11h. 29′ 42″ the altitude was 26° 0′. Required the true apparent time when the greateſt altitude was obſerved?

```
Time per watch. |
 9h. 51′ 58″
10   48 54Diff.=56′ 56″Half=28′ 28″In deg. 7° 7′arc 1ſt } 12° 13′
11   29 42Diff.=40 48″Half=20  24 In deg. 5  6 arc 2d } =arc 3d.
Alt.=21°  11  N. ſine 36135
        24  40  N. ſine 41734 diff.=7702=arc fourth.
        26   0  N. ſine 43837 diff.=2103=arc fifth.
Arc fourth=7702 ar-co-log. 1.11340 arc firſt 5° 6′  ſine    8.94887
Arc fifth  =2103        log. 8.32284 arc ſec. 7  7  co-ſine 9.99664
Arc third  =12° 13′   ſine  9.32553 arc ſev. 8821    log.  8.94551
Arc ſixth  =     —          8.76177  —     5778
                             Difference  —  3043ar-co-lo. 1.51670
                             Arc firſt   —   5° 6′ ſine — 8.94887
                             Arc ſecond, — 7   7 . ſine — 9.09304
                             Arc eighth - 19 54 - tang. - 9.55861
                             Arc third  -  12 13
                             Merid. diſt. - 7 41 in time=30′ 44″
                             Apparent time     —      11  29  16
                             Time per watch    —      11  29  42
                             Watch faſt     ——      ——        26
```

In the preceding problems, for computing the apparent time from the obſervation of the altitude of a celeſtial object, the Earth was ſuppoſed ſpherical, and the computations were performed agreeable to that hypotheſis. But becauſe the figure of the Earth is that of an oblate ſpheroid, the latitude of a place, as inferred directly from obſervation, is therefore greater than the angle at the Earth's center, contained between the equatorial radius, and a line joining the Earth's center and the place of obſervation. This reduction of latitude, according to Sir Iſaac Newton's hypotheſis, is contained in table xxxvi.

As the altitude of a celeſtial object is obſerved from the viſible horizon, whoſe pole is the apparent zenith; therefore, in order to make an allowance for the ſpheroidal figure of the Earth, a correction muſt be applied to the obſerved altitude, to refer it to the reduced horizon. Table xxvi. contains this reduction. Since the reduced zenith is always more diſtant from the elevated pole than the apparent zenith, the above correction is therefore additive, when the azimuth reckoned from the meridian is leſs than 90°, but ſubtractive, when it exceeds that quantity.

In order to illuſtrate the above, the following example of the computation of the apparent time, from an obſervation of the Sun's altitude, is inſerted.

June 18, 1792, in latitude 57° 8′.9 N. in the afternoon the altitude of the Sun's lower limb was 42° 24′, and height of the eye 13 feet. Required the apparent time of obſervation?

Altitude ——		42°	24′	Latitude ——		57° 8′.9
Semidiameter —	+		15.8	Reduction —	−	13.7
Dip —	−		3.4			
Correction —	−		.9	Reduced latitude		56 55.2
Corrected alt.		42	35.5			
Red. tab. xxxvi.	+		5.7			

Reduced altitude		42	41.2			
Polar diſtance		66	33.2	—	co-ſecant	0.0374267
Latitude —		56	55.2	—	ſecant —	0.2629589
Sum —		166	9.6	—		
Half —		83	4.8	—	co-ſine -	9.0809269
Remainder —		40	23.6	—	ſine —	9.8115960
						19.1929085
		23	15.5	—	ſine —	9.5964542
			8			
Apparent time,	3	6	4			

CHAP.

CHAP. IV.

Of the Methods of clearing the Apparent Diſtance between the Moon and the Sun or a Fixed Star, from the effects of Refraction and Parallax.

SINCE the obſerved altitude of a celeſtial object is affected by two phyſical cauſes, the refraction and parallax, whoſe effects are produced in a vertical direction; it is therefore obvious, that the obſerved diſtance between any two objects will be alſo affected by theſe cauſes. Indeed with regard to the fixed ſtars, the parallax vaniſhes; and therefore thoſe objects are affected by refraction only. But in obſervations of the Moon particularly, the effect of parallax is very ſenſible, upon account of its proximity to the earth. Therefore, by reaſon of the above cauſes, the true diſtance between the Moon and any celeſtial object is, for the moſt part, conſiderably different from that obſerved.

Let Z. (fig. 24.) repreſent the zenith, *s* the apparant place of the Sun, and *m* that of the Moon; the arch *sm* will therefore be the apparent diſtance between theſe objects. Alſo ZS, ZM, being vertical circles, paſſing thro' the centers of the Sun and Moon, the true and apparent places of theſe objects will be found therein.

Now ſince the refraction is ever greater than the parallax of the Sun at the ſame altitude, the true place of the Sun will therefore be lower than the apparent place, which let be S, and becauſe the moon's parallax at any given altitude, is greater than the refraction at that altitude, its true place will therefore be higher than the apparent place, which let be M, and SM will be the true diſtance.

The method of reducing the apparent to the true diſtance, or in other words, that of clearing the apparent diſtance from the effects of refraction and parallax, being the moſt tedious part of the calculus for aſcertaining the longitude, when the calculation is performed by the common ſpherical analogies; many eminent aſtronomers and mathematicians have therefore given compendiums to facilitate the ſolution of this problem; among which are thoſe by the Chevalier de Borda, the Abbe de la Caille,* Meſſrs Dunthorn, Emerſon, Jeaurat, Krawft, de la Lande, Lyons, Maſkelyne, Robertſon, Romme, Witchel, Vince, &c.—but the largeſt and moſt elaborate work, that has hitherto appeared for the purpoſe of correcting the apparent diſtance, is the *Cambridge Tables.* †

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* Mr James Ferguſſon's Parallactic Rotula is conſtructed on the ſame principles as the Abbe de la Caille's *Chaſſis de Reduction.*

† Theſe tables are adapted to give the correction, anſwering to the diſtance between the Moon and a ſtar only; therefore, when the true diſtance between the Sun and Moon is required, it becomes neceſſary to apply another correction depending on the Sun's

All the methods that have hitherto been given, for the purpoſe of reducing the apparent to the true diſtance, depend on one or other of the two following principles; of which, the firſt appears to be the moſt ſimple and accurate.

Firſt General Principle.

With the apparent zenith diſtances *Zm*, *Zs* (fig. 24.) and the apparent diſtance between the objects *ms*, compute the vertical angle mZs: with which, and the true zenith diſtances, *ZS*, *ZM*, the true diſtance *MS*, may be found.

Second General Principle.

Let Z (fig. 25.) be the zenith; *Zm*, *Zs*, the apparent zenith diſtances of the Moon and ſtar, and *ſm* the apparent diſtance. Let *Ss*, *mn* be the refractions in altitude of thoſe objects reſpectively. Join *Sn*, and draw *ſa mb* perpendicular thereto; and *Sa nb* will be the effects of refraction: which therefore being applied to the apparent diſtance, will give *Sn*, the diſtance corrected by refraction.

Again, let *nM* be the parallax of the Moon in altitude, and *SM* being joined, will be the true diſtance. From *M* draw *Mc* perpendicular to *Sn*, and *cn* will be the principal effect of parallax in diſtance; which being applied to the diſtance corrected by refraction, will give the arc *Sc*. Now in the right angled ſpherical triangle *ScM*, *Sc* and *Mc* being given, SM may be found, or rather the difference between *SM* and *Sc* may be computed, which being applied to *Sc* will give the true diſtance *SM*.

If the object with which the Moon is compared be the Sun, another correction, depending on the parallax of that object, is neceſſary. This correction may be computed on the ſame principles, as the effect of the Moon's parallax.

The firſt of the following methods is perhaps as eaſy a ſolution of this problem as has hitherto appeared, eſpecially when the table of natural verſed ſines, which accompanies this work, is uſed. The ſecond is another ſolution, which will be found extremely eaſy, if Taylor's log. tables be uſed. Both theſe depend on the firſt general principle. The other methods are deduced from the ſecond general principle.

PROBLEM I.

The Apparent Diſtance between the Moon and the Sun, or a Fixed Star, together with the Altitude of each being given, to find the True Diſtance.

ME-

Sun's parallax. This correction may be taken from tables v. and vi. ſupplemental of the firſt edition of the Requiſite Tables. Mr Margett's Longitude Tables, are deduced from the Cambridge Tables.

METHOD I.

Of reducing the Apparent to the True Diſtance.

RULE.

Take the correction of the Moon's altitude from table IX. to which add the correction* of the Sun's altitude, or the refraction of the ſtar. Now this ſum added to, or ſubtracted from, the difference of the apparent altitudes, according as the Moon is higher or lower than the Sun or ſtar, will give the difference of the true altitudes.

From the natural verſed ſine of the obſerved diſtance, ſubtract the natural verſed ſine of the difference of the apparent altitudes, and to the log. of the remainder add the log. from table XLII. anſwering to the Moon's apparent altitude and horizontal parallax. Now the natural number anſwering to the ſum of theſe two logs. being added to the natural verſed ſine of the difference of the true altitudes, will give the natural verſed ſine of the true diſtance.

EXAMPLES.

I. Let the apparent diſtance between the centers of the Sun and Moon be 81° 23′ 38″, the apparent altitude of the Sun 27° 43′, the apparent altitude of the Moon 48° 22′, and the Moon's horizontal parallax 58′ 45″. Required the true diſtance?

App. diſtance=	81° 23′ 38	N.V.S.=	850359		
Diff. app. alt.=	20 39 0	N.V.S.=	64248	log. t. XLII.	9.994623
Corr. ☽'s alt.=	+ 38 26	Diff. =	786111	- log. —	5.895484
⊙ s alt.=	+ 1 40	N. No.=	776440	——	5.890107
Diff. true alt.=	21 19 6	N.V.S.=	68425		
True diſt.	81 4 31	N.V.S.=	844865		

II. Let the apparent diſtance between the centers of the Sun and Moon be 72° 21′ 40″, the apparent altitude of the Moon 19° 19′, that of the Sun 25° 16′, and the Moon's horizontal parallax 56′ 32″. Required the true central diſtance?

App. diſtance=	72° 21′ 40″	N.V.S.=	696983		
Diff. app. alt.=	5 57 0	N.V.S.=	5387	log. t. XLII.	9.997818
Corr. ☽'s alt.=	— 50 40	Diff. =	691596	— log. -	5.839852
⊙'s alt.=	— 1 53	N. No.=	688130	——	5.837670
Diff. true alt.=	5 4 27	N.V.S.=	3919		
True diſtance=	72 3 51	N.V.S.=	692049		

ME-

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* The difference between the refraction and parallax of the Sun in altitude.

METHOD II.

Of reducing the Apparent to the True Diſtance.

RULE.

Add together, the apparent diſtance and the apparent altitndes of the centers of the Sun and Moon; find the difference between half the ſum and the apparent diſtance.

Take the log. from table XLII. anſwering to the Moon's apparent altitude and horizontal parallax, to which add the log. coſines of the above half ſum and difference, and half the ſum of theſe three logs. will be the log. ſine of an arc.

Now add together, the log. coſines of the ſum and difference of this arc, and half the ſum of the true altitudes; then will half the ſum of theſe two logs. be the log. ſine of half the true diſtance.

EXAMPLES.

I. Let the apparent diſtance of the Sun and Moon's centers be 38° 45′ 40″, the apparent altitude of the Moon's center 29° 31′, that of the Sun's center 35° 43′, and the Moon's horizontal parallax 57′ 43″. Required the trne diſtance?

		Sum of apparent altitudes	65° 14′ 0″
		Correction ☽'s altitude	+ 48 33
		Correction ☉'s altitude	− 1 11
App. diſtance =	38° 45′ 40″		
App. altitude ☽ =	29 31 0	Snm of true altitudes -	66 1 22
App. altitude ☉ =	35 43 0	Half ſum true altitudes -	33 0 41
Sum —	103 59 40	Log. diff. tab. XLII. XLIII.	9.996580
Half —	51 59 50	— — — co-ſine —	9.789369
Difference —	13 14 10	— — — co-ſine -	9.988307
Half ſum true alt.	33 0 41		19.774256
Arc —	50 27 20	— — — ſine —	9.887128
Sum —	83 28 1	— — — co-ſine —	9.056052
Difference —	17 26 39	— — — co-ſine —	9.979552
			19.035604
	19 14 9½	— — — ſine —	9.517802
True diſtance —	38 28 19		

II. The apparent diſtance between the Moon's center and α Arietis is 64° 36′ 40″, their apparent altitudes 44° 33′ and 11° 51′ reſpectively, and the Moon's horizontal parallax 61′ 10″. Required the true diſtance?

				Sum of apparent altitudes =	56° 24′ 0″
				Correction of ☽'s altitude +	42 38
				Refraction of *'s altitude −	4 26
Apparent diſtance	64°	36′	40″		
Apparent alt. ☽	44	33	0	Sum of true altitudes —	57 2 12
Apparent alt. *	11	51	0	Half ſum of true altitudes	28 31 6
Sum —	121	0	40	Log. diff. tab. XLII. and XLIV.	9.994750
Half —	60	30	20	— — — co-ſine —	9.692264
Difference —	4	6	20	— — — co-ſine —	9.998884
Half ſum true alt.	28	31	6		19.685898
Arc —	44	9	1	— — — ſine —	9.842949
Sum —	72	40	7	— — — co-ſine —	9.474067
Difference —	15	37	55	— — — co-ſine —	9.983632
					19.457699
	32	23	7	— — — ſine —	9.728849
True diſtance —	64	46	14		

When the diſtance is given in degrees, minutes, and ſeconds, and the computation performed by means of the common log. tables, it will facilitate the practice, to omit the ſeconds till the operation is finiſhed, and then add them to the computed diſtance. It may alſo be obſerved, that if the unit figure in the minutes of the ſum of the apparent altitudes and diſtance be an odd digit, it will be proper to increaſe that ſum by one minute; and in this caſe, the complement of the omitted ſeconds to 60″ muſt be deducted from the computed diſtance.

EXAMPLE.

Let the apparent altitude of the Sun's center be 27° 43′, that of the Moon's 46° 18′, the apparent diſtance 54° 57′ 35″, and the Moon's horizontal parallax 59′ 33″. Required the true diſtance?

Sum of apparent altitudes = 74° 1′ 0″
Correction of ☽'s altitude = + 40 14
Correction of ☉'s altitude = − 1 40

App. diſtance − 35″ =	54°	57′			
Apparent altitude ☽	46	18		Sum of true altitudes —	74 39 34
Apparent altitude ☉	27	43		Half ſum true altitudes	37 19 47
Sum — —	128	58		Log. diff. tab. XLII. XLIII. =	9.994730
Half — —	64	29		— — — co-ſine —	9.634249
Difference — —	9	32		— — — co-ſine —	9.993960
Half ſum true altitudes	37	19	47		19.622939
Arc — —	40	22	45	— — ſine —	9.811469
Sum —	77	42	32	— — co-ſine —	9.328132
Difference —	3	2	58	— — co-ſine —	9.999384
					19.327516
	27	27	20	— — ſine —	9.663758
Computed diſtance	54	54	40		
Seconds omitted —	+		35		
True diſtance —	54	55	15		

The two preceding methods, being derived from the firſt general principle, are ſtrictly accurate; the rules are alſo eaſily remembered, upon account of the uniformity of the mode of calculation. In order to illuſtrate the ſecond general principle, the following method is ſubjoined, which, though only an approximation to the truth, will generally give the true diſtance within a few ſeconds.

METHOD III.

Of reducing the Apparent to the True Diſtance.

RULE.

To the log. co-tangent of half the ſum of the apparent altitudes, add the log. tangent of half their difference, and the log. co-tangent of half the apparent diſtance; the ſum will be the log. tangent of arc *firſt*.

If the Moon's altitude is greater than that of the ſtar, the ſum of arc firſt and half the apparent diſtance will be arc ſecond, and their difference arc third; but if leſs, the difference is arc ſecond, and ſum, arc third. Now find the refraction correſponding to the complements of arc ſecond and third, and their ſum, if arc firſt is leſs than half the apparent diſtance, otherwiſe their difference, will be the firſt correction of diſtance, being always additive to the apparent diſtance.

To the log. co-tangent of arc third, add the log. co-ſecant of the

Moon's altitude corrected by refraction, and the prop. log. of the Moon's horizontal parallax; the ſum will be the prop. log. of the ſecond correction, which is additive, when arc firſt is greater than half the diſtance, the Moon's altitude at the ſame time being greater than that of the ſtar. In every other caſe, this correction is ſubtractive.

Take from table xxii. the numbers anſwering to the ſupplement of twice the diſtance, and the parallaxes in altitude* and diſtance; half the difference of theſe numbers is to be added to, or ſubtracted from the corrected diſtance, according as it is leſs or greater than 90°, and the true diſtance between the Moon and a ſtar will be obtained.

But if the object with which the Moon is compared be the Sun, another correction, depending on the parallax of that object, is neceſſary. This may be found by an operation ſimilar to that employed for computing the ſecond correction, by uſing arc ſecond, the Sun's altitude and horizontal parallax; or find the parallax anſwering to the complement of the Sun's altitude. Now, in the traverſe table, the departure, correſponding to the above parallax found in a latitude column, arc ſecond being taken as a courſe, will be the effect of the Sun's parallax in diſtance, which is to be applied with a contrary ſign to part firſt of the firſt correction.

EXAMPLE.

Let the apparent diſtance between the Moon's center and a ſtar be 73° 46′ 14″, the apparent altitude of the ſtar 53° 48′, that of the Moon's center 20° 16′, and horizontal parallax 59′ 19″. Required the true diſtance?

Moon's app. alt =	20° 16′				
Star's app. alt. =	53 48				
Sum —	74 4	Half =	37° 2′	co-tangent — —	0.1224
Difference —	33 32	Half =	16 46	tangent — —	9.4790
Apparent diſtance	73 46 14	Half =	36 53	co-tangent — —	0.1247
Firſt correction	+ 2 10	Arc firſt =	28 1	tangent — —	9.7261
Second correction	— 43 48	Arc ſec. =	8 52	complt. 81° 8′ refract. =	0 9″
Corrected diſtance	73 4 36	Arc third =	64 54	complt. 25.6 refract. =	2 1
In table xxii. to ſup. twice diſt. &				firſt correction —	+ 2 10
par. in alt. 55′ = 16″		Arc third =	64 54	co-tangent —	9.6706
par. in diſt. 44 = 10		☽'s alt. — ref.	20 14	co-ſecant —	0.4611
Difference	6 half + 3	☽'s hor. par.	59′ 19″	P. L. —	0.4821
True diſtance	73 4 39	Second correct.	43 48	P. L. —	0.6318

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* The parallax of the Moon in altitude may be found by the rule given in page 62. Or it may be found with ſufficient accuracy for this porpoſe, by entering a traverſe table, with the Moon's altitude taken as a courſe; and the difference of latitude, anſwering to the horizontal parallax found in a diſtance column, will be the parallax in latitude.

Becauſe, in this example, the altitude of the Moon is leſs than that of the ſtar, the difference between half the diſtance and arc firſt is arc ſecond, and their ſum is arc third. Alſo, ſince arc firſt is leſs than half the diſtance, the ſum of the refractions, anſwering to the complements of theſe arcs, is the firſt correction of diſtance, being always additive. And altho' arc firſt is leſs than half the diſtance, yet the Moon's altitude being leſs than that of the ſtar, the ſecond correction is therefore ſubtractive. The third correction is additive, becauſe the diſtance is leſs than 90°.

If the Moon had been compared with the Sun, the effect of the parallax of that object in diſtance ſhould be found, and applied to the diſtance previouſly corrected. Thus, in the preceding example, the Sun's altitude corrected by refraction is 53° 47′; hence its complement is 36′ 13″, and the parallax of the Sun in altitude anſwering thereto is 7″, which being found in a latitude column, under 9° the ſecond arc, the correſponding departure 1″ is the effect of the Sun's parallax in diſtance; which is ſubtractive, becauſe part firſt of the firſt correction is additive. Hence, in this caſe, the true diſtance would be 73° 4′ 38″.

The firſt correction is liable to a ſmall error, which ariſes from the aſſumption of the refraction being as the co-tangent of the altitude.

METHOD IV.

Of reducing the Apparent to the True Diſtance.

This is performed by means of plate v.* and a circular piece of paper, whoſe diameter is equal to that of the interior circle, (fig. 26.) in which, a line is drawn from the center to the circumference, called the *vertical* or *perpendicular*. In the circle, fig. 26. the altitudes and diſtance of the objects are to be found. The ſcale, No. 1. gives the principal effect of the Moon's parallax in diſtance, and the ſcales No. 4. and 5. exhibit the ſecond correction of the Moon's parallax. The effect of refraction in diſtance is found by means of ſcale No. 2. and that of the Sun's parallax, by ſcale No. 5. By this plate, the whole operation of finding the longitude of a ſhip may be performed. This will be explained afterwards.

RULE.

Set the vertical on the moveable circle to the zenith or 90°; lay a ruler over the given degree of the Moon's altitude, and draw a line which will repreſent the Moon's parallel of altitude; at the interſection of this line with the vertical, put the character ☽.

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* This plate is a miniature ſketch of a large plate drawn with the utmoſt accuracy, by which the true diſtance may be found within one or two ſeconds. It is intended to publiſh this plate at as moderate an expence as poſſible, immediately after the publication of this work.

Then, ſet the vertical to the apparent diſtance of the objects, and draw a line from the center to the zenith; draw alſo the parallel of altitude of the Sun or ſtar. At the interſection of theſe lines put the character ∗, and at that of the two parallels, let the character ⊙ be marked.

Now the interval ⊙ ☽ being applied to the line repreſenting the Moon's horizontal parallax, in ſcale No. 1. will give the principal effect of the Moon's parallax in diſtance; and to the ſcale No. 2. on the line expreſſing the Moon's altitude, will give the effect of the Moon's refraction in diſtance. The firſt of theſe corrections is additive, when ⊙ is to the right of ☽, and the ſecond ſubtractive. The contrary rule is to be applied, when ⊙ is to the left of ☽.

Apply the interval ⊙ ∗ to the line repreſenting the altitude of the ſtar, ſcale No. 2. and the correction depending on the ſtar's refraction will be known. If the object with which the Moon is compared be the Sun, then the above extent being applied to the ſcale No. 3. will give the effect of the Sun's parallax. The firſt of theſe corrections is additive or ſubtractive, according as ⊙ is to the right or left of ∗. The other correction is to be applied with a contrary ſign.

With the Moon's apparent altitude and horizontal parallax, find the parallax in altitude†. Now from the ſcale No. 4. take the interval between the parallaxes in altitudes and diſtance, on the line repreſenting the diſtance, which being applied to the ſcale No. 5. will give the final correction of diſtance, which is additive when the diſtance is leſs than 90°, otherwiſe ſubtractive.

EXAMPLE.

Let the apparent diſtance between the ſun and moon be 68° 35′ 40″, the apparent altitude of the moon 30° 44′, that of the ſun 33° 31′, and the moon's horizontal parallax 59° 17′. Required the true diſtance?

The parallel of altitude and the Sun's vertical being drawn, and the points of interſection marked, as directed, the meaſures from the ſeveral ſcales will be as follow:

Apparent diſtance		68°	35′ 40″
Interval ⊙ ☽ applied to ſcale N°. 1. gives	−	23	17
—— —— —— N°. 2.	+		44
Interval ⊙ ∗ applied to ſcale N°. 2. gives	+		34
—— —— —— N°. 3.	−		3
Interval from ſcale N°. 4, between 51′ and 23′, the parallaxes in alt. and diſt. applied to ſcale N°. 5, gives	+		7
True diſtance		68	13 45

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† See note, page 116.

In the methods hitherto given, for correcting the apparent diſtance between the Moon and the Sun, or a fixed ſtar, the figure of the Earth has been ſuppoſed ſpherical; but it has been formerly mentioned, that the figure of the Earth is that of an oblate ſpheroid; and ſince the neceſſary obſervations are taken on its ſurface, it is hence evident, that a farther correction is neceſſary, upon account of the deviation of the Earth's figure from a ſphere. This correction may be computed by a fluxional equation, and applied to the true diſtance found as formerly; but the moſt natural method for effecting this, is that of previouſly reducing the altitudes to thoſe which would have been obſerved, if the figure of the Earth had been ſpherical, and adapting the Moon's horizontal parallax to the latitude of the place of obſervation.

Therefore, in order to obtain the true diſtance, the apparent altitude of each object's center muſt be corrected by the quantity from table XXVI. anſwering to the azimuth of the object, and latitude of the place of obſervation. The equatorial horizontal parallax of the Moon, as given in the Nautical Almanac, muſt be diminiſhed by the equation from table XXXII. anſwering to the latitude. Now, with theſe quantities thus adapted to a ſpherical Earth, and the apparent diſtance, the true diſtance is to be found by any of the methods given for that purpoſe.

EXAMPLE.

Let the apparent diſtance between the centers of the ſun and moon be 72° 19′ 40″: the apparent altitudes of the ſun and moon 25° 11′ 1 and 19° 22′.2 reſpectively. Azimuths $16^\circ\frac{1}{4}$ and $95^\circ\frac{1}{5}$; the latitude of the place 57° 9′, and moon's horizontal parallax per Naut. Alm. 56′ 42″. Required the true diſtance?

☉'s ap. alt. =	25° 11′ 1	☽'s app. alt. =	19° 22′.2	☽'s hor. par. =	56′ 42″
N°. tab. xxvi.	+ 12.7	N°. tab. xxvi. —	1.2	Red. tab. xxxii.	— 10
☉'s red. alt.	25 23.8	☽'s red. alt.	19 21.0	☽'s red. par.	56 32
☽'s red. alt.	19 21.0				

Difference 6 2.8 = 6° 2′ 48″
App. diſt. = 72° 19′ 40″ N. V. S. = 696429
Diff. app. alt^s. 6 2 48 N. V. S. = 5563 Log. diff. 9.997812

Corr. ☽'s alt. — 50 40 690866 — log. — 5.839394

Corr. ☉'s alt. — 1 52 687395 — log. — 5.837206

Diff. true alt^s. 5 10 16 N. V. S. = 4070

True diſtance 72 1 44 N. V. S. = 691465

When the greateſt preciſion is required, the mean refraction, as given in table VI. muſt be reduced to the true refraction, by applying the change anſwerable to the ſtate of the barometer and thermometer, from table VIII. The true diſtance between the obſerved limbs of the objects ſhould be computed; for this purpoſe, the altitudes of thoſe points of the limbs which were obſerved in contact, and the corrected refractions anſwering thereto, ought to be employed. To which diſtance, the apparent ſemidiameters of the objects being applied, will give the true central diſtance.

The difference of altitude between the center of the object, and the point of contact may eaſily be eſtimated, or it may be found by firſt computing the angle at its center, contained between the diſtance and vertical diameter; and the co-ſine of this angle, will be the difference of altitude, the ſemi-diameter of the object being radius.

The contraction of ſemidiameter is contained in table XXXI. the arguments of which are, the apparent altitude of the object, and the angle at its center contained between the diſtance and vertical diameter.

PROBLEM II.

To find the Time at Greenwich, anſwering to a given Diſtance between the Moon and the Sun, or one of the Stars uſed in the Naut. Almanac.

RULE.

If the given diſtance is found in the Nautical Almanac, in either of pages VIII. IX. X. XI. of the month, oppoſite to the given day, or to that which immediately precedes or follows it; the time is found at the top of the page. But if this diſtance is not found exactly in the ephemeris; then, ſubtract the prop. log. of the difference between the two contiguous diſtances, one of which being greater, and the other leſs than the given diſtance; from the prop. log. of the difference between the given and preceding diſtances, the remainder will be the prop. log. of the exceſs of the time, above that anſwering to the preceding diſtance; hence, the apparent time is known.

EXAMPLE.

♀ September 21, 1792, the true diſtance between the centers of the ſun and moon was 68° 13′ 8″. Required the apparent time at Greenwich?

Given diſtance = 68° 13′ 8″
Diſt. at IX hours = 67 53 27 Differ. = 0° 19′ 41″ P. Log. = 9612
Diſt. at XII hours = 69 30 6 Differ. = 1 36 39 P. Log. = 2701

Exceſs — — — 0 36 39 P. Log. 6911
Preceding time — — 9 0 0

Apparent time at Greenwich 9 36 39

CHAP.

CHAP. V.

Of finding the Longitude at Sea or Land, by Lunar Obſervations.

PROBLEM I.

The Latitude of a Place, and its Longitude by account, being given, together with Diſtance between, and the Altitudes of the Moon and the Sun, or one of the Stars in the Nautical Almanac—to find the correct Longitude of the Place of Obſervation.

RULE.

Reduce the eſtimate time of obſervation to the meridian of Greenwich by prob. III. page 73. To this time, take from the Nautical Almanac, page VII. of the month, the Moon's horizontal parallax and ſemidiameter. Increaſe the ſemidiameter by the augmentation from table XXX. anſwering to the Moon's altitude.

Find the apparent and true altitudes of each object's center, and the apparent central diſtance, with which let the true diſtance be found, by any of the methods given for that purpoſe, and find the apparent time at Greenwich anſwering thereto, by the laſt problem.

If the Sun or ſtar be at a proper diſtance from the meridian, when the diſtance was obſerved, compute the apparent time at the ſhip. If not, the error of the watch may be found from obſervations, taken either before or after that of the diſtance. Or, the apparent time may be inferred from the Moon's altitude, by prob. IX. page 100.

The difference between the apparent times of obſervation at the ſhip and Greenwich will be the longitude of the ſhip in time, which is eaſt or weſt, according as the time at the ſhip is later or earlier than the Greenwich time.

EXAMPLES.

March 17, 1792, in latitude 34° 53′ N. and longitude 27° W. by account; about 9h. A. M. the diſtance between the neareſt limbs of the Sun and Moon was 68° 3½′, the altitude of the Sun's lower limb was 38° 18′, the altitude of the Moon's upper limb 31° 3′, and the height of the eye 12 feet. Required the true longitude of the ſhip?

Time at ſhip - - = 9h. 0′ A. M.
Long. in time = 1 . 48

Eſt. time at Green. 10 48 A. M.
Alt. ☽'s up. limb = 31 3
Semidiameter — 16.3
Dip — — 3.3

App. alt. ☽'s cent. 30 43.4
App. alt. ☉'s cent. 33 30.8

Diſt. ☉ & ☽'s near. l. 68° 3′ 15″
Sun's ſemidiameter + 16 6
Moon's ſemidiam. + 16 10
Augmentation, + 9

App. central diſtance 68 35 40
Alt. ☉'s lower limb 33 18
Semidiameter — + 16.1
Dip — — 3.3

App. alt. ☉'s center, 33 30.8

Diff. app. altitudes 2 47.4
App. diſtance — 68° 35′ 40″ N.V.S. = 635033
Diff. app. alt. — 2 47 24 N.V.S. = 001185 log. diff. 9.996335

Cor. ☽'s alt. — 49 26 Differ. = 633848 — log. 5.801985

Cor. ☉'s alt. — 1 18 N. N°. = 628521 — — 5.798320

Diff. true altitudes 1 56 40 N.V.S. = 000576

True diſtance — 68 13 43 N.V.S. = 629097
Diſt. at XXI. hours 69 11 20 Differ. = 0h. 57′ 37″ P. L. 4947
Diſt. at noon — 67 32 38 Differ. = 1 38 42 P. L. 2610

Exceſs — — 1 45 6 P. L. 2337
Preceding time, — 21 — —

Apparent time at Greenwich 22 45 6

Sun's apparent alt. 33° 30′.8
Correction — — 1.3

Sun's true altitude 33 29.5
Sun's polar diſtance 90 57.9 — — — co-ſecant 0.00006
Ship's latitude — 34 53.0 — — — ſecant - 0.08602

Sum — 159 20.4
Half — 70 40.2 — — — co-ſine - 9.25362
Difference — 46 10.7 — — — ſine — 9.85824

19.19794
23 24 — — — ſine — 9.59897
8

Sun's mer. diſt. 3 7 12 Hence ap. time is 20h. 52′ 48″
Ap. time at Green. 22 45 6

Longitude in time 1 52 18 = 28°4½′W.

November 4, 1791, in latitude 41° 7′ S. and longitude 176° W. by account; about half paſt three, the following obſervations were made, the height of the eye being 14 feet. Required the ſhip's true longitude?

	Diſt. near. l. ⊙ & ☽	Alt. ☽'s upper limb	Alt. ⊙'s l. limb
Eſtim. time 3h. 30′	115° 49′ 10″	21° 0′	38° 30′
Lon. in time 11 44 W.	50 0	20 38	38 51
	51 0	20 15	39 13
Red. time 15 14	51 50	19 53	39 34
	53 0	19 30	39 55
	5 0	76	63
Mean	115 51 0	20 15.2	39 12.6
Sun's ſemidiameter	+ 16 12	Semidiameter — 15.6	Semidia. + 16.2
Moon's ſemidiameter	+ 15 30	Dip — 3.6	Dip — 3.6
Augmentation	+ 6		
		App. altitude 19 56.0	App. alt. 39 25.2

Apparent diſtance - 116 22 48 N. V. S. = 1444323
Diff. apparent altitudes 19 29 12 N. V. S. = 057281 Log. diff. 9.997723

Correction Moon's alt. — 50 24 Diff. = 1387042 Log. — 5.142090

Correction Sun's alt. — 1 2 N. No. = 1379788 — — 5.139813

Diff. true altitudes — 18 37 16 N. V. S. = 52349

True diſtance — 115 35 49 N. V. S. = 1432137
Diſtance at xv. hours 115 31 30 Diff. = 0 4 19 — P. L. 1.6201
Diſtance at xviii. hours 117 1 7 Diff. = 1 29 37 — P. L. 0.3029

Proportional part — — 0 8 40 — P. L. 1.3172
Preceding time — — 15 0 0

Apparent time at Greenwich, — 15 8 40
Sun's apparent altitude = 39° 25′.2
Correction, — — 1.0

Sun's true altitude — 39 24.2
Sun's polar diſtance — 74 20.4 — — — co-ſecant 0.01643
Ship's latitude — 41 7.0 — — — ſecant - 0.12299

Sum — — 154 51.6
Half — — 77 25.8 — — — co-ſine - 9.33772
Difference — — 38 1.6 — — — ſine — 9.78960

19.26674
25 27.7 — — — ſine — 9.63337
8

Apparent time at ſhip 3 23 41
App. time at Greenw. 15 8 40

Longitude in time — 11 44 59 176° 14¾′ W.

September 1, 1792, in latitude 13° 57′ N. and longitude by account 56° E. the following obſervations were made, the height of the eye being 13 feet. Required the ſhip's true longitude?

Time per watch	Diſt. ☽ & Altair	Alt. ☽'s l. limb	Alt. Altair.
1h. 15′ 1″ A.M.	58° 43′ 30″	70° 52′	26° 25′
16 55	44 20	44	25 57
18 57	45 25	34	25 27
21 0	46 30	24	24 58
23 2	47 25	12	24 30
44 55	130	166	137
1 18 59	58 45 26	70 33.2	25 27.4
Long.=3 44	aug. ſem. + 16 28	ſemid. + 16.5	dip − 3.4
		dip − 3.4	
Eſt. time 9 35	Cent. diſt. 59 1 54		ap. alt. 25 24.0
		app. alt. 70 46.3	70 46.3
			diff. 45 22.3

Apparent diſtance = 59° 1′ 54″ N.V.S. = 485436

Diff. app. altitudes = 45 22 18 N.V.S. = 297495 log. diff. 9.993082

Correction ☽'s alt. = + 19 15 Diff. = 187941 log. - 5.274021

Refraction ∗ — = + 2 0 N.N°. = 184971 — 5.267103

Difference true alt. = 45 43 33 N.V.S. = 301907

True diſtance — = 59 7 41 N.V.S. = 486878

Diſt. at IX. hours = 58 51 17 Diff. = 0° 16′ 24″ P. L. 1.0404

Diſt. at XII. hours = 60 24 34 Diff. = 1 33 17 P. L. 0.2855

Proportional part — 0 31 19 P. L. 0.7549

Preceding time — 9 0 0

Apparent time at Greenwich 9 31 19

Apparent alt. Altair = 25° 24′

Refraction — = − 2

True altitude — 25 22

Polar diſtance — 81 40.2 — — — co-ſecant 0.00461

Latitude — 13 57 — — — ſecant - 0.01300

Sum — — 120 59.2

Half — — 60 29.6 — — — co-ſine 9.69243

Difference — 35 7.6 — — — ſine — 9.75996

19.47000

		32	54·3	— — — ſine — 9·73500
			8	
Altair's mer. diſt.	4	23	14	
Altair's right aſc.	19	40	40	
Right aſc. mer. -	0	3	54	
Sun's right aſc.	10	46	17	
Ap. time at ſhip	13	17	37	
— at Greenw.	9	31	37	
Longitude in time	3	46	0	= 56° 30′ E.

As a ſmall error in the altitude of an object near the meridian may produce a conſiderable error in the apparent time; and becauſe the altitude of a ſtar cannot always be ſo accurately obſerved as is neceſſary for eſtabliſhing the apparent time; nor can the time, inferred from the Moon's altitude, be depended on, unleſs its declination and right aſcenſion are reduced to the time of obſervation, by applying the equation of ſecond difference—it therefore ſeems moſt eligible to uſe the Sun for this purpoſe. Hence, if this object is too near the meridian at the time of obſervation of the diſtance, it becomes neceſſary to obſerve its altitude, when in a more proper ſituation, either previous, or ſubſequent thereto. Hence the error of the watch will be known, which being applied to the time of obſervation of the diſtance, will give the apparent time of obſervation, agreeable to the meridian for which this error was determined. Or, if the change of longitude in time, between the meridians of the places where the obſervations were made for aſcertaining the time and longitude, be applied to the above found apparent time, by addition or ſubtraction, according as the ſhip's courſe made good has been in the eaſtern or weſtern quarter, the apparent time will hence be known, for the meridian of the place where the obſervations were made for determining the longitude. If the watch gains or loſes conſiderably, allowance muſt be made for its rate in the above interval.

October 17, 1786, in latitude 36° 12′ S. and longitude, by account, 48° 15′ W. the following obſervations were taken in order to determine the error of the watch, the height of the eye being 17 feet.

	Time per watch.		Alt. ☽'s l. limb.
	7° 51' 10" A. M.	—	28° 55'.0
	52 40	—	29 12.5
	54 17	—	29 32.0
	55 48	—	29 50.0
	57 5	—	30 5.0
	21 0		154.5
Mean —	7 54 12	Mean —	29 30.9
Long. in time	3 13	Semidiam. +	16.1
Red. time	11 7	Dip - - —	4.0
		Correction —	1.5

True alt. 29 41.5

Polar diſt. 80 35.5 — co-ſecant 0.00588

Latitude 36 12.0 — ſecant — 0.09315

Sum — 146 29.0

Half — 73 14.5 — co-ſine - 9.45990

Difference 43 33.0 — ſine — 9.83821

19.39714

29 58.1 — ſine — 9.69857

8

⊙'s mer. diſt. } 3 59 45. hence ap. time is } 8h. 0' 15"

Time p. watch 7 54 12

Watch ſlow - 6 3

Towards noon, the following obſervations were made, the height of the eye being 16 feet.

Time p. watch	Diſt. ⊙ & ☽'s n.l.	Alt. ☽'s u. l.	Alt. ⊙'s l. limb.
11h. 45' 28"	50° 48' 20"	29° 15'	62° 54'
47 13	47 30	28 57.5	62 56
48 48	46 50	28 42	62 57
50 34	46 0	28 24	62 57
51 52	45 20	28 11	62 58
53 27	44 30	27 55	62 58
297 22	38 30	204.5	41

Mean

Mean	11 49 34		50 46 35		28 34.1		62 56.7
Error	+ 6 3	ſem. ⊙	+ 16 7	ſemid.	— 15.3	ſem.	+ 16.1
		ſem. ☽	+ 15 11	dip	- — 3.8	dip	- — 3.8
Ap. time	11 55 37	augm.	+ 7	ap. alt.	28 15	ap. alt.	63 9

Apparent central diſt. 51 18 2 N.V.S. 374765
Difference app. altitudes 34 54 0 N.V.S. 179848 log. diff. 9.996830

Correction ☽'s alt. — 47 26 Diff. = 194917 log. - 5.289850

Correction ⊙'s alt. — 25 N. N°. 193500 — 5.286680

Difference true altitudes 34 6 9 N.V.S. 171965

True diſtance — 50 36 52 N.V.S. 365465
Diſtance at III. hours 50 38 39 Diff. - 0° 1′ 47″ P. L. 2.0040
at VI. hours 49 12 44 Diff. - 1 25 55 P. L. 0.3212

Proportional part — — — 0 3 44 P. L. 1.6828
Preceding time — — 3 0 0

Apparent time at Greenwich, — 3 3 44
Apparent time at ſhip — 11 55 37

Longitude in time — — 3 8 7 = 47° 1¾′ W.

Auguſt 9, 1792, in latitude 63° 28′ N. and longitude by account 56° W. the following obſervations were made, in order to determine the ſhip's correct longitude.

Time per watch	Diſt. ⊙ & ☽'s l.	Alt. ☽'s u. l.	Alt. ⊙'s l. l.
11h. 3′ 16″	90° 2′ 18″	14° 3′	40° 52′
5 12	90 1 20	14 16	40 56
7 20	90 0 13	14 31	41 1
9 13	89 59 30	14 44	41 5
61	3 21	94	234

Mean

Mean -	11h. 6′ 15″		90° 0′ 50″		14° 23′.5		40° 58′.5
Long. in time	3 48	☉'s ſem. +	15 50	☽'s ſem. —	15.2	☉'s ſem. +	15.8
Red. time	2 54 15	☽'s ſem. +	15 10	dip - —	3.3	dip - - —	3.3
		augm. +	4	ap. alt.	14 5	ap. alt.	41 11

Apparent central diſtance	90 31 54	N.V.S.	1009279		
Difference app. altitudes	27 6 0	N.V.S.	109787	log diff.	9.998466
Correction of ☽'s alt. —	50 13	Diff. =	899492	log.	5.953997
Correction of ☉'s alt. —	58	N. N°.	896320	—	5.952463
Difference true altitudes	26 14 49	N.V.S.	103103		
True diſtance —	89 58 1	N.V.S.	999423		
Diſtance at noon —	91 22 48	Diff. =	1° 24′ 47″	P. L.	3270
Diſtance at III. hours —	89 57 12	Diff. =	1 25 36	P. L.	3228
Apparent time at Greenwich — —	2 58 17			P. L.	0042

Since the apparent time at the ſhip cannot be inferred with ſufficient accuracy from the Sun's altitude, taken at the ſame time with the diſtance, becauſe of the proximity of that object to the meridian; the following altitudes of the Sun were therefore obſerved in the afternoon, the ſhip's latitude reduced to that time being 63° 13′ N. and height of the eye 13 feet.

Time per watch =	3h. 0′ 44″	Alt. ☉'s l. l.	33° 1′
— —	1 28	— —	32 53
— —	2 20	— —	32 43
— —	3 0	— —	32 36
— —	3 50	— —	32 26
— —	11 22		219
Mean —	3 2 16	—	32 43.8
T. p. w. of ob. diſt.	11 6 15	Sun's ſemidia. +	15.8
Interval —	3 56 1	Dip - - - —	3.4
Ap. t. at Green. of obſ. diſt.	2 58 17	Correction - —	1.3
Ap. t. at Green. of obſ. alt.	6 54 18	Sun's true alt.	32 54.9

Sun's alt.	32° 54′.9			
Polar diſt.	74 28.1	—	co-ſecant	0.01616
Latitude	63 13.0	—	ſecant -	0.34619
Sum —	170 36.0			
Half —	85 18.0	—	co-ſine	8.91349
Difference	52 23.1	—	ſine -	9.89880
				19.17464
	22 44.8	—	ſine -	9.58732
	8			
Apparent time at ſhip —	3 1 58			
Apparent time at Greenwich	6 54 18			
Longitude in time — —	3 52 20 = 58° 5′ W.			

PROBLEM II.

To find the Longitude of a Place, by means of Plate V.

RULE.

Reduce the apparent to the true diſtance, by the rule given for that purpoſe in page 117.

Draw a line over the Moon's relative motion in three hours, found in the ſide ſcales N°. 7. Take the difference between the true and preceding diſtances from the ſcale N°. 9, and lay it off on the above line from the right hand; then a line drawn from the upper right hand corner thro' this point, and produced, will give the proportional part on the ſcale of hours N°. 8, which being added to the preceding time, the ſum will be the apparent time at Greenwich.

Put the vertical to 90°, and draw the parallel of declination, and put *m* at the interſection of theſe two lines; then put the vertical to the latitude of the place found in the quadrant to the right, and draw the parallel of altitude of the object, and at its interſection with the parallel of declination put the character ∗. Now the interval between *m*∗, being meaſured on the parallel of declination ſcale N°. 10, will give the time from noon or midnight, according as ∗ is to the left or right of *m*. Hence the apparent time at the place of obſervation will be known; the difference between which, and the time at Greenwich, will be the required longitude in time.

EXAMPLE.

In latitude 39° 32′ S. long. by account 115° W. July 8, 1786, about three quarters paſt XI. P. M. the diſtance between the Moon's remote limb and Fomalhaut was 69° 43¼′, the altitude of the Moon's upper limb 66° 38′, that of the ſtar 43° 7′, and height of the eye 20 feet. Required the true longitude?

Eſtim. time	11h. 45′	Diſt. ☽ & ✱	69° 43′ 15″	Alt. ☽ u. l.	66° 38′	Alt. ✱	43° 7′
Long. in time	7 40	Moon's ſem. —	15 20	Moon's ſem. —	15.3	Dip - —	4.3
Red. time -	19 25	App. diſt.	69 27 55	Dip - - - —	4.3	Ap. alt.	43 2.7
				App. alt. -	66 18.4	Refr. - —	1.0
						True alt.	43 1.7

The vertical in the moveable circle, being directed to the zenith, and diſtance, and the parallels of altitude drawn, the ſeveral corrections from the ſcales will be as follow:

Apparent diſtance — — —		69°	27′	55″
Interval between ⊙ ☽ N°. 1. — —	−		21	24
— — N°. 2. — —	+			24
Interval between ⊙ ✱ N°. 2. — —	+		1	0
Int. bet. par. in alt. and diſt. ſcales N°. 4. and 5.				0
True diſtance — — — —		69	7	55

Now draw a line over 1° 23′ 38″, the interval between the preceding and following diſtances, in the ſide ſcales N°. 7; take 38′ 23″, the difference between the true diſtance, and that which immediately precedes it, from the ſcale N°. 9, and lay it off from the right upon this line; and a line drawn from the upper corner to the right, thro' this point, will interſect the ſcale N°. 8, in the point repreſenting 1h. 22′ 37″, which being added to 18h. the time anſwering to the preceding diſtance, gives 19h. 22′ 37″ the apparent time at Greenwich.

Direct the vertical to the zenith, and draw the parallel of the ſtar's declination; then direct the vertical to the latitude, and draw the parallel of the ſtar's altitude; now the interval between *m* ✱, being applied to the parallel of declination*, on the ſcale No. 10, will give 3h. 49′ 40″, the meridian diſtance of the ſtar, which ſubtracted from its right aſcenſion 22h. 45′ 49″, the remainder 18h. 56′ 9″ is the right aſcenſion of the meridian, from which the Sun's right aſcenſion 7h. 14′ 15″ being ſubtracted, the remainder 11h. 41′ 54″ is the apparent time at the ſhip—the difference between which, and 19h. 22′ 37″ the time at Greenwich, is 7h. 40′ 43″, the longitude in time, which reduced to degrees is 115° 10¾′ W.

The ſcale N°. 6. is intended to find the change of altitude anſwering to a given interval of time, independent of the rule in page 55,

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* As the ſtar's declination is 30° 45′, its parallel is therefore a little without the limits of the ſcale. This however can eaſily be eſtimated.

55, and is therefore very convenient for reducing the altitudes to the time of obſervation of the diſtance, when theſe obſervations are made by the ſame perſon. The change of altitude is found thus:

Take the neareſt diſtance between the point of interſection of the latitude and azimuth, and the neareſt horizontal line at the top of the ſcale, and apply it to the ſcale N°. 9, as often as there are minutes in the given interval, and the change of altitude will be known.

C H A P. VI.

Of finding the Longitude at Sea or Land,

The neceſſary data being,

The obſerved Diſtance between the Moon and the Sun, or a Fixed Star, the Apparent Time, the Latitude of the Place of Obſervation, and its Longitude by account.

INTRODUCTION.

THE diſtance between the Moon and a ſtar, and the altitnde of the Moon, may be very accurately obſerved at the ſame time, that the horizon under the ſtar is ſo indiſtinct, as to render it impoſſible to obſerve its altitude to any tolerable degree of accuracy. In this caſe, therefore, rather than loſe an opportunity of determining the ſhip's longitude, when the diſtance and altitude of the Moon only can be obſerved, it will be proper to compute the true and apparent altitudes of the ſtar. The remaining part of the operation is to be performed as uſual. At the time of obſervation of the diſtance between the Sun and Moon, the horizon under one or both of theſe objects may be obſcured by fog, which is often the caſe at the entrance of the Channel, and in high latitudes; or the ſhip may be cloſe to the land, ſo that it may not be ſafe to make any allowance for dip. It hence becomes neceſſary to compute the true and apparent altitudes of the objects.

This method will alſo be found particularly uſeful at land, eſpecially when the obſerver is not provided with a proper artificial horizon.

PROBLEM I.

Given the Apparent Time, at a known Place, to find the True and Apparent Altitudes of a known celeſtial Object.

RULE.

Find the meridian diſtance of the given object, which, in caſe of the Sun, is the interval between noon and the given apparent time. But if the object is the Moon or a ſtar, add the Sun's right aſcenſion to the apparent time; the ſum will be the right aſcenſion of the meridian, the difference between which, and the right aſcenſion of the object, will be its meridian diſtance.

To the log. riſing,* anſwering to the meridian diſtance of the object, add the log. coſines of the object's declination, and of the latitude of the place of obſervation. Find the natural number anſwering to the above ſum, which being added to the natural verſed ſine of the difference or ſum of the latitude and declination, according as they are of the ſame, or of a contrary name, will give the natural co-verſed ſine of the true altitude of the object's center.

If the object is the Sun, the quantity from table x. anſwering to the true altitude, being added thereto, will give the apparent altitude. In the caſe of the Moon, take the reduction from table xi. anſwering to its horizontal parallax and true altitude, which ſubtracted therefrom gives the apparent altitude. The apparent altitude of a ſtar is found, by adding the quantity from table xii. anſwering to its true altitude.

EXAMPLES.

I. Required the true and apparent altitudes of the Sun's center 24th Auguſt 1792, at 4h. 26′ 20″ P. M. in latitude 46° 20′ N. and longitude 30° W.?

Apparent time =	4h.	26′	20″		Sun's declination at noon	10° 48′.4
Long. in time =	2	0	0		Equation, table XIII. —	5.6
Reduced time =	6	26	20		Reduced declination ——	10 42.8
Apparent time	4	26	20	—	Riſing —— —	4.78002
Declination -	10	42.8	N.	—	Co-ſine ——	9.99238
Latitude —	46	20.0	N.	—	Co-ſine ——	9.83914
Difference —	35	37.2		N. V. S. = 18710		
				N. N°. 40883	—— —	4.61154
True altitude	23	50	0	N.Co-v.S. 59593		
Equ. per t.x. = +		2	1			
Apparent altit.	23	52	1			

II. Re-

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* The log. riſing may be taken from table xvi. of the ſecond ed. of the Requiſite Tables. However, as that table does not extend beyond ix. hours, it may therefore be found in all caſes, as follows:

Reduce

II. Required the true and apparent altitudes of Regulus, March 28, 1792, at 11h. 34′ 52″ P. M. apparent time, in latitude 32° 14′ S. and longitude 26° 40′ W.?

Apparent time —	11h.	34′	52″	Apparent time	11h. 34′ 52″
Sun's right aſcenſion	0	33	19	Long. in time	1 46 40
Right aſc. of meridian	12	8	11	Reduced time	13 21 32
Right aſc. of Regulus	9	57	17		
Merid. diſt. of Regulus	2	10	54	— Riſing ——	4.20064
Declination of Regulus	12	58	.5 N.	— Co-ſine —	9.98877
Latitude ——	32	14	.0 S.	— Co-ſine —	9.92731
Sum ——	45	12	.5	N. V. S. = 29547	
				N. N°. - - 13083 -	4.11672
True altitude —	35	0	31	N. Co-v. S. = 42630	
Equation, table XII. = +		1	21		
Apparent altitude -	35	1	52		

III. Required the true and apparent altitudes of the Moon, December 25, 1792, at 10h. 40′ 27″ P. M. in latitude 21° 28′ S. and longitude 55° 48′ E.?

Apparent time ——	10h.	40′	27″	Apparent time	10h. 40′ 27″
Sun's right aſcenſion	18	21	0	Longit. in time	3 51 12
Right aſc. of meridian	5	1	27	Reduced time -	6 49 15
Moon's right aſcenſion	4	21	21		
Moon's merid. diſtance	0	40	6	— Riſing ——	3.18378
Moon's declination —	16	32 N.		— Co-ſine — —	9.98166
Latitude — ——	21	18 S.		— Co-ſine — —	9.96878
Sum — — —	38	0		N. V. S. = 21199	
				N. N°. = 1362 -	3.13422
True altitude ——	50°	45′	0	N. Co-v. S. = 22561	
Equation, table XI. - - —		34	48		
Apparent altitude —	50	10	22		

PROB.

Reduce the meridian diſtance to degrees, and the logarithm of the natural verſed ſine anſwering thereto, will be the log. riſing.

PROBLEM II.

Given the apparent Diſtance between the Moon and the Sun or a fixed Star, the apparent Time, the Latitude and Longitude by account; to find the true Longitude.

RULE.

Compute the true* and apparent altitudes of each object's center by laſt problem; with which and the apparent diſtance, find the true diſtance, and hence the apparent time at Greenwich as uſual. Then will the difference between the apparent time at the place of obſervation and Greenwich, be the longitude in time, which is weſt or eaſt, according as the time at the place is earlier or later than that at Greenwich.

EXAMPLES.

Auguſt 23, 1792, in latitude 48° 54′ S. and longitude by account 25° 40′ W. at 1h. 15′ 18″ apparent time, the diſtance between the neareſt limbs of the Sun and Moon was 68° 21′ 20″. Required the true longitude?

Apparent time —	1h.	15′	18″	Apparent time —	1h.	15′	18″
Sun's right aſcenſion	10	12	25	Longitude in time -	1	42	40
Right aſcenſion mer.	11	27	43	Reduced time —	2	57	58
Moon's right aſcenſion	14	33	55				
Moon's mer. diſtance	3	6	12	— — Riſing —			4.49455
Moon's declination -	11	10	S.	— — Co-ſine —			9.99170
Latitude — —	48	54	S.	— — Co-ſine —			9.81781
Difference —	37	44	—	N.V.S. - 20913			
				N.No. — 20140 —			4.30406
True altitude —	36	7	—	N.Co-v.S. 41053			
Reduction —	—	45	48				
Apparent altitude —	35	21	12				

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* It is ſufficient in moſt caſes, to compute the true altitude to the neareſt minute; but the reduction of altitude muſt be taken out to the neareſt ſecond.

Sun's meridian diſtance 1h. 15′ 18″ — — Riſing — 3.72829
Sun's declination — 11 6.5 N. — — Co-ſine — 9.99178
Latitude — 48 54.0 S. — — Co-ſine — 9.81781

Sum — — 60 0.5 N.V.S. - 50013
N.N°. - - 3450 — 3.53788

Sun's true altitude - 27 45 N.Co-v.S. 53463
Reduction — + 1 40

Sun's apparent alt. - 27 46 40
Obſerved diſtance 68° 21′ 20″ Appar. altitude of ☽ 35° 21′ 12″
Sun's ſemidiam. + 15 53 Appar. altitude of ⊙ 27 46 40
Moon's ſemidiam. + 15 45
Augmentation - + 8 Diff. of app. alt. — 7 34 32

Appar. diſtance 68 53 6 N.V.S. 639759
Diff. app. altitudes 7 34 32 N.V.S. 008728 Log. diff. 9.995947

Moon's true alt. 36 7 Diff. - 631031 Log. — 5.800051

Sun's true altitude 27 45 N.N°. 625170 — — 5.795998
N.V.S. 010643
Diff. true altitudes 8 22
True diſtance — 68 38 33 N.V.S. 635813
Diſtance at noon 67 6 52 Diff. 1° 31′ 41″ — — P. L. 2930
Diſt. at III. hours 68 39 55 Diff. 1 33 3 — — P. L. 2866

Apparent time at Greenwich, 2 57 22 — — P. L. 0064
Apparent time at place of obſ. 1 15 18

Longitude in time — — 1 42 4 = 25° 31′ W.

June 20, 1791, in latitude 80° 37′ N. and longitude by account 17° 30′ W. at 14h. 6′ 48″ apparent time, the diſtance between the neareſt limbs of the Sun and Moon was 115° 14′ 52″ the apparent altitude of the Sun's center 15° 28⅓′. Required the true longitude?

Apparent time, - 14° 6′ 48″ App. alt. Sun's center 15° 28′ 20″
Longitude in time 1 10 0 W. Correction — — 3 16

Reduced time — 15 16 48 True alt. Sun's center 15 25 4
Apparent time - 14 6 48 Obſ. diſt. ⊙ & ☽'s lim. 115 14 52
Sun's right aſcen. 5 58 12 Sun's ſemidiameter - + 15 47

Right

Right aſcen. mer.	20 5 0	Moon's aug. ſemidiam. +	15 43	
Moon's right aſc.	22 17 54	App. central diſtance -	115 46 22	
Mer. diſtance of ☽	2 12 54	— Riſing — —		4.21345
Moon's declination	6 54 —S.	— Co-ſine — —		9.99699
Latitude —	80 38 —N.	— Co-ſine — —		9.21153
Sum —	87 22	N.V.S. -	95406	
		N.No. - -	02642	— — 3.42197
Moon's true altitude	1° 7′ 6″	N.Co-v.S.	98048	
Reduction, tab. XI.	— 30 1			
Moon's app. altitude	0 37 5	— Secant — —		0.0000253
Sun's app. altitude	15 28 20	— Secant — —		0.0160311
Difference — —	14 51 15	N.V.S. -	033418	
App. diſtance -	115 46 22	N.V.S. -	1434803	
			1401385	Log. 6.1465574
Moon's true alt.	1 7 6	— Co-ſine — —		9.9999173
Sun's true altitude	15 25 4	— Co-ſine — —		9.9840829
Difference —	14 17 58	N.V.S. -	030981	
			1401567	— 6.1466140
True diſtance -	115 37 46	N.V.S.	1432548	
Diſt. at XV. hours	115 46 22	Diff. -	0° 8′ 36″	P. L. 1.3208
Diſt. at XVIII. h.	114 14 56	Diff. -	1 31 26	P. L. 0.2942
Proportional part —	0 16 56	— —		P. L. 1.0266
Preceding time —	15 0 0			
App. time at Greenwich	15 16 56			
App. time at ſhip —	14 6 48			
Longitude in time —	1 10 8 = 17° 32′ W.			

The logarithmic difference in table XLII. is not carried lower down than 3 degrees; therefore as the Moon's altitude in the preceding example is leſs than that quantity, an operation for clearing the apparent diſtance is uſed, which differs in part from that formerly given. This difference is however obvious on inſpection.

CHAP.

CHAP. VII.

A New Method of finding the Longitude and Latitude of a Ship at Sea.*

INTRODUCTION.

IN the preceding methods of finding the longitude at ſea, the neceſſary elements, beſide the obſervations, are, the latitude of the ſhip, and its longitude by account, together with the time at the ſhip nearly. It may however happen, that the latitude is not accurately known; hence the apparent time at the ſhip cannot be computed, and conſequently the ſhip's longitude will remain unknown. To obviate this, the following method was invented, wherein neither the latitude, longitude, nor time, are required as neceſſary data, but are found directly from the ſame ſet of obſervations.

PROBLEM.

The Apparent Diſtance between the Moon and the Sun or a Fixed Star, together with the Altitude of each being given, to find the Latitude and Longitude of the Place of Obſervation.

RULE.

Take the ſemidiameters of the Sun and Moon, and the Moon's horizontal parallax, from the Nautical Almanac for the noon of the given day, and let the Moon's ſemidiameter be increaſed by its augmentation from table xxx.

Reduce the obſerved altitude of each object's limb to the apparent central altitude, and find the correction of altitude anſwering to each; from whence, and the central diſtance of the objects, compute the correct diſtance as formerly, and find the correſponding time at Greenwich.

To this time, let the Moon's ſemidiameter and horizontal parallax be again taken from the Nautical Almanac, and find the ſum of the corrections of the altitudes of both objects. Then to the ar-co-P. L.

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* This method was tranſmitted to Dr Maſkelyne, ſeveral years after the author diſcovered it; in conſequence of which, he received a letter from Dr Maſkelyne, dated Royal Obſervatory, Greenwich, May 22, 1787, from which the following is extracted.

"Laſt Sunday ſe'night, a paper of yours was left with me, intitled, *A New Method of finding the Longitude and Latitude of a Ship at Sea*, which I was deſired to preſent to the Board of Longitude, if I thought it deſerving of that honour. I ſhall be ready to preſent it to the Board at their next meeting, as I think it ingenious, and that you might reap the credit you deſerve. Your method of correcting the errors ariſing from a wrong aſſumption of horizontal parallax of the Moon and ſemidiameter, are indeed ingenious, and your modes of calculation for finding the latitude, time, and longitude, are neat."

of the ſum of the eſtimate corrections, add the P. Log. of the ſum of the true corrections, and the P. Log. of the difference between the apparent and computed diſtances; the ſum will be the P. Log. of the correction of diſtance—which correction, together with the change of the Moon's ſemidiameter, between noon and the Greenwich time of obſervation, being applied to the apparent central diſtance, will give the true diſtance; from which, the apparent time at Greenwich is to be re-computed.*

From the N. V. ſine of the zenith diſtance of the object neareſt the meridian, ſubtract the N. V. ſine of the difference between the true diſtance, and the zenith diſtance of the other object. To the log. of the remainder, add the log. co-ſecants of the two laſt quantities; the ſum of theſe three logs. being found in column of riſing, gives *arc firſt.*

From the N. V. ſine of the polar diſtance of the object neareſt the meridian, ſubtract the N. V. ſine of the difference between the true diſtance, and the polar diſtance of the other object; to the log. of the remainder, add the log. co-ſecants of the two laſt diſtances; the ſum, being found in column of riſing, gives *arc ſecond.* The difference between arcs firſt and ſecond is *arc third.*

Take the log. of arc third from column of riſing, to which add the log. ſines of the zenith and polar diſtances of the object fartheſt from the meridian; ſubtract the natural number anſwering to this ſum, from the N. V. ſine of the difference between the above zenith and polar diſtances; the remainder will be the coverſed ſine of the latitude of the place of obſervation.

Take the log. of arc third from column of half elapſed time, to which add the log. co-ſine of the latitude, and the log. co-ſecant of the zenith diſtance of the object fartheſt from the meridian; the ſum being found in column of half elapſed time, gives the diſtance of that object from the meridian at the time of obſervation.

Now, if the object whoſe meridian diſtance is computed, be the Sun, the apparent time at the ſhip will be known—the difference between which, and the Greenwich time, will be the longitude of the ſhip from Greenwich. But if that object is a ſtar, or the Moon, let the right aſcenſion of the meridian be found, by applying the object's meridian diſtance to its right aſcenſion, reduced to the Greenwich time of obſervation, by addition or ſubtraction, according as it is to the weſt or eaſt of the meridian; from which ſubtract the Sun's right aſcenſion, and the remainder will be the apparent time at the ſhip. Now the difference between the apparent times of obſervation at the

ſhip

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* If the difference between the computed and true diſtances is a very ſmall quantity, the difference between the approximate; and the apparent time at Greenwich, expreſſed in ſeconds, will be nearly equal to twice the number of ſeconds in that quantity.

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ſhip, and Greenwich, will be the longitude of the ſhip in time; which is eaſt, if the time at the ſhip is later than at Greenwich, but weſt if earlier.

Remark. If the object from which the time at the ſhip is inferred, be the Moon, or a ſtar, and its altitude imperfectly obſerved, it will be proper to take altitudes of the Sun for that particular purpoſe, either previous, or ſubſequent to the obſervations for the longitude. In this caſe, the computed latitude muſt be reduced to the place of obſervation of the Sun's altitude, by applying thereto, the change of latitude in the interval between the obſervations; and the longitude found by the compariſon of the time thus computed, with the Greenwich time, will be that of the meridian of the place where the obſervations for the apparent time were made.

Examples.

Being in ſouth latitude ⊙ June 24, 1792, in the afternoon, the diſtance between the neareſt limbs of the Sun and Moon was 55° 48′ 34″, the altitude of the Moon's lower limb 43° 23′, that of the Sun's 17° 40′, and the height of the eye 12 feet. Required the latitude and longitude of the place of obſervation?

Obſ. diſt. n. l.	55°48′34″	Alt. ☽'s l. l.	43°23′ 0″	Alt. ⊙'s l. l.	17°40′ 0″
⊙'s ſemidia. +	15 47	Semidiam. +	15 7	Semidiam. +	15 47
☽'s ſemidia. +	14 58	Dip - - - −	3 18	Dip - - - - −	3 18
Augment. - +	9	Appar. alt.	43 34 49	Appar. alt.	17 52 29
Ap. cent. diſt.	56 19 28	Correction +	38 47	Correction −	2 47
		True alt.	44 13 36	True alt. -	17 49 42

Moon's app. alt.	43° 34′ 49″				
Sun's app. alt. -	17 52 29				
Difference —	25 42 20	N.V.S.	098965		
Appar. diſtance	56 19 28	N.V.S.	445510	Log. diff.	9.995399
Moon's true alt.	44 13 36	Diff. -	346545	Log. —	5.539760
Sun's true alt. -	17 49 42	N. N°.	342893	— —	5.535159
Difference —	26 23 54	N.V.S.	104276		
Computed diſt. -	56 26 19	N.V.S.	447169		
Diſt. at III. hours	55 51 53	Diff.	0° 34′ 26	— — P. L.	7183
Diſt. at VI. hours	57 15 18	Diff.	1 23 25	— — P. L.	3349
Proportional part	— —		1 14 27	— — P. L.	3834

Proportional part — —	1 14 27	— —	P. L. 3834
Preceding time — —	3 0 0		
Approximate time at Greenw.	4 14 27		

The Moon's hor. par. at the Greenwich time is 54′ 59″. Now the

Eſtimate	38′ 47″	True corr.	38′ 51″	App. diſtance	56° 19′ 28″
corrections	2 47		2 47	Comp. diſt.	56 26 19
Sum —	41 34	Sum —	41 38	Difference -	6 51

Sum of eſtimate corrections —	41′ 34″	Ar-co-P. L.	9.3635
Sum of true corrections — —	41 38	— P.L.	0.6358
Difference between app. and comp. diſt.	6 51	— P.L.	1.4196
Difference between app. and true diſt.	6 52	— P.L.	1.4189

Moon's ſemid. at noon =	14′ 58″
— — at comp. time	14 59
Difference — —	1
Apparent diſtance — —	56 19 28
True diſtance — —	56 26 21

The difference between the true and computed diſtances being only 2″, the error of the computed time will therefore be 4″; hence the apparent time at Greenwich is 4h. 14′ 31″. Moreover the Moon's true altitude will be 44° 13′ 41″.

True diſtance -	56° 26′ 21″	— Co-ſecant — —	0.07920
Sun's zen. diſt.	72 10 18	— Co-ſecant — —	0.02138
Difference —	15 43 57	N.V.S. 03746	
Moon's zen. diſt.	45 46 19	N.V.S. 30248	
		Diff. - 26502 — Log. —	4.42328
Arc firſt —	3h. 12 59	— Riſing — —	4.52386
True diſtance -	56 26 21	— Co-ſecant — —	0.07920
Sun's pol. diſt.	113 24 42	— Co-ſecant — —	0.03731
Difference —	56 58 21	N.V.S. 45496	
Moon's pol. diſt.	99 13 0	N.V.S. 116017	
		Diff. - 70521 — Log. —	4.84832
Arc ſecond —	5 42 9	— Riſing — — -	4.96483
Arc third —	2 29 10		

Arc

Arc third —	2	29	10	— — Riſing — —	4·31056
Sun's zen. diſt.	72	10	18	— — Sine — — —	9·97862
Sun's pol. diſt.	113	24	42	— — Sine — — —	9·96269
Difference —	41	14	24	N.V.S. = 24804	
				17859 —	4·25185
Latitude —	34	59	7	N.Co-v.S. 42663 - Coſine -	9.91344
Sun's zen. diſt.	72	10	18	— — Co-ſecant — —	0.02138
Arc third —	2	29	10	— — H. E. T. — —	0.21762
App. time at ſhip	2	58	59	— — H. E. T. — —	0.15244
Ap. t. at Greenw.	4	14	31		
Longitude in time	1	15	32	= 18° 53′ W.	

In north latitude, January 10, 1791, about one o'clock P. M. the following obſervations were taken, the height of the eye being 16 feet. Required the latitude and longitude of the place of obſervation?

	Diſt. ☉ and ☽'s n. limbs		Alt. ☽'s low. limb		Alt. ☉'s low. limb.
	70° 10′ 30″	—	17° 48′	— —	9° 50′
	11 40	—	18 17	— —	45
	13 0	—	18 49	— —	39
	14 10	—	19 19	— —	32
	9 20		133		166
Mean —	70 12 20		18 33 15		9 41 30
Sun's ſemidiam.	+ 16 19	Semid.	+ 15 6	Semidiam.	+ 16 19
Moon's ſemid.	+ 15 2	Dip -	− 3 49	Dip -	− 3 49
Augmentation -	+ 4				
		App. alt.	18 44 32	App. alt.	9 54 0
Apparent diſt. -	70 43 45	Correct.	+ 49 30	Correction	− 5 9
Moon's app. alt.	18 44 32				
Sun's app. alt. -	9 54 0	True alt.	19 34 2	True alt. -	9 48 51

Difference - - -	8 50 32	N.V.S. = 011884	
Apparent diſt. -	70 43 45	N.V.S. = 669966 - Log. diff.	9.997941
Moon's true alt.	19 34 2		
Sun's true alt. -	9 48 51	Diff. - - 658082 - Log. —	5.818280
		N. N°. - 654970	
Difference —	9 45 11	N.V.S. - 014453 —	5.816221
Correct diſtance	70 41 46		
		N.V.S. - 669423	

Correct diſt. — 70° 41′ 46″
Diſtance at xxi. h. 69 35 20 Diff. - 1° 6′ 26″ — P. L. 4329
Diſtance at noon - 70 59 0 Diff. - 1 23 40 — P. L. 3327

Proportional part — — 2 22 55 — P. L. 1002
Preceding time — — — 21 0 0

Time at Greenwich — — 23 22 55

The time at Greenwich being ſo near noon, the correction ariſing from the change of the Moon's parallax and ſemidiameter, to be applied to the computed diſtance, is therefore almoſt inſenſible.

True diſtance - 70° 41′ 46″ — — Co-ſecant — — 0.02513
Moon's zen. diſt. 70 25 58 — — Co-ſecant — — 0.02583

Difference — 0 15 48 N.V.S. = 00001
Sun's zenith diſt. 80 11 9 N.V.S. = 82955

Diff. - - 82954 - Log. — 4.91884

Arc firſt — 5 44 36 — — — — Riſing - 4.96980

True diſtance - 70 41 46 — — Co-ſecant — — 0.02513
Moon's pol. diſt. 87 34 0 — — Co-ſecant — — 0.00039

Difference — 16 52 14 N.V.S. = 04304
Sun's pol. diſt. 111 55 47 N.V.S. = 137347

Diff. - - 133043 - Log. — 5.12399

Arc ſecond — 7 27 4 — — Riſing — — 5.14951

Arc third — 1 42 28 — — Riſing — — 3.99252
Moon's zen. diſt. 70 25 58 — — Sine — — — 9.97417
Moon's pol. diſt. 87 34 0 — — Sine — — — 9.99961

Difference — 17 8 2 N.V.S. = 04438
N.N°. = 09253 — — 3.96630

Latitude — 59 40 N.Co-v.S. 13691 Co-ſine - 9.7332
Moon's zen. diſt. 70 25 58 — — Co-ſecant — — 0.02583
Arc third — 1 42 28 — — H. E. T. — — 0.36417

Moon's mer. diſt. 3 35 5 — — H. E. T. — — 0.09332

Moon's

Moon's mer. diſt.	3°	35′	5″
Moon's right aſc.	0	0	20
Right aſc. merid.	20	25	15
Sun's right aſc.	19	27	48
App. time at ſhip	0	57	27
Ap. t. at Green.	23	22	55

Longitude in time 1 34 32 = 23° 38′ W:

The preceding method of computation would be facilitated, by taking the Moon's horizontal parallax and ſemidiameter from the Nautical Almanac, agreeable to the time anſwering to the apparent central diſtance. This time may be known with ſufficient accuracy for the above purpoſe, by inſpection. Moreover, if the latitude of the place of obſervation, and the azimuths of the obſerved objects, are known nearly, allowance may be made in the computation for the ſpheroidal figure of the earth. In this caſe, the computation gives the reduced latitude; and therefore the reduction from table XXXVI. is to be added thereto. The following example is inſerted, in order to illuſtrate the above.

March 3d, 1789, the central diſtance of the Sun and Moon, deduced from ſeven obſervations, and corrected by the index error, and the error of the line of collimation, was 72° 19′ 40″, the Sun's apparent central altitude was 25° 11′.1, that of the Moon 19° 22′.2, and the azimuths of theſe objects were 16°¼ and 95°⅓ reſpectively; the eſtimate latitude was 57° N. Required the true latitude and longitude of the place of obſervation.

The time, anſwering to the central diſtance, is a little before noon; and at this time the Moon's horizontal parallax is 56′ 42″ and ſemidiameter 15′ 27″.

The reduced and corrected altitudes of the Sun's center are 25° 23′.8, and 25° 22′.0 reſpectively; and thoſe of the Moon 19° 21′ and 20° 11′.7, and the Moon's reduced parallax 56′ 32″—from whence and the apparent diſtance, the true diſtance is 72° 1′ 44″.—See page 119.

True diſtance -	72°	1′	44″						
Diſt. at XXI. hours	70	57	50	Diff. =	1°	3′	54″	—	P. L. 4498
Diſtance at noon	72	26	28	Diff. =	1	28	38	—	P. L. 3077
Proportional part — —					2	9	46	—	P. L. 1421
Preceding time — —					21	0	0		
Apparent time at Greenw.					23	9	46		

True

True diſtance - =	72° 1'.7	— —	Co-ſecant —	— 0.02172
Moon's zen. diſt. =	69 48.3	— —	Co-ſecant —	— 0.02756
Difference —	2 13.4	N.V.S. = 00075		
Sun's zenith diſt. -	64 38.0	N.V.S. = 57159		
		Diff. - = 57084	- Log. —	4.75651
Arc firſt — —	4 35 27	— —	Riſing — — -	4.80579
True diſtance —	72 1.7	— —	Co-ſecant —	— 0.02172
Moon's P. diſtance	70 19.6	— —	Co-ſecant —	— 0.02612
Difference —	1 42.1	N.V.S. = 00044		
Sun's P. diſtance -	96 32.4	N.V.S. = 111390		
		Diff. - = 111346	Log. —	5.04667
Arc ſecond —	6 56 17	— —	Riſing — — -	5.09451
Arc third —	2 20 50	— —	Riſing — — -	4.26231
Moon's Z. diſtance	69 48.3	— —	Sine — — —	9.97244
P. diſtance	70 19.6	— —	Sine — — —	9.97388
Difference —	0 31.3	N.V.S. = 00004		
		16167	— —	4.20863
Reduced latitude -	56 57.6	N.Co-v.S. 16171	Co-ſine -	9.73658
Moon's Z. diſt. -	69 48.3	— —	Co-ſecant —	— 0.02756
Arc third —	2 20 50	— —	H. E. T. —	— 0.23916
Moon's merid. diſt.	5 31 47	— —	H. E. T. —	— 0.00330
Moon's right aſcen.	3 31 40		Reduced latitude	56 57.6
			Reduct. t. XXXVI. +	13.8
Right aſc. merid.	21 59 53			
Sun's right aſcen.	22 58 44		Latitude —	57 11.4 N.
Apparent time -	23 1 9			
Ap. time at Green.	23 9 46			
Longitude in time	8 37 = 2° 9¼' W.			

CHAP.

CHAP. VIII.

Of finding the Longitude at Sea or Land,

By

An Obſervation of the Diſtance between the Moon and a Star, not uſed in the Nautical Almanac.

INTRODUCTION.

THE diſtances between the Moon and the Sun, and ten of the brighteſt fixed ſtars neareſt the Moon's path, are given in the Nautical Almanac, when the Moon is in a proper poſition with reſpect to thoſe objects. It may however happen, that ſome other ſtar is in a more favourable poſition for obſervation, than any of thoſe given in the Ephemeris; the ſhip's longitude might therefore be determined by ſuch an obſervation, when perhaps it would otherwiſe be impoſſible.

The difference of longitude between the Moon, and the ſtar with which it is to be compared, muſt not however be leſs than a certain quantity, otherwiſe the Moon's relative motion will be too ſlow, for the purpoſe of determining the longitude. Several other elements enter alſo into this method, which makes it neceſſary to treat of it in a particular manner.

PROBLEM I.

To find that Quantity, which the Difference between the Longitudes of the Moon and the given Star muſt exceed, that the Moon's Relative Motion may not be too much diminiſhed.

RULE.

Enter table XVII. with the difference between the latitudes of the Moon and the propoſed ſtar, if both latitudes are of the ſame name, but with their ſum, if of contrary names, and find the correſponding quantity. Now if this quantity is leſs than the difference between the longitudes of the Moon and given ſtar, their diſtance may be obſerved, for the purpoſe of finding the longitude of the place of obſervation; but if it is greater than that difference, the Moon's relative motion will be too ſlow, to derive any benefit for the above purpoſe, from ſuch an obſervation.

EXAMPLE.

Is it proper to compare the Moon with α Orionis, in order to determine the longitude, 21 November 1792, about 10h. P. M. reduced time?

Latitude of Betelgueſe =	16°	4′	S.	Long. of Betelgueſe =	2^{s}	25°	52′
Latitude of the Moon	1	3	N.	Long. of the Moon	11	10	4
Difference of latitude -	17	7		Diff. of longitude —	3	15	48

Now in table XVII. oppoſite to the diff. of latitude 17° 7′ is 33° 48′; which being leſs than the diff. of longitude 105° 48′; that ſtar may, therefore, in the preſent caſe, be employed for the purpoſe of finding the longitude.

PROBLEM II.

Given the true Diſtance between the Moon and a fixed Star, together with the Latitude of each, to find the Moon's Longitude.

RULE.

To the true diſtance, add the latitudes of the Moon and ſtar, and find the difference between the half ſum and diſtance.

Now to the log. ſecants of the latitudes of the Moon and ſtar, add the log. co-ſines of the half ſum and difference, if the latitudes are of the ſame name, or the log. ſines if of a contrary name; half the ſum of theſe four logs. will be the log. co-ſine, or ſine of half the difference of longitude, according as the latitudes of the Moon and ſtar are of the ſame or of a different name.

To the apparent longitude of the ſtar, add the difference of longitude, if the Moon be eaſt of the ſtar, otherwiſe ſubtract it, and the ſum or remainder will be the true longitude of the Moon.

REMARK. It will be ſufficient in moſt caſes to take out the latitudes of the Moon and ſtar to the neareſt minute.

EXAMPLE.

Let the true diſtance between α Orionis and the Moon's center be 105° 26′ 14″, November 21, 1792, at 10 hours reduced time. Required the Moon's longitude?

True diſtance —	105°	26′	14″		
Latitude of Moon -	1	3	— N. — —	Secant	0.0000729
Latitude of α Orionis	16	3	— S. — —	Secant	0.0172672
Sum — —	122	32	14		
Half — —	61	16	7 — —	Sine -	9.9429416
Difference — —	44	10	7 — —	Sine -	9.8430908
					19.8033725
	52	53	3 — —	Sine -	9.9016862
Difference of long. -	105	46	6		

Mean

Mean longitude of α Orionis, 21 Nov. 1792,	-	2^s	25°	51′	40″
Equation of equinoxes, table LXI. — —	—				3
Aberration — — — —	+				19
Apparent longitude of α Orionis — —		2	25	51	56
Difference of longitude, the ☽ being weſt of	*	3	15	46	6
Moon's longitude — — —		11	10	5	50

PROBLEM III.

Given the True Longitude of the Moon, to find the Apparent Times at Greenwich.

RULE.

From the Nautical Almanac, take four longitudes of the Moon, two of which immediately preceding, and two following the given longitude. Find the difference between each pair ſucceſſively; find alſo the ſecond difference, and let their mean be taken.

Now, to the conſtant log. 2.8573325, add the ar-co. of the log. of the variation of the Moon's longitude in 12 hours, reduced to ſeconds, and the log. of the difference between the given and preceding longitudes in ſeconds; the ſum, rejecting radius, will be the log. of the approximate time in minutes, to be reckoned from the preceding noon or midnight.

Take the equation of ſecond difference from table XXXVI. anſwering to the approximate time, and the mean ſecond difference, with which enter table XXXVII. at the top, and find the equation correſponding thereto, and the Moon's motion in longitude in 12 hours, in the ſide column; which being applied to the approximate time, by addition or ſubtraction, according as the firſt difference of the Moon's motion is increaſing or decreaſing, will give the apparent time at Greenwich.

EXAMPLE.

21 November 1792, the longitude of the Moon deduced from obſervation was 11^s 10° 5′ 50″. Required the apparent time at Greenwich?

☽'s lon.	20h.	at midn.	10^s	27°	22′	0″			
	21	at noon	11	4	18	52	Diff. 6° 56′ 52″		
	21	at midn.	11	11	12	0	6 53 8	Diff. 3′ 44″	Mean
	22	at noon	11	18	1	27	6 49 27	= 3 41	=3′ 42″.5

☽'s long. 21h. at noon	11ˢ 4° 18′ 52″		
Given longitude —	11 10 5 50		
Difference —	5 46 58 = 20818″	Log. —	4.3184390
Var. ☽'s long. in 12 hours	6 53 8 = 24788″	Ar-co-log.	5.6057584
Conſtant log. — — — — —			2.8573325
Approximate time —	10 4 41 = 604′.68	—	2.7815299
The eq. from tab. xxxvii. anſwer. to 15″ the eq. of 2d. diff. & 6° 53′ is —	26		
App. time at Greenwich -	10 4 15		

Problem IV.

Given the Latitude and Eſtimate Longitude of the Place of Obſervation, the Diſtance between the Moon and a Fixed Star, together with the Altitude of each, to find the True Longitude of that Place.

Rule.

With the given latitude, the corrected altitude and declination of the ſtar, compute the apparent time of obſervation by prob. vii. page 95. Reduce this time to the meridian of Greenwich, and find the Moon's ſemidiameter and horizontal parallax agreeable thereto.

With the true and apparent altitudes, and the apparent central diſtance, compute the true diſtance; with which, and the latitudes of the Moon and ſtar, find their difference of longitude, and from thence find the apparent time at Greenwich, by the laſt problem. Now the difference between the apparent time at the place of obſervation, and that at Greenwich, will be the longitude of the place in time, as formerly.

Example.

November 21, 1792, in latitude 48° 50′ N. longitude by account 19° W. a little before 9h. P. M. the diſtance between the Moon's weſt limb and Betelgueſe was 106° 2′ 24″, the altitude of Betelgueſe 14° 44′, and that of the Moon's lower limb 26° 38′, height of the eye 10 feet. Required the true longitude?

Obſerved

Obſ. alt. Betelgueſe =	14°	44′			
Dip - - - - - - - -	—	3			
Apparent altitude - -	14	41			
Refraction - - - - -	—	3	35		
True altitude —	14	37	25		
North polar diſtance	82	39		— — Co-ſecant —	0.00358
Latitude —	48	50		— — Secant —	0.18167
Sum — —	146	6			
Half — —	73	3		— — Co-ſine —	9·46469
Difference —	58	26		— — Sine — —	9·93046
					19·58034
	38	5		— — Sine — —	9·79017
		8			

Mer. diſt. Betelgueſe	5	4	40
R. A. Betelgueſe -	5	43	57
R. A. of meridian	0	39	17
Sun's R. aſcenſion	15	51	2
Approximate time	8	48	15
Eq. to approx. time tab. XVII. - - -	—	1	32
— to long. 19° W.	—		13
Apparent time —	8	46	30
Longitude in time -	1	16	
Reduced time —	10	2	30
Obſerved diſtance	106	2	24
Augm. ſemidiam. —		16	6
Apparent diſt. -	105	46	18
Apparent alt. ☽	26	51	
Apparent alt. ✱	14	41	

Obſ. alt. ☽'s l. limb		26°	38′	
Semidiameter — dip -	+		13	
Apparent altitude -		26	51	
Correct. to red. par.	+		50	18
True altitude ☽ =		27	41	18
✱ =		14	37	25
Sum —		42	18	43
Half — —		21	9	21

Sum —	147	18	— — Log. diff. -	9·996844
Half — —	73	39	— — Co-ſine —	9·449485
Difference —	32	7	— — Co-ſine —	9·927867
				19·374196

Half ſum true alt.	21 9 21		
Arch — —	29 6 43	— — Sine —	9.687098
Sum — —	50 16 4	— — Co-ſine —	9.805637
Difference —	7 57 22	— — Co-ſine —	9.995799
			19.801436
	52 42 58	— — Sine —	9.900718
Computed diſtance	105 25 56		
Seconds omitted =	+ 18		
True diſtance —	105 26 14		

The Moon's longitude, inferred from the true diſtance, and the latitudes of the Moon and ſtar, by prob. II. page 146, is 11s 10° 5′ 50″, and the apparent time at Greenwich anſwering thereto, by prob. III. page 147, is — — = 10h. 4′ 15″
And the app. time at the place of obſer. is 8 46 30

Hence the longitude of that place is — 1 17 45 = 19° 26¼′ W.

C H A P. IX.

Of finding the Longitude at Sea or Land,

By

An Obſervation of the Diſtance between the Moon and a Planet.

INTRODUCTION.

THIS method of determining the longitude at ſea will, for the moſt part, be attended with the deſired ſucceſs, provided the Moon is compared with Venus, Mars, Jupiter, or Saturn. The proximity of Mercury to the Sun, and the ſmallneſs of the apparent magnitude of the Georgian planet, prevent them from being of any ſervice for this purpoſe.

Venus and Jupiter may be compared with the Moon, even when the twilight is pretty ſtrong, and in this caſe their altitudes may be very accurately obſerved. Hence the longitude being computed from the mean of ſeveral ſets of obſervations, may be ſafely depended on.

If the obſerver is provided with a Circular Inſtrument, he may meaſure the diſtance between the Sun and Moon when it exceeds 120°. As however, the diſtance when above that quantity, is not given in the Nautical Almanac, the following method of computation is to be uſed, in place of that formerly given. In this caſe, the difference of longitude between the Sun and Moon may be found thus:

To the log. ſecant of the Moon's latitude, add the log. coſine of the true diſtance; the ſum, rejecting radius, will be the log. coſine of the difference of longitude, of the ſame affection with the diſtance.

PROBLEM.

Given the Latitude of a Place, and its Longitude by account, the obſerved Diſtance between the Moon and a Planet,[*] *and their Altitudes, to find the true Longitude of the Place of Obſervation.*

RULE.

Let the error of the watch be inferred from the altitude of the Sun, obſerved the preceding evening, or from that of a fixed ſtar, or from the altitude of the Moon or planet, taken at the ſame time with the diſtance. Hence the apparent time of obſervation will be known—to which the longitude by account being applied, the reduced time will be obtained.

Find the apparent and true altitudes of the Moon and planet, with which, and their apparent central diſtance, compute the true diſtance. Now, with the true diſtance, and the latitudes of the Moon and planet, find their difference of longitude, by prob. II. page 146, which being added to, or taken from the geocentric longitude of the planet, found in the Nautical Almanac page IV. of the month, according as the Moon is eaſt or weſt of the planet, and the Moon's true longitude will be obtained. Find the time at Greenwich anſwering to this longitude, by prob. III. page 147, the difference between which, and the time at the place of obſervation, will be the longitude of that place in time.

EXAMPLE.

April 2, 1792, in latitude 35° 10′ S. and longitude by account, 42° E. at 7h. 24′ P. M. per watch, the diſtance between the Moon's weſt limb and Jupiter's center was 71° 12′ 8″, the altitude of the

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[*] The diſtance between the Moon's limb, and the center of the planet, is to be obſerved. In the rules, no allowance is made for the parallax of the planet, which in many caſes is inſenſible. Thoſe who wiſh to be rigorous in their calculations, may compute the Geocentric place of the planet, its ſemidiameter and horizontal parallax, from the beſt aſtronomical tables. M. de Lambre's tables of Jupiter and Saturn, and thoſe of the other planets by M. de la Lande, will be proper for this purpoſe.

Moon's lower limb 41° 18', that of Jupiter's center 11° 13', and height of the eye 13 feet. Required the true longitude?

Time p.w. 7h. 24' Obſ. diſt. ☽'s l. from ♃ 71° 12' 8" ☽'s h. par. 55' 35"
Long. in t. 2 48 Augm. ſemidiameter — 15 19 Reduction — 5

Red. time 4 36 App. central diſtance 70 56 49 Red. h. par. 55 30
Obſ. alt. ☽'s l. limb = 41° 3'   Obſerved altitude of ♃ 11° 13'
Semidiameter — + 15   Dip — — — 3
Dip — — — 3
  Apparent altitude - - 11 10
Apparent altitude - 41 15   Refraction — — 4 42
Correction — + 40 38
  True altitude of ♃ = 11 5 18
True alt. of ☽'s center 41 55 38   — — — 41 55 38
App. diſtance + 11" 70 57
Apparent altitude ☽ - 41 15   Sum — — 53 0 56
Apparent altitude ♃ - 11 10   Half — — 26 30 28

Sum — — 123 22 — — Log. diff. 9.995559
Half — — 61 41 — — Co-ſine - 9.676094
Difference — 9 16 — — Co-ſine - 9.994295

Half ſum true altitudes 26 30 28   19.665948
Arch — — 42 54 2 — — Sine — 9.832974

Sum — — 69 24 30 — — Co-ſine - 9.546179
Difference — 16 23 34 — — Co-ſine - 9.981977

19.528156
35 30 42 — — Sine - - 9.764078

Computed diſtance - - 71 1 24
Seconds added — — 11

True diſtance — 71 1 13
Latitude of ☽ — 3 48 S. — — Secant - 0.000956
Latitude of ♃ — 1 32 N. — — Secant - 0.000156

Sum — — 76 21
Half — — 38 10 30 — — Sine — 9.791034
Difference — 32 50 30 — — Sine — 9.734255

19.526401

35

	35 25 45½	— Sine — —	9.763201
	70 51 31		
Seconds omitted - -	+ 13		
Differ. of longitude -	70 51 44 = 2^{s} 10° 51′ 44″		
Geoc. long. of ♃ per Ephemeris —	6 28 16 30		
Moon's true longitude — —	4 17 24 46		
Moon's long. 2d April at noon —	4 15 3 20		
Difference of longitude — —	2 21 26		
☽'s long. 1ſt at midn.	= 4^{s} 8° 54′ 30″		
2d. at noon	= 4 15 3 20 + 6° 8′ 50″		
2d. at midn.	= 4 21 16 5 + 6 12 45 + 3′ 55″		
3d. at noon	= 4 27 33 8 + 6 17 3 + 4 18 + 4′ 6″½		
Conſtant logarithm — — —		—	2.8573325
Var. ☽'s long. in 12 hours	6° 12′ 45″ = 22365″	ar-co-log.	5.6504311
Diff. of long. of ☽ and ♃	2 21 26 = 8486	log. —	3.9287030
Approximate time —	4 33 11 = 273′.19	log. —	2.4364666
Eq. from tab. XXXVIII. -	+ 56		
App. time at Greenwich	4 34 7		
Altitude of Jupiter —	11° 5′		
South polar diſtance of ♃	80° 33	— Co-ſecant —	0.00593
Latitude of ſhip —	35 20	— Secant —	0.08752
Sum — —	126 48		
Half — —	63 24	— Co-ſine —	9.65140
Difference — —	52 19	— Sine —	9.89840
			19.64289
	41 31½	— Sine —	9.82144
	8		
Meridian diſtance ♃	5 32 10		
Paſſ. ♃ over mer. ſhip	12 55 3		
Apparent time at ſhip -	7 22 53		
App. time at Greenw.	4 34 7		
Longitude in time —	2 48 46 = 42° 11½′ E.		

X CHAP.

C H A P. X.

Of finding the Longitude at Sea or Land.

The neceſſary data being

The obſerved Altitude of either limb of the Moon, the Apparent Time at the Place of Obſervation, together with its Latitude, and Longitude by account.

INTRODUCTION.

A Like degree of accuracy is not to be expected in this, as in any of the former methods. In thoſe, it was ſufficient to take the altitude to the neareſt minute, but in this, the altitude ſhould be taken to the neareſt ſecond, and the apparent time of obſervation, determined to the tenth of a ſecond. The Moon's right aſcenſion and declination ſhould be reduced to the apparent time of obſervation, by the method of *Interpolation;* and if the computed longitude differs conſiderably from that which was aſſumed, ſo as to make an alteration in the Moon's declination, the operation muſt be repeated.

PROBLEM.

Given the obſerved Altitude of the Moon's limb, to find the Longitude of the Place of Obſervation.

RULE.

Reduce the apparent time of obſervation to the meridian of Greenwich, by prob. III. page 73. To this time, take the Moon's declination from the Nautical Almanac; with which, the correct altitude of the Moon, and the latitude of the place, find the horary diſtance of the Moon from the meridian, by prob. IX. page 100. Now to the Moon's right aſcenſion at the preceding noon, add its meridian diſtance, if in the weſtern hemiſphere—otherwiſe ſubtract it; and from this ſum or difference, increaſed by 360° if neceſſary, ſubtract the Sun's right aſcenſion at the preceding noon; reduce the remainder to minutes, and find its logarithm.

From the ſum of 180°, and the variation of the Sun's right aſcenſion in 12 hours, ſubtract the variation of the Moon's right aſcenſion in 12 hours; and find the arithmetical complement of the logarithm of the remainder. Now the ſum of theſe two logarithms, and the conſtant quantity 2.8573325, will be the log. of a certain quantity expreſſed in minutes.

From the P. Log. of the difference between this time, and the apparent time at the place of obſervation, ſubtract the P. Log. of the difference between the variation of the right aſcenſions of the Sun and Moon in 12 hours, and the remainder will be the P. Log. of the longitude in minutes and ſeconds, which are to be eſteemed degrees and minutes.

EXAMPLES.

October 17, 1791, in latitude 43° 18′ N. and longitude by account 24° 30′ W. at 11h. 37′ 53″ apparent time, the altitude of the Moon's lower limb was 23° 31′, and the height of the eye 12 feet. Required the true longitude of the place of obſervation?

Obſerved diſt. ☽'s l. l.	23° 31′ 0″	App. time of obſ.	11h. 37′ 53″
Augm ſemidiameter +	14.9	Longitude in time	1 38 0 W.
Dip — — −	3.3		
		Eſt. time at Green.	13 15 53
Apparent alt. ☽'s center	23 42.6		
Correction — +	47.6		
True altitude ☽'s center	24 30.2		
Moon's polar diſtance	71 41	— Co-ſecant —	0.02258
Latitude —	43 18	— Secant — —	0.13800
Sum — —	139 29.2		
Half — —	69 44.6	— Co-ſine —	9.53936
Difference — —	45 14.4	— Sine — —	9.85130
			19.55124
	36 37.2	— Sine — —	9.77562
Moon eaſt of meridian	73 14.4		
Moon's right aſcenſion	83 36.0		
Difference — —	10 21.6		
Sun's right aſcenſion	202 16.2		
Remainder —	168 5.4	= 10085.′4 log. —	4.0036931
180°+var. ☉'s R.A. in 12 h. — —	180 28.1	Conſtant log. —	2.8573325
Var. ☽'s R.A. in 12h.	6 14.0		
Difference — —	174 14.1	= 10454.′1 ar-co-log.	5.9807133
	11 34 36	= 694.6 — —	2.8417389
Apparent time of obſer.	11 37 53		
Difference — —	3 17	— P. Log. —	1.7389
Diff. var. ☽ & ☉'s R. A. in t.	23 3.6	— P. Log. —	0.8924
Longitude — — —	25° 38′	— — P. Log. —	8465

Which is weſt, becauſe the time at the ſhip is later than the computed time.

December 26, 1792, in latitude 45° 28′ N. and longitude by account 38° 10′ E. at 14h. 20′ 10″ apparent time, the altitude of the Moon's lower limb was 39° 2′.9, and height of the eye 16 feet. Required the true longitude?

Obſerved alt. ☽'s l. limb	39° 2′.9	Apparent time —	14h. 20′ 10″
Semidiameter — —	+ 15.0	Longitude in time	2 32 40
Dip — —	− 3.8		
		Reduced time —	11 47 30
Apparent alt. ☽'s center	39 14.1		
Correction — —	+ 41.3		
True altitude — —	39 55.4		
Polar diſtance —	71 50.6	— Co-ſecant —	0.02218
Latitude — —	45 28.0	— Secant — —	0.15408
Sum — —	157 14.0		
Half — —	78′ 37.0	— Co-ſine —	9.29529
Difference —	38 41.6	— Sine — —	9.79599
			19·26754
	25 29.2	— Sine — —	9.63377
Moon weſt of meridian	50 58.4		
Moon's right aſcenſion -	74 25.0		
Sum — —	125 23.4		
Sun's right aſcenſion -	276 2.1		
Remainder — —	209 21.3	= 12561.3 — —	4.0990346
180° + var. ⊙'s R. A. in 12h. — —	180 33.2	Conſtant log. —	2.8573325
Var. ☽'s R. A. in 12h.	6 21		
Difference — —	174 12.2	= 10452.2 ar-co-log.	5.9807923
	14 25 17	= 865.28 — —	2.9371594
Apparent time at ſhip -	14 20 10		
Difference — —	5 7	— — P. L. —	1.5463
Diff. var. ⊙ and ☽'s R. A. in 12h. —	23 11.2	— — P. L. —	8900
Longitude — —	39° 43′	— — P. L. —	6563

Which is eaſt, becauſe the apparent time at the ſhip is earlier than the computed time.

BOOK

BOOK IV.

CONTAINING

Various other Methods of finding the Longitude of Places.

CHAP. I.

Of finding the Longitude of a Place,

By

An Obſervation of the Moon's Tranſit over the Meridian.

INTRODUCTION.

THIS method appears to have been firſt mentioned in Purchas' account of Hall's diſcovery of Greenland, and ſoon after it was clearly explained in Carpenter's Geography, printed at Oxford in 1635. It is alſo mentioned by Dr. Maſkelyne in his inſtructions, relative to the tranſit of Venus in 1769. By the Abbe Toaldo, in a pamphlet publiſhed in 1784, and by Mr. Pigott in the Phil. Tran. for 1786 and 1790, who very ſtrongly recommends it, "being convinced, "that in a very ſhort time it muſt be univerſally adopted, having every "advantage over Jupiter's firſt ſatellite, and but little inferior in pre-"ciſion to occultations."

The longitude of a place may be very accurately determined by this method at land; and a tolerable degree of preciſion may be obtained at ſea, by obſerving equal altitudes of the Moon, as accurately as poſſible, and making an allowance in the time of tranſit, deduced from the equal altitudes, for the change of declination between the obſervations.

As the obſervation for the above purpoſe happens ſo very often, it therefore ſeems to demand the particular attention of the practical aſtronomer, as well as of thoſe who travel to diſtant countries. The Marquis de Chabert, in his travels, uſed this method with ſucceſs. In 1753, he determined the longitude of Carthagena; and in order to facilitate the practice of it, he deſcribed, in the *Memoires de l'Academie* for the year 1766, a ready and ſimple method of placing and verifying the poſition of the Tranſit Inſtrument.

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The neceſſary obſervation, for determining the longitude by this method, is the apparent time of the Moon's paſſage over the meridian. Or, in order to attain the utmoſt accuracy of which it is ſuſceptible, the paſſage of a ſtar, having the ſame declination nearly, is alſo to be obſerved; both at the place whoſe longitude is wanted, and at a place whoſe ſituation is known. It is not abſolutely neceſſary, that the ſame ſtar be obſerved at both places. In this caſe, it will be proper to uſe thoſe ſtars only, whoſe places are accurately determined. The inſtruments neceſſary for this purpoſe at land are, a tranſit inſtrument, and an aſtronomical clock. At ſea, the equal altitudes may be obſerved with a ſextant for the ſake of greater accuracy.

The meridian altitude of the Moon being obſerved at the ſame time with the tranſit, will afford another method of determining the longitude; and which, when the change of the Moon's declination is a maximum, or nearly ſo, will be found tolerably accurate. This method, however, can only be practiſed with ſucceſs in an obſervatory.

PROBLEM I.

Given the Apparent Time of the Tranſit of the Moon's Limb over the Meridian, to find the Longitude of the Place of Obſervation.

RULE.

Reduce the apparent ſemidiameter of the Moon to time, by tables XXXIX. and XL. which being applied to the apparent time of tranſit of the Moon's limb, by addition or ſubtraction, according as the weſtern or eaſtern limb was obſerved; the ſum or difference will be the apparent time of the tranſit of the Moon's center.

From the right aſcenſion of the Moon in time, ſubtract that of the Sun, the remainder will be the approximate time of the Moon's paſſage over the meridian of Greenwich. Now, to the log. of the difference between the change of the Moon's right aſcenſion in 12 hours, and that of the Sun's in the ſame time, reduced to ſeconds, add the ar-co-log. of the difference between the above quantity and 43200". and the log of the approximate time in ſeconds; the natural number anſwering to the ſum of theſe three logs, will be the correction in ſeconds, of the approximate time; which being added thereto, will give the apparent time of the Moon's paſſage over the meridian of Greenwich.

From the P. Log. of the difference between the apparent times of tranſit at Greenwich and the place of obſervation, ſubtract the P. Log. of the difference between the variations of the right aſcenſions of the Sun and Moon in 12 hours; the remainder will be the P. Log. of the longitude of the place of obſervation in minutes and ſe-

ſeconds, which are to be eſteemed degrees and minutes. If the time of tranſit at the place of obſervation is earlier than that at Greenwich, the longitude is eaſt, otherwiſe it is weſt.

EXAMPLES.

July 2, 1792, the apparent time of the tranſit of the Moon's weſtern limb was 10h. 43′ 58″. Sought the longitude of the place of obſervation?

☽'s R. A. at noon	16h. 58′ 20″	☽'s R. A. at noon - -	16° 58′ 20″	App. time of tran. ☽'s w. l.	10h. 43′ 58′
☉'s R.A. at noon	6 48 30.4	☽'s R. A. at midnight	17 30 0	☽'s eq. ſemid tab. xxxix.	+ 1 9.7
				Increaſe of ſem. diam. t. xl.	+ 3.6
Approx. time	10 9 49.6	Var. ☽ R A. in 12 h.	31 40		
		Var. ☉ R.A. in 12 h.	2 3.7	App. time of tr. ☉'s center	10 45 11.3

Difference — — — —	29 36.3 =	1776″.3	log. -	3.249516
Conſtant quantity — — — —		43200		
Difference — — — — —		41423.7	ar-co-log	5.382751
Approximate time of tranſit —	10 9 49.6 =	36589.6	log. -	4.563358
Correction — — —	+ 26 9.0	1569.0	— —	3.195625
Apparent time of tranſit at Green.	10 35 58.6			
App. time of tran. at place of obſ.	10 45 11.3			
Difference — — —	9 11.7	—	P. L. —	1.2909
Diff. of var. R. A. ☉ and ☽ in 12 h.	29 36.3	—	P. L. —	0.7839
Longitude — — —	56° 0′	— —	P. L. —	0.5070

Which is weſt, becauſe the time of tranſit by obſervation is later than the time at Greenwich.

December 30, 1792, the Moon's eaſtern limb was obſerved to paſs the meridian at 13h. 53′ 33″.8. Required the longitude of the place of obſervation?

☽'s R. A. at noon	8h. 42′ 20″	☽'s R. A. at noon	8h. 42′ 20″	Appar. time of tranſit ☽'s E. l.	13h. 53′ 33″.8
☉'s —	18 44 4.4	at midnight	9 6 12	Eq. ſem. t. xxxix. —	1 0.8
				Increaſe, tab. xl. —	2.0
Aprox. time	13 58 15.6	Var. ☽'s R. A. in 12 h.	23 52		
	12	☉'s — —	2 12.6	Appar. time of tran. ☽'s cent.	13 52 31.0
	1 58 15.6				

Difference —	21 39.4 =	1299″.4	log.	3.113743
Conſtant quantity — — — —		43200		
Difference — — — — —		41900.6	ar-co-log.	5.377780

Ap-

Approximate time of tranſit	1 58 15.6 A.M.	7095.6	log.	3.850989
Correction	+ 3 40.0	= 220.0		2.342512
Apparent time of tranſit	2 1 55.6 A. M.			
Apparent time by obſervation	1 52 31.0 A. M.			
Difference	9 24.6		P. L.	1.2817
Diff. var. R. A. ☉ and ☽ in 12 hours	21 39.4		P. L.	9197
Longitude	78° 13′ E.		P. L.	3620

The above is a very ready method of finding the longitude of a place, eſpecially as a ſingle obſervation only is neceſſary. The longitude, deduced by it, is however affected by the error of the Moon's place in the Nautical Almanac. The poſition of the tranſit inſtrument muſt alſo be perfectly rectified, or allowance made in the obſerved time of tranſit for its deviation from the meridian, otherwiſe the concluſions cannot be depended on. In the following method, the error in longitude, ariſing from an error in the Moon's place, is avoided; and if the horizontal deviation of the inſtrument from the meridian does not exceed a few ſeconds of time, a very accurate ſolution may be obtained, provided the declinations of the Moon and ſtar be nearly the ſame.

PROBLEM II.

Given the Intervals of Time between the Tranſits of the Moon's Limb and a Fixed Star over two different Meridians, to find the Difference of Longitude between the Places of Obſervation.

RULE.

The difference or ſum of the given intervals, according as the Moon is on the ſame, or on oppoſite ſides of the ſtar, at the places of obſervation, reduced to ſiderial time, will be the increaſe of the Moon's right aſcenſion, anſwering to the difference of meridians of thoſe places. To the log. of which expreſſed in ſeconds, add the ar-co-log. of the increaſe of the Moon's right aſcenſion in 12 hours in minutes, taken from the Nautical Almanac, and the conſtant log. 5.20951; the ſum will be the log. of the difference of longitude in ſeconds.

The weſternmoſt place of obſervation anſwers to the greater or leſs interval, according as the Moon is to the eaſt or weſt of the ſtar. If the Moon be on oppoſite ſides of the ſtar, that place will be weſternmoſt, where the Moon was obſerved to precede the ſtar.

EXAMPLE.

Let the interval in ſiderial time, between the tranſits of the Moon's weſt limb and δ Scorpionis, be 1 h. 24′ 3″.7, and the interval in ſiderial time, as obſerved at another place, be 1h. 21′ 40″.3. —the increaſe of the Moon's right aſcenſion in 12 hours being 6° 12′. Required the longitude of the place of obſervation?

Obſ. interval at Greenw.	1 h. 24′ 3″.7			
Obſ. inter. at other place	1 21 40.3			
Difference — —	2 23.4	= 143″.4	log. —	2.15655
Increaſe ☽'s R. A. in 12h.	6° 12′	= 372′	ar-co-log.	7.42946
Conſtant log. — — — — —				5.20951
Longitude —	17° 20′ 50″	= 62450″	— —	4.79552

CHAP. II.

The Method of finding the Longitude of a Place at Sea or Land,

By

An Eclipſe of the Moon.

INTRODUCTION.

THIS method of finding the longitude of a place was mentioned by Hipparchus, about 150 years before the Chriſtian æra. It is particularly deſcribed in Apian's Coſmography, printed at Antwerp in 1550; and ſince, has been recommended by ſucceeding writers on this ſubject.

This is, perhaps, the moſt eaſy of the aſtronomical methods that have been propoſed, to find the longitude at ſea; as it is neither affected by refraction nor parallax, to which ſome of the other methods are liable. However, as lunar eclipſes happen ſo very ſeldom, the longitude cannot be found by this method ſo often as is neceſſary; yet ſuch obſervations ſhould not, if poſſible, be neglected.

The beginning and end of the eclipſe are the principal phaſes, from which the longitude is to be found. If the obſerver is provided with a ſextant, theſe phaſes may be obſerved with tolerable accuracy, with the teleſcope belonging to that inſtrument. The obſervations may alſo be multiplied, by meaſuring the verſed ſines of the enlightened

ened part of the Moon's diſc, for which purpoſe this inſtrument is well adapted at ſea. At land, a different method is commonly uſed.

PROBLEM I.

To find if a Lunar Eclipſe will be viſible at a given Place.

RULE.

To the times of the beginning and end of the eclipſe, as given in the Nautical Almanac, let the longitude by account be added or ſubtracted, according as it is eaſt or weſt, and the correſpondent inſtants of theſe phaſes by the meridian of the place will be obtained.

Now, compute the times of Sun-riſing and ſetting by means of a table of ſemidiurnal arcs or otherwiſe; and if the reduced times of theſe phaſes happen while the Sun is above the horizon, the eclipſe will not be viſible; but one or both phaſes will be viſible, if the Sun is under the horizon, and may therefore be obſerved, provided the Moon is not obſcured by clouds or fog.

EXAMPLE.

Was the lunar eclipſe of October 11, 1791, viſible at Cape Comorin, in latitude 7° 56′N. and longitude 78° 5′ E.

App. time beginning at Green.	12h.	0′	App. time of end	15h.	11″
Long. Cape Comorin in time	5	12 E.	— —	5	12
Reduced time, October 11,	17	12	— — —	20	23
or October 12,	5	12 A. M.	—	8	23 A. M.

Now, to latitude 7° 56′N. and declination of the Sun 7° 25′ S. the ſemidiurnal arc is about 5h. 56′. The Sun therefore riſes at 6h. 4′. Hence the beginning of the eclipſe was viſible; but its end was inviſible, as at that time the Sun was conſiderably above the horizon, and therefore the Moon nearly as much below.

PROBLEM II.

To obſerve a Lunar Eclipſe, and from thence to determine the Longitude of the place of obſervation.

RULE.

Apply a ſuitable magnifying power to the teleſcope, and adjuſt it to diſtinct viſion, by means of the lunar ſpots, or by any other convenient object. Put the watch in a proper place, or give it to an aſſiſtant to hold, who alſo may be provided with materials to mark down the times of obſervation. Direct the teleſcope to that part of the Moon which, it is expected, will firſt touch the ſhadow, ſome minutes before the time of the beginning of the eclipſe, if that phaſe be viſible; and as the extremity of the ſhadow is not well defined, a certain degree of ſhade muſt be fixed on; which may be determined ſoon after the Moon has entered the penumbra. The end of the eclipſe is alſo to be obſerved, the ſame degree of ſhade being uſed.

Reduce the times per watch of the beginning and end of the eclipſe to apparent time; the mean of theſe will be the apparent time of the middle of the eclipſe—the difference between which, and that given in the Nautical Almanac, will be the longitude in time of the place of obſervation, which is eaſt or weſt, according as the time at the place of obſervation is later or earlier than the Greenwich time.

If only the beginning or end of the eclipſe be obſerved, it muſt be compared with the correſpondent phaſe in the Nautical Almanac, to find the longitude. If the eclipſe is total, it will be proper to obſerve the immerſions of both limbs of the Moon, and alſo their emerſions. The mean of the firſt immerſion and laſt emerſion is to be taken, and alſo that of the laſt immerſion and firſt emerſion; and the mean of theſe will be the time of the middle of the eclipſe, being more to be depended on, than that deduced from a ſingle pair of obſervations. Now this mean being compared with the time of the middle of the eclipſe, as given in the Nautical Almanac, will give the longitude as before.

If the obſerver is provided with a ſextant, it will conduce to greater accuracy to meaſure the verſed ſines of the uneclipſed part of the Moon, as often as is poſſible, at the ingreſs; and to take correſpondent obſervations at the egreſs.

EXAMPLES.

October 11, 1791, the beginning of the lunar eclipſe was obſerved at 8h. 40½′ P. M. per watch, and the end was obſerved at 11h 52′ P. M. the error of the watch for apparent time was 13¼′ ſlow. Required the longitude of the place of obſervation?

Beginning of the eclipſe per watch	8h.	40½′
End — — —	11	52
		32½
Middle of eclipſe per watch —	10	16¼
Error of the watch —	+	13¼
Apparent time of middle of eclipſe	10	29½
— — per Naut. Almanac	13	35⅔
Longitude in time —	3	6⅙ = 46° 32′½ W.

April 18, 1791, being in lat. 11° 48′ N. and long. by account 80½ E. the undermentioned obſervations of the lunar eclipſe were taken; and towards the middle of the eclipſe, the following altitudes of Antares were obſerved, in order to aſcertain the error of the watch, the height of the eye being 14 feet. Required the longitude of the place of obſervation?

Time p. watch	9°	53′	18″	Alt. =	11°	43′
		54	20		11	56
		55	12		12	8
		56	14		12	21
		19	4			8
Mean —	9	54	46	Mean	12	2
Dip and refraction ——					—	8

Altitude corrected — —		11	54			
Polar diſtance — — —		115	57	—	Co-ſecant	0.04616
Latitude — — —		11	48	—	Secant —	0.00928
Sum — — —		139	39			
Half — — —		69	49½	—	Co-ſine -	9.53768
Difference — — —		57	55½	—	Sine —	9.92806
						19.52118
		35	11½	—	Sine —	9.76059
			8			
Antares' meridian diſtance —	4	41	29			
——— right aſcenſion —	16	16	38			
Right aſcenſion of meridian	11	35	9			
Sun's right aſcenſion —	1	45	23			
Approximate time — —	9	49	46			
Equation to approximate time —		1	31			
Equation to longitude E. — +			50			
Apparent time — —	9	49	5			
Time per watch — —	9	54	46			
Watch faſt — — —		5	41			

Ob-

Obſervations of the Eclipſe.

Beginning	8h.	44′	43″	V.S. unecl. part.	End	11h.	37′	25″
	8	58	30	28′		11	23	30
	9	9	34	22½		11	12	40
	9	21	50	18		11	0	20
	9	35	40	13½		10	46	16
	9	49	50	10		10	32	30
		100	7				32	41
	9	16	41⅙			11	5	26⅚
						9	16	41⅙
						20	22	8
Time per watch of middle of eclipſe —						10	11	4
Watch faſt — — —							5	41
Apparent time of middle — —						10	5	23
— — per Nautical Almanac —						4	42	20
Longitude in time — — —						5	23	3=80°45¾′E.

If the eclipſe be partial, obſervations of the verſed ſines of the enlightened part of the Moon, taken near the middle, will be of no ſervice in determining the inſtant of that phaſe, as the change is then too ſlow.— The following method of obſerving a lunar eclipſe is commonly practiſed in obſervatories.

Set the aſſiſtant clock agoing exactly with the tranſit clock; then, the teleſcope being prepared for obſervation, direct it to the Moon as before, and, as it difficult to aſcertain the preciſe inſtant of the beginning of the eclipſe, a certain degree of ſhade is to be fixed on, after the Moon has entered the penumbra. Obſerve alſo the immerſion of both edges of all the remarkable ſpots that ly in the path of the ſhadow. In like manner, the emerſions of the ſame ſpots, and the end of the eclipſe, are to be obſerved; the ſame degree of ſhade being ſtill uſed.

Reduce the times per clock of obſervation to apparent time; compare the inſtants of the beginning and end of the eclipſe by obſervation, with thoſe given in the Nautical Almanac, and their difference will be the longitude of the place of obſervation in time.

If the eclipſe has been obſerved at any other place, compare the inſtants of each correſponding pair of obſervations with each other; their difference will be that of the meridians in time. Take the mean of all the differences of longitude, deduced from the ingreſs, and alſo the mean of thoſe inferred from the egreſs; and the mean of both will be correct difference of longitude. Hence the longitude of either place with reſpect to ſome given meridian may be obtained, provided the longitude of the other place is known.

DE-

DETERMINATION

Of the Longitude of the Obſervatory at Aberdeen, by obſervations of the lunar eclipſe of 10th September 1783, made at Aberdeen, and at Chiſlehurſt in Kent, 19″ in time Eaſt of the Royal Obſervatory at Greenwich.

Clock faſt 10th Sept. at noon	10′ 9″.5	Equation of time at noon ——	3′ 11″.7
and gained daily ——	5 5	and daily increaſe —	20. 5
Hence clock faſt at ix hours =	10 11 3	Hence, eq. of time at ix hours =	3 19.4
x ——	10 11.5	x ——	3 20.2
xi ——	10 11.8	xi — —	3 21.1
xii —	10 12.0	xii ——	3 21.9
xiii —	10 12.2	xiii — —	3 22.8
xiv ——	10 12.5	xiv ——	3 23.7

Names of Spots.	Time p. Cl.	Err. Cl.	Eq.time	Appar. Time.	Ap.t. at Ch.	Dif Mer.
			I N G R E S S.			
		—	+			
Ariſtarchus —	9h. 49′ 34″	10′ 11″.5	3′ 20″.0	9h. 42′ 42″.5	9h. 50′ 55″	8′ 12″.5
Kepler — —	9 51 1	10 11. 5	3 20. 1	9 44 9. 6	9 52 20	8 10. 4
Copernicus —	10 1 0	10 11. 5	3 20. 2	9 54 8. 7	10 2 24	8 15. 3
Manilius covered	10 14 27	10 11. 6	3 20. 4	10 7 35. 8	10 15 30	7 54. 2
Tycho covered	10 15 49	10 11. 6	3 20. 4	10 8 57. 8	10 17 5	8 19. 2
Menelaus cov.	10 17 43	10 11. 6	3 20. 5	10 10 51. 9	10 19 10	8 18. 1
Dionyſius cov.	10 20 38	10 11. 6	3 20. 5	10 13 46. 9	10 21 38	7 51. 1
Plinius covered	10 21 47	10 11. 6	3 20. 5	10 14 55. 9	10 22 40	7 44. 1
Mare Criſ E.end	10 32 42	10 11. 6	3 20. 6	10 25 51. 0	10 34 34	8 43. 0
W.end	10 37 44	10 11. 7	3 20. 7	10 30 53. 0	10 39 45	8 52. 0
Total darkneſs	10 43 30	10 11. 7	3 20. 7	10 36 39. 0	10 46 34	9 55. 0
						254.9
				Mean ——	——	8 23.17
			E G R E S S.			
Ariſtarchus —	12h. 31′ 54″	10′ 12″.1	3′ 22″.4	12h. 24′ 4.″3	12h. 33′ 52″	9′ 47.″7
Kepler ——	12 33 55	10 12. 1	3 22. 4	12 27 5. 3	12 37 26	10 20. 7
Copernicus —	12 42 24	10 12. 1	3 22. 5	12 35 34. 4	12 45 52	10 18. 6
Plato, E. end -	12 45 22	10 12. 2	3 22. 6	12 38 32. 4	12 47 22	8 49. 6
Tycho, E. end	12 46 54	10 12. 2	3 22. 6	12 40 4. 4	12 48 30	8 25. 6
W. end	12 47 55	10 12. 2	3 22. 6	12 41 5. 4	12 49 58	8 53. 6
Menelaus —	12 59 50	10 12. 2	3 22. 8	12 53 0. 6	13 1 40	8 39. 4
Dionyſius —	13 1 35	10 12. 2	3 22. 8	12 54 45. 6	13 3 18	8 32. 4
Plinius ——	13 3 38	10 12. 2	3 22. 8	12 56 48. 6	13 5 40	8 51. 4
MareCriſ. E. end	13 14 0	10 12. 3	3 23. 0	13 7 10. 7	13 16 35	9 24. 3
W end	13 19 10	10 12. 3	3 23. 1	13 12 20. 8	13 20 53	8 32. 2
						95.5

Mean per Egreſs —— —— —— ——	9 8.68
per Ingreſs —— —— —— ——	8 23.17
	17 31.85
Difference meridians of Aberdeen and Chiſlehurſt in time nearly	8 45.92
Longitude of Chiſlehurſt in time E. — —— ——	— 19
Longitude of Aberdeen nearly —— — ——	8 26.92
Change of equation of time in 8′ 27″ ——	— .12
Longitude Obſervatory at Aberdeen in time —— ——	8 26.8 W

C H A P. III.

The Method of finding the Longitude of a Place
By
An Eclipſe of the Sun.

Introduction.

THIS and the following methods are the moſt accurate of any that have hitherto been employed for the purpoſe of finding the longitude of a place. The difference of the meridians of two places may be found to the neareſt ſecond of time, by comparing correſpondent obſervations of the ſame eclipſe. Theſe obſervations are therefore eagerly deſired by the practical aſtronomer. The calculation is indeed longer than that in any of the former methods. This ariſes from parallax—the effect of which may be found by various methods; however, that by means of the *Nonageſimal* appears to be the eaſieſt.

As the ecliptic is a great circle, one half of it is always above the horizon, and the other half underneath it; the middle of the elevated part is therefore 90° diſtant from its interſection with the horizon, and is hence called the Nonageſimal. The elevation of the nonageſimal above the horizon is called its Altitude, and its diſtance from the equinoctial point aries is called its Longitude.

Problem I.

Given the Apparent Time at a Place, its Latitude, and the Obliquity of the Ecliptic, to find the Altitude and Longitude of the Nonageſimal.

Rule.

From the given latitude, ſubtract the number anſwering thereto in table xxxvi. the remainder will be the reduced latitude.

To the apparent time, add the Sun's right aſcenſion reduced to the given time and place; the ſum will be the right aſcenſion of the meridian.

Let the difference between the right aſcenſion of the meridian, and 0.xii or xxiv. hours, according to which it is neareſt, be reduced to degrees, and called arc firſt.

Then to the log. co-tangent of the reduced latitude, add the log. ſine of arc firſt; the ſum will be the log. tangent of arc ſecond. Now, if arc firſt is between 0 and 180°, the ſum of arc ſecond and the obliquity of the ecliptic is arc third, otherwiſe their difference.

To the log. tangent of arc firſt, add the log. co-ſecant of arc ſecond, and the log. ſine of arc third; the ſum is the log. tangent of arc fourth; which, when the right aſcenſion of the meridian is between 0 and VI. hours, is the longitude of the nonageſimal, or its ſupplement if between VI. and XII. hours. But arc fourth being added to 180°, if the right aſcenſion of the meridian is between XII. and XVIII. hours, or ſubtracted from 360°, if between XVIII. and XXIV. hours, will give the longitude of the nonageſimal.

To the log. tangent of arc third, add the log. co-ſecant of arc fourth, the ſum will be the log. tangent of the altitude of the nonageſimal.

EXAMPLE.

Required the altitude and longitude of the nonageſimal, in latitude 57° 8′.9 N. and longitude in time 8′ 40″ W. Nov. 26, 1787, at 11h. 18′ 8″ apparent time*?

Apparent time — 11h. 18′ 8″
☉'s R. A. at red. time 16 10 57

Latitude = 57° 8′.9
Reduction — 13.8 R. A. of meridian — 3 29 5 = 52° 16′¼

Red. lat. - 56 55.1 Co-tang. 9.81387
Arc firſt - 52 16.¼ Sine — 9.89813 tang. 0.11143

Arc ſecond 27 15.½ Tangent 9.71200 co-ſec. 0.33913
Obliq. ecl. 23 28.0

Arc third - 50 43.½ — Sine — — 9.88881 tang. 0.08737

Long. non. 65 24.¼ — Tangent — 0.33937 coſec. 0.04131

Altitude nonageſimal 53° 22′ tang. 0.12868

The altitude and longitude of the nonageſimal may alſo be readily found by the following rule.

Find the reduced latitude of the place, and the right aſcenſion of the meridian as before. Then, if the right aſcenſion of the meridian is leſs than VI. hours, increaſe it thereby, and the ſum will be arc firſt; but if it exceeds VI. the difference between it and XVIII. hours is arc firſt.

Take the log. correſponding to arc firſt from column of riſing, to which add the log. co-ſine of the reduced latitude, and the log. ſine of the obliquity of the ecliptic; the ſum will be the log. of a natural number, which being ſubtracted from the natural ſine of the ſum of the reduced latitude and obliquity of the ecliptic, will give the nat. co-ſine of the altitude of the nonageſimal.

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* It is ſufficient in moſt caſes to find the altitude and longitude of the nonageſimal to the neareſt minute.

Take the log. correſponding to arc firſt, or its ſupplement, if it exceeds VI. hours from column of half elapſed time; to which add the log. ſecant of the reduced latitude, and the log. ſine of the altitude of the nonageſimal; the ſum will be the log. ſecant of the longitude of the nonageſimal, if the right aſcenſion of the meridian is leſs than VI. hours. If between VI. and XII. hours, its ſupplement is the longitude of the nonageſimal; but being added to 180°, if the right aſcenſion of the meridian is between XII. and XVIII. hours, or ſubtracted from 360°, if above XVIII. hours, will give the longitude of the nonageſimal.

Remark. If arc firſt exceeds the limits of the tables, take the log. riſing anſwering to its ſupplement, to which add the log. co-ſine of the latitude, and the log. ſine of the obliquity of the ecliptic; the natural number, anſwering to the ſum of theſe three logs. being added to the natural ſine of difference of the latitude and obliquity of the ecliptic, will give the natural co-ſine of the altitude of the nonageſimal.†

Example.

Required the altitude and longitude of the nonageſimal at Greenwich, June 3, 1788, at 19h. 24′ 56.″5 apparent time?

Apparent time =	19h.	24′	56″.5	Lat. of Greenwich	51°	28′	40″ N.
Sun's right aſc.	4	51	30.5	Reduction —		14	33
R. A. meridian -	0	16	27.0	Reduced latitude -	51	14	7
	6						

Arch firſt —	6	16	27	— —	Riſing —	5.03007
Latitude —	51	14	—	— —	Co-ſine —	9.79668
Obliq. ecliptic	23	28	—	— —	Sine — —	9.60012
Sum — —	74	42	—	N. S. = 96456		
				26722	— —	4.42687
Alt. nonageſimal	45	47	—	N. co-ſine 69734 -	Sine —	9.85534
Suppl. arc firſt -	5	43	33	— —	H. E. T. —	0.00112
Reduced latitude	51	14	—	— —	Secant —	0.20332
Long. nonageſ.	29	23	—	— —	Secant —	0.05978

The

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† The table of natural verſed ſines may be uſed in the computation inſtead of that of natural ſines.

The effect of parallax depends on the altitude of the nonageſimal, and on the diſtance of the object therefrom. When the object is in the nonageſimal, its latitude only is affected; but in other poſitions, both the latitude and longitude of the object are altered by parallax. Theſe effects may be computed as follows.

PROBLEM II.

Given the Altitude and Longitude of the Nonageſimal, together with the Moon's Latitude, Longitude, and Horizontal Parallax, to find the Parallaxes in Latitude and Longitude.

RULE.

From the horizontal parallax of the Moon, as given in the Nautical Almanac, ſubtract the number from table XXXII. and add that from table XXXIII. anſwering to the latitude of the place; call the aggregate the *Reduced Parallax.*

To the log. co-ſecant of the altitude of the nonageſimal, add the log. co-ſecant of the Moon's diſtance from the nonageſimal, the log. co-ſine of the Moon's latitude, and the prop. log. of the reduced horizontal parallax; the ſum will be the approximate parallax in longitude, which being added to the difference between the longitudes of the Moon and nonageſimal, the ſum will be the apparent difference of longitude nearly—with which, the altitude of the nonageſimal, the Moon's latitude and reduced parallax, repeat the proceſs, and the reſult will be part firſt of the parallax in longitude, which is additive to the Moon's longitude, when that object is eaſt of the nonageſimal, otherwiſe ſubtractive.

Take the greateſt effect of part ſecond of the parallax in longitude from table XXXV. anſwering to the reduced parallax and latitude—to the log. of which, add the log. co-ſine of the Moon's longitude; the ſum will be the log. of part ſecond of the parallax in longitude, which is additive or ſubtractive, according as the Moon is in the aſcending or deſcending ſigns, the latitude being north; the contrary rule is to be applied, when the latitude is ſouth. Hence the Moon's apparent longitude will be obtained.

To the P. Log. of the Moon's reduced parallax, add the log. ſecant of the altitude of the nonageſimal, and the log. ſecant of the Moon's latitude; the ſum will be the approximate parallax in latitude

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The altitude and longitude of the nonageſimal may be found independent of each other, by the following formulæ.

$$\left.\begin{array}{l}\text{Co-ſine alt. nonageſimal} = \text{ſine ob. ecl. coſ. lat. ſine R.A.M. fm. near.} \\ \quad \text{eq.} + \text{coſ. ob. ecl. ſine lat.} \\ \text{Tang. long. non. fm. neareſt eq.} = \text{ſine ob. ecl, ſecant R. A. M. tang.} \\ \quad \text{lat.} - \text{coſ. ob. ecl. tan. R. A. M.}\end{array}\right\}\text{to radius 1.}$$

tude, which being added to the Moon's diſtance from the elevated pole of the ecliptic, will give its apparent polar diſtance nearly; and hence the Moon's approximate apparent latitude will be known; to the log. ſecant of which, add the log. ſecant of the altitude of the nonageſimal, and the P. L. of the Moon's reduced parallax; the ſum will be the P. L. of arc firſt.

Then, to the log. co-ſecant of the Moon's approximate latitude, add the log. co-ſecant of the altitude of the nonageſimal, the P. L. of the reduced parallax, and the log. ſecant of the Moon's apparent diſtance from the nonageſimal; the ſum will be the P. Log. of arc ſecond. If the Moon is between the ecliptic and its elevated pole, the difference between arcs firſt and ſecond will be the principal effect of parallax in latitude, otherwiſe their ſum, the Moon being above the horizon; and which is ſubtractive or additive, according as the Moon and the elevated pole of the ecliptic are on the ſame, or on oppoſite ſides of the ecliptic. Part ſecond of the parallax in latitude is found in table XXXIV. the arguments of which are, the latitude of the place of obſervation, and the Moon's horizontal parallax. It is additive or ſubtractive, according as the latitudes of the Moon and given place are of the ſame, or of a contrary name.

EXAMPLE.

Let the altitude of the nonageſimal be 45° 47′, its longitude 29° 23′, the Moon's longitude 2^s 13° 19′ 41″, latitude 0° 20′ 9″ N. equatorial horizontal parallax 60′ 33″. and the latitude of the place 51° 14′ N. Required the parallaxes in longitude and latitude?

Equat. hor. par. ☽ 60′ 33″
Reduction — − 9
Augmentation - + 19

Moon's longitude — 2^s 13° 20′
Long. of nonageſimal - - 29 23
Difference of longitude 1 13 57

Reduced parallax 60 43 - P. Log. 0.4719
Alt. nonageſ. 45° 47 — Co-ſec. 0.1447
Moon's latitude - 20 — Co-ſine 0.0000

0.6166 — — 0.6166
Diff. of long. 43 57 — Co-ſec. 0.1586

Appr. par. lon + 30 — P. Log. 0.7752

Appr. app. D. long. 44 27 — — — — Co-ſine — 0.1547

Part firſt of parallax in long. + 30′ 28″ — P. L. — 0.7713
Moon's true longitude — 2 13 19 41 p. t. XXXV. 9″.8 log. 0.9912

Moon's long. corrected - 2 13 59 9 — Co-ſine — 9.4447

Moon's long. corrected -	2 13 50 9		—	Co-ſine —	9.4447
Part ſecond of par. in long. +	3		—	Log. —	0.4359
Moon's apparent longitude	2 13 50 12				
Reduced parallax	60′ 43″	P. L. 0.4719			
Alt. nonageſ.	45 47 —	Secant 0.1565			
☽'s latitude -	0 20 9	Secant 0.0900			
Approx. parallax	42 21	P. L. 0.6284			
Appr. app. lat.	0 22 12	Secant 0.0000	—	Co-ſecant -	2.1899
Alt. nonageſ.	45 47 —	Secant 0.1565	—	Co-ſecant -	0.1447
☽'s reduced par.	60 43	P. L. - 0.4719	—	— —	0.4719
	42 21	P. L. - 0.6284	App. ☽. long. 44° 27′ -	Sec.	0.1464
	12	—	—	— P. L. -	2.9529
Part 1ſt par. in lat.	42 9				
Part 2d par. in lat. +	23				
Moon's true lat. -	20 9 N.				
Moon's app. lat.	21 37 S.				

PROBLEM III.

To find the Apparent Time at Greenwich of the ecliptic Conjunction of the Sun and Moon.

RULE.

Find the difference between the longitudes of the Sun and Moon, at the noon or midnight immediately preceding the conjunction, and alſo at that which follows it. To the log. of the firſt of theſe differences, expreſſed in ſeconds, add the ar-co-log. of the ſum of the two differences in ſeconds, and the conſtant log. 2.8573325, the ſum will be the log. of the approximate time in minutes, reckoned from the preceding noon or midnight.

Take the equation from table XXXVIII. anſwering to the ſum of the two differences, and to the mean ſecond difference, which being applied to the approximate time, by addition or ſubtraction, according as the Moon's relative motion in longitude is increaſing or decreaſing; hence the apparent time of conjunction will be obtained.

Ex-

Example.

Required the apparent time of the conjunction of the Sun and Moon, June 3, 1788?

	Moon's long.	Sun's long.	Difference.	Firſt diff.	Second diff.	Mean
June 3, noon	2s 1° 29′ 3″	— 2 13 26 43	— 11 57 40			
midn.	2 8 47 5	— 2 13 55 26	— 5 8 21	+ 6 49 19		
4, noon	2 16 8 50	— 2 14 24 8	+ 1 44 42	+ 6 53 3	+ 3 44	
midn.	2 23 33 29	— 2 14 52 50	+ 8 40 39	+ 6 55 57	+ 2 54	+ 3 19

Preceding difference	= 5° 8′ 21″ = 18501″	— log.	4.2671952
Sum preced. & foll. diff.	6 53 3 = 24783	— ar-co-log.	5.6058461
Conſtant log.	— — — —	—	2.8573325
Approximate time	— 8 57 29.6 = 537″.494	log. —	2.7303738
Equation, tab. XXXVIII	+ 32.8		
Ap. t. conj. paſt midn.	8 58 2.4 or June 3, at 20h. 58′ 2″.4		

Problem IV.

Given the Apparent Times of the Beginning and End of a Solar Eclipſe to find the Longitude of the Place of Obſervation.

Rule.

Reduce the apparent times of the beginning and end of the eclipſe, to the meridian of Greenwich, by applying thereto the eſtimate longitude, by prob. III. page 73; and to theſe times, let the Moon's horizontal parallax and ſemidiameter, the Sun's longitude and right aſcenſion, be taken from the Nautical Almanac. Increaſe the Moon's ſemidiameter by the augmentation anſwering to its altitude, from which ſubtract the quantity of inflexion, which is about 3 ſeconds. Hence the reduced parallax and apparent ſemidiameter of the Moon, and the right aſcenſion of the meridian, will be known. Diminiſh the latitude of the place by the quantity from table XXXVI. and from the Sun's ſemidiameter, ſubtract the quantity of irradiation, which is about 4 ſeconds.

Compute the altitude and longitude of the nonageſimal, and thence the parallaxes in latitude and longitude at the inſtants of obſervation, uſing the difference between the Moon's reduced parallax, and the Sun's horizontal parallax, and the difference between the longitudes of the Sun and nonageſimal.

From the Moon's relative motion in longitude during the obſerved interval, ſubtract the difference between the parallaxes in longitude, if the Sun is on the ſame ſide of the nonageſimal at the beginning, and at the end of the eclipſe; but if on oppoſite ſides, ſubtract the ſum of the parallaxes, and the remainder will be the apparent difference of longitude.

If the Moon is in the aſcending node, find the difference between the parallax in latitude at the end of the eclipſe, and the ſum of the parallax in latitude at the beginning, and the change of the Moon's latitude in the obſerved interval; but if the Moon is towards the deſcending node, find the difference between the parallax in latitude at the beginning, and the ſum of the change of the Moon's latitude, and the parallax in latitude at the end. Hence the apparent difference of latitude will be known.

From the P. Log. of the apparent difference of longitude, its index being increaſed by 10, ſubtract the P. Log. of the apparent difference of latitude, the remainder will be the log. tangent of the apparent inclination—to the log. coſine of which, add the P. Log. of the apparent difference of longitude, and the ſum will be the apparent motion of the Moon in its relative orbit.

Find the ſum of the apparent ſemidiameters of the Sun and Moon at the beginning and end of the eclipſe, and from the P. Log. of their ſum, its index being increaſed by 10, ſubtract the P. Log. of the Moon's relative motion in its proper orbit; the remainder will be the log. co-ſine of the central angle.

Now when the Moon is in the aſcending node, the difference between the central angle, and the apparent inclination, will be arc firſt, and their ſum arc ſecond. But if the Moon is in the deſcending node, the ſum of the above angles is arc firſt, and their difference arc ſecond. In the firſt caſe, the ſum of the change of the Moon's latitude and parallax, at the beginning of the eclipſe, is ſuppoſed to be greater than the parallax in latitude at the end; and in the ſecond caſe, the ſum of the change of latitude and parallax at the end is ſuppoſed greater than the parallax in latitude at the beginning; if otherwiſe, the contrary rules muſt be applied.

To the log. ſecant of arc firſt, add the P. Log. of the ſum of the apparent ſemidiameters of the Sun and Moon, at the beginning of the eclipſe; the ſum will be the P. Log. of arc third; and to the log. ſecant of arc ſecond, add the P. Log. of the ſum of the apparent ſemidiameters of the Sun and Moon at the end; the ſum is the P. Log. of arc fourth.

Now, if the Moon is eaſt of the nonageſimal, to arc third add the parallax in longitude at the beginning of the eclipſe; and from arc fourth ſubtract the parallax in longitude at the end. If the Moon is weſt of the nonageſimal, apply the contrary rules. Then to the P. Logs. of each of theſe quantities, add the ar-co. of the P. Log. of the Moon's relative motion in longitude, and the P. Log. of the obſerved interval; the ſums will be the P. Logs. of the intervals in time, between the ecliptic conjunction, and the beginning and end of the eclipſe reſpectively. Hence the apparent time of the ecliptic conjunction at the place of obſervation will be known.

Compute the apparent time of the ecliptic conjunction of the Sun and Moon at Greenwich by laſt problem; the difference between which, and the apparent time of conjunction at the place of obſervation, will be its longitude in time as uſual.

If the apparent time of conjunction at Greenwich, found directly from obſervation, be uſed inſtead of that given by laſt problem, the longitude of the place will be accurately determined.

Example.

June 3, 1788, in latitude 57° 9′ N. and eſtimate longitude 8′ 36″ in time, the beginning of the ſolar eclipſe was obſerved at 19 h. 33′ 19″ apparent time, and the end at 20 h. 49′ 29″. Required the true longitude of the place of obſervation?

App. time beg. eclipſe	19h. 33′ 19″	App. time of end	20h. 49′ 29″
Eſtimate longitude -	+ 8 36	— — —	8 36
Reduced time —	19 41 55	— — —	20 58 5
Moon's horizontal parallax	60 33	— — — —	60 34
Reduction, tab. XXXII.	− 11	— — — —	− 11
Augmentation, t. XXXIII.	+ 22	— — — —	+ 22
Reduced parallax —	60 44	— — — —	60 45
Sun's horizontal parallax	− 9	— — — —	− 9
Difference — —	60 35	— — — —	60 36
Moon's ſemidiameter —	16 30	— — — —	16 30
Augmentation, tab. XXX.	+ 9	— — — —	+ 12
Inflexion —	− 3	— — — —	− 3
Apparent ſemidiameter —	16 36	— — — —	16 39
Sun's longitude —	2^{s} 14° 14′	— — —	2^{s} 14° 17
Semidiameter — —	− 16	— — —	+ 16
Long. ⊙'s w. l. at begin.	2 13 58	Long. ⊙'s eaſt l. at end	2 14 33
Sun's right aſcenſion -	4h. 51′ 34″	— — —	4h. 51′ 47″
Apparent time —	19 33 19	— — —	20 49 29
R. A. merid. at begin.	0 24 53	R. A. meridian at end	1 41 16

Lati-

Reduced lat. = 56° 55′ co-tang. 9.81390
Arc firſt — 6 13¼ ſine — 9.03487 tang. 9.03743

Arc ſecond — 4 2.3 tangent 8.84877 coſec. 1.15229
Obliq. ecliptic 23 28

Arc third — 27 30.3 — ſine — 9.66448 tang. 9.71657

Long. nonageſ. 35 33.5 — tangent - 9.85420 coſe. 0.23543

Long. ☉'s w. l. 73 58 — Alt. nonageſimal 41° 50′⅔ tang. 9.95200

App. ☽. long. 38 24.5 co-ſec. 0.2067
Alt. nonageſ. 41 50.4 co-ſec. 0.1758 ſecant — — 0.1278
Diff. parallax - - 60′ 35″ P.L. 0.4729 — — — — 0.4729

Part 1ſt par. in lon. 25 7 P.L. 0.8554 P. 1ſt. par. lat. 45′ 8″ } = P. L. 0.6007
Part 2d par. long. — 3 — P. 2d. par. — 24 }

Parallax in long. - 25 4* Parallax in lat. 44 44

Reduced lat. 56° 55′ cotang. 9.81390
Arc firſt - 25 19 ſine - 9.63106 tang. 9.67491

Arc ſecond 15 34 tangent 9.44496 coſec. 0.57128
Obliq. eclipt. 23 28

Arc third - 39 2 — — ſine — 9.79918 tang. 9.90880

Long. nonag. 47 59 — — tangent - 0.04537 coſec. 0.12904

Lon. ☉'s e. l. 74 33 — altitude nonageſ. 47° 30′ tang. 0.03793

Diff. of long. 26 34 co-ſec. 0.3495
Alt. nonageſ. 47 30 co-ſec. 0.1324 — — ſecant — 0.1703
Diff. parallax 60 36 P.L. 0.4728 — — — — 0.4728

P. 1ſt. par. long. 19 59 P.L. 0.9547 P. 1ſt par. lat. 40′ 56″ P.L. 0.6431
Part 2d. — — 3 — Part 2d. - - — 24

Parallax in long. 19 56 Parallax in latitude — 40 32

Ap-

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* In computing the parallaxes in latitude and longitude, the Moon's latitude may be ſuppoſed = O, by which means the computation will be rendered more ſimple, and the accuracy at the ſame time will not be diminiſhed.

Par. in lat. 40′ 32″
Change lat. 4 18
Par. lon. at beg. 25′ 4″
at end 19 56 Sum — 44 50
Difference — 5 8 Par. lat. at beg. 44 44
Rel. mot. long. 43 46 Difference 6 P. L. 3.2553

Ap. mot. in lon. 38 38 — P. L. + 10 = 10.6683 — 0.6683

App. inclinat. 0 9 — — Tangent — 7.4130 co-ſine 0.0000

Apparent motion in proper orbit — — 38′ 38″ P. L. 0.6683

☉'s ſem. at beg. 15′ 44″* at end 15′ 44″
☽'s ſemidiam. 16 36 — — 16 39

Sum — 32 20 + 32 23 = 64′ 43″ — P. L. 10.4442
Moon's apparent motion in proper orbit 38 38 — P. L. 0.6683

Central angle — — — 53 21 — co-ſine 9.7759
Apparent inclination — — 0 9

Arc firſt — — — 53 30
Arc ſecond — — — 53 12

Arc firſt 55° 30′ Secant - 0.2256 Arc 2d 53° 12′ ſec. 0.2226
Sum ſem. beg. 32 20 P. L. - 0.7456 Sum ſem. at end 32 23 P. L. 0.7449

19 14 P. L. - 0.9712 19 24 P. L. 0.9675
Par. in long. at beg. 25 4 Par. lon. at end - 19 56

Sum — 44 18 P. L. - 0.6080 Difference 32 P. L. 2.5283
☽'s relative mot. long. 43 46 ar-co-P. L. 9.3859 — — — 9.3859
Obſ. inter. 1 16 10 P. L. - 0.3735 — — — 0.3735
Int. bet. beg. and conjun. 1 17 5 P. L. - 0.3683 Int. betw. end and conjun. 56″ P. L. 2.2877
Beginn. 19 33 19 End 20 49 29
App. t. of con. 20 50 24 Ap. t. conj. 20 50 25

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* The Sun's ſemidiameter is diminiſhed by the quantity of irradiation.

Ap. time conj.	20	50	24
T. con. at Gr. by laſt prob.	20	58	2
Long. in time -		7	38 W.

But the time of conjunction at Greenwich,† inferred from obſervation, was 20h. 58′ 47″, the difference between which and 20h. 50′ 24″ is 8′ 23″, the longitude in time.

PROBLEM V.

Given the Apparent Time of the Beginning, or of the End of a Solar Eclipſe, to find the Longitude of the Place of Obſervation.

RULE.

Reduce the apparent time of obſervation to the meridian of Greenwich; to which time, find the Moon's ſemidiameter, horizontal parallax and latitude; and the Sun's ſemidiameter, longitude, and right aſcenſion; find alſo the difference between the Moon's reduced parallax, and the Sun's horizontal parallax.

Now, compute the altitude and longitude of the nonageſimal, and the parallaxes in latitude and longitude as uſual. Hence the Moon's apparent latitude will be known; to the P. Log. of the ſum of which and the ſum of the ſemidiameters of the Sun and Moon, add the P. Log. of their difference; half the ſum will be the P. log. of arc firſt.

If the Moon is eaſt of the nonageſimal, and the obſervation that of the beginning of the eclipſe, the ſum of the above arc and the parallax in longitude is to be taken; but if the end of the eclipſe be obſerved, find their difference. If the Moon is weſt of the nonageſimal, uſe the contrary rules. Then, to the P. Log. of this ſum or difference add the ar-co. of the P. Log. of the Moon's relative motion in longitude, and the conſtant log. 0.4771; the ſum will be the P. Log. of an arc.

Now, if the Moon is eaſt of the nonageſimal, the above arc muſt be added to the time of the beginning of the eclipſe, or ſubtracted from that of the end, if the parallax in longitude is leſs than arc firſt; otherwiſe added. If the Moon is weſt of the nonageſimal, the above arc muſt be applied to the apparent time of the beginning, by addition or ſubtraction, according as it is greater or leſs than the parallax in longitude; but if the obſervation is that of the end of the eclipſe, it muſt be ſubtracted therefrom, and the ſum or difference will be the apparent time of conjunction at the place of obſervation; the difference between which and that by the meridian of Greenwich,

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† Phil. Tran. vol. LXXIX. part firſt.

wich, inferred from the Nautical Almanac, by prob. III. or from obſervation as above, will be the longitude of the place in time.

EXAMPLE.

At the obſervatory at Aberdeen, the author obſerved the end of the ſolar eclipſe of 15 June 1787 at 5h. 32′ 2″ apparent time, and at the Royal Obſervatory at Greenwich, Dr Maſkelyne obſerved the beginning at 4h. 10′ 58″ apparent time.† (The beginning of the eclipſe could not be obſerved at Aberdeen, nor the end at Greenwich, by reaſon of clouds.) Required the longitude of Aberdeen?

Ap. time of end at Abd.	5h. 32′ 2″	☽'s ſemidiam.	16′ 45″	⊙'s ſemidiam.	15′ 47″
		Augmentation +	7	Irradiation - —	4
Eſt. long. in t. +	8 36	Inflexion - - —	3		
				Corr. ſemid. -	15 43
Reduced time	5 40 38	Corr. ſemid.	16 49	— —	16 49
				Sum —	32 32
☽'s eq. hor. par.	61 26	Moon's lat. June 15. at noon	1° 13′ 6″ N.		
Reduction - —	11	Prop. part to 5h. 40′ 38″ - —	19 29		
Augmentation +	22	Equation of ſecond differ. - +	7		
Reduced parallax	61 37	Moon's latitude at red. time	0 53 44 N.		
Sun's hor. par. —	9				
Difference —	61 28				
Sun's right aſcen.	5h. 35′ 39″				
Apparent time -	5 32 2				
R. A. meridian	11 7 41 18				
Arc firſt —	6 52 19	— — Riſing — —			5.08859
Latitude —	56 55	— — — Co-ſine — —			9.73708
Obliq. ecliptic	23 28	— — — Sine — —			9.60012
Sum — —	80 23	— N. Sine -	98595		
			26656	— —	4.42579
Alt. nonageſimal	44 0	— N. Coſine	71939		

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† In Dr. Maſkelyne's letter to the author, dated Royal Obſervatory, Greenwich, December 29th, 1787, he ſays, "I obſerved the beginning of the eclipſe of the ſun, " June 15 laſt, at 4h. 11′ 3″ apparent time, or rather a ſmall impreſſion of ☽ on ⊙; " the true beginning may be reckoned 5″ ſooner, or at 4h. 10′ 58″. The end was " hid by clouds."

Alt. nonageſimal 44° 0′ — — — Sine — 9.84177
Supplement arc firſt 5h. 7′ 41 — — H. E. T. — 0.01141
Latitude — — 56 55 — — — Secant — — 0.26292

40 3 — — — Secant — 0.11610

Long. nonag. 139 57
Long. ⊙'s E. l. 84 25

Difference — 55 32 — Co-ſec. 0.0838
Alt. nonageſ. 44 0 — Co-ſec. 0.1582 — Secant — 0.1431
Difference of par. 61 28 P. L. 0.4666 — — — 0.4666

P. 1ſt par. long. — 35 13 P. L. 0.7086 P. 1ſt P. lat. 44′ 13″ } P. L. 0.6097
Part 2d — + 1 Part 2d — 24

Parallax in long. 35 12 Parallax in lat. - 43 49
Moon's latitude — — — 53 33

Apparent latitude — — — 9 44
Sum of ſemidiameters — — 32 32

Sum — — — — — 42 16 P. L. — 6293
Difference — — — — 22 48 P. L. — 8973

1.5266
Arc firſt — — — — 31 3 P. L. — 7633
Parallax in longitude — — 35 12

Sum — — — — 66 15 P. L. — 0.4341
☽'s relative horary motion — — 35 34 ar-co-P.L. 9.2958
Conſtant log. — — — — — 0.4771

Difference between end and conjunction 1 51 46 P. L. — 0.2070
Apparent time of end — — 5 32 2

Apparent time of conjunction — 3 40 16

Computation of the Apparent Time of Conjunction at Greenwich.

Moon's ſemid. 16′ 45″ Sun's ſemid. 15′ 47″ Moon's hor. par. 61′ 27″
Augmentation + 10 Irradiation - — 4 Reduction - — 10
Inflexion - - — 3 Augmentation - + 19
Red. ſemid. 15 43
Red. ſemidiam. 16 52 — — 16 52 Reduced parall. 61 36

Sum 32 35

Apparent time - 4^{h} 10′ 58″
Sun's R. A. — 5 35 24

R. A. meridian - 9 46 22
18

Reduced parall. 61′ 36″
Sun's hor. par. 9

Difference — 61 27

Arc firſt — 8 13 38 — — Riſing — — 5.19049
Reduced lat. 51 14 — — — Co-ſine — — 9.79668
Obliq. ecliptic 23 28 — — — Sine - — — 9.60012

Sum — — 74 42 — N. Sine - 96456
38663 — — 4.58729

Alt. nonageſ. - 54 42 — N. Co-ſine 57793 Sine — 9.91176
Suppl. arc firſt 3 46 22 — — H. E. T. — — 0.07844
Latitude — 51 14 — — — Secant — — 0.20332

50 11 — — — Secant — — 0.19352

Long. nonag. 129 49
Sun's long. - 84 21

Difference — 45 28 — Co-ſec. 0.1470
Alt. nonageſ. 54 42 — Co-ſec. 0.0882 — — Sec. 0.2382
Diff. of parall. - 61 27 P. L. - 0.4668 — — P. L. 0.4668

P. 1ſt par. in long. 35 45 P. L. - 0.7020 P. 1ſt P. lat. 35 30 } P. L. 0.7050
Part 2d — — 1 Part 2d — 23

Parallax in long. 35 44 Parallax in lat. 35 7
Moon's true lat. - 58 43

Moon's app. lat. - 23 36
Sum of ſemidiam. 32 35

Sum — — 56 11 - P. L. 0.5056
Difference — 8 59 - P. L. 1.3018

1.8074
Arc — — 22 28 - P. L. 0.9037
Parallax in long. 35 44

Difference — 13 16

Diff.

Difference	—	13′	16″	P. L.	1.1325
☽'s rel. hor. mot.		35	34	ar-co-P.L.	9.2958
Conſtant log.		—		—	0.4771
Diff. begin. & conj.		22	22	- P. L.	0.9054
Beg. at Greenw.	4	10	58		
App. time conj.	3	48	36		
App. time Abd.	3	40	16		
Long. in time		8	20 W.		

The apparent time of conjunction at Greenwich, computed by prob. III. is 3h. 49′ 28″, and hence the longitude in time is 9′ 12″. Now the mean of this and 7′ 38″, the longitude by laſt example, deduced from a like compariſon with the Nautical Almanac, is 8′ 25″.

Since the obſervations of the above eclipſe were, the beginning at one place, and the end at the other, the longitude deduced is therefore affected by the ſum of the errors, ariſing from the error of the Moon's latitude. If both obſervations had been the beginning, or the end, the reſulting longitude would be affected by the difference of theſe errors; and conſequently would be nearer the truth.

An obſervêr at land, provided with proper inſtruments, beſides obſerving the beginning and end of the eclipſe, takes as many meaſures as poſſible of the verſed ſines of the lucid part of the Sun's diſc, or of the diſtance between the cuſps. Hence by the method of interpolation, he can find the time of the middle of the eclipſe, and the neareſt approach of the centers of the Sun and Moon; from whence, the error of the Moon's latitude, and the longitude of the place of obſervation, are determined.

C H A P. IV.

The Method of finding the Longitude of a Place,

By

An Occultation of a Fixed Star by the Moon.

INTRODUCTION.

THE obſervations neceſſary for finding the longitude of a place by this method are, the inſtant when the Moon's eaſtern limb covers a ſtar, called the *Immerſion*, and that of the re-appearance of the ſtar from behind the Moon's weſt limb, called the *Emerſion*.

The immerſion of a ſtar is eaſily obſerved, as the ſight may be directed to the ſtar, till that obſervation happens. In an emerſion, the obſerver ſhould direct his ſight to that part of the Moon's limb, from which the ſtar is expected to emerge, ſome minutes before the ſuppoſed time of emerſion, and continue looking till the ſtar appears. This time may be found nearly, by adding to the obſerved time of immerſion, the time required by the Moon to paſs over a ſpace, equal to the eſtimated chord of the ſegment cut off by the ſtar.

In the calculations, the altitude and longitude of the nonageſimal, and the parallaxes in latitude and longitude, are neceſſary, as in a ſolar eclipſe. Theſe are therefore calculated by prob. I. and II. of laſt chapter.

PROBLEM I.

To find the Apparent Time at Greenwich of the ecliptic Conjunction of the Moon and a Fixed Star.

RULE.

Reduce the mean longitude of the ſtar, to its apparent longitude, by the rule given for that purpoſe in page 15. vol. II.

From the Nautical Almanac, page v. of the month, take the longitudes of the Moon, immediately preceding and following that of the ſtar; find alſo the mean ſecond difference of the Moon's place.

Now to the ar-co. of the log. of the change of the Moon's longitude in XII. hours, add the log. of the difference between the ſtar's longitude and the preceding longitude of the Moon, and the conſtant log. 2.8573325; the ſum will be the log. of the approximate time in minutes.*

Take the equation of ſecond difference from table XXXVII. anſwering to the approximate time, and the mean ſecond difference—with which enter table XXXVIII. at the top, and the change of the Moon's longitude in twelve hours in the ſide column, and take out the correſponding equation; which being added to the approximate time, if the Moon's motion in XII. hours is increaſing, or ſubtracted if decreaſing, will give the apparent time at Greenwich of the ecliptic conjunction of the Moon and ſtar.

EXAMPLE.

Required the apparent time of the ecliptic conjunction of the Moon and η ♊, 26 November 1787?

Mean

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* When the proportional part is leſs than three hours, the computation may be facilitated, by uſing P. logarithms. In this caſe, the conſtant log. is 1.1761; and the degrees and minutes in the change of the Moon's longitude are to be eſteemed minutes and ſeconds.

Mean long. η ♊ at giv. t.	3s	0°	28′	45″						
Equat. of equinoxes +				18						
Aberration — +				18						
Apparent longitude —	3	0	29	21	Conſtant log. — —					1.1761
Moon's long. at midn.	3	0	13	39	Diff.	0	15	42	P. L.	1.0594
— — at noon	3	7	43	20	Diff.	7	29	41	ar-co-P.L.	8.6195
						0	25	8	P. L.	0.8550
						12				
Approximate time — —						12	25	8		
The mean 2d diff. is 1′ 57″, hence in tab. XXXVII. the equat. of 2d diff. is 1″.9, to which & 7° 30′ in t. XXXVIII. the correction is — —						—		3		
Apparent time of conjunction —						12	25	5		

PROBLEM II.

To find the Longitude of a Place by an Occultation of a fixed Star by the Moon, when both the Immerſion and Emerſion are obſerved.

RULE.

Reduce the apparent times of obſervation to the meridian of Greenwich, by applying thereto the eſtimated longitude. Find the Moon's horizontal parallax and ſemidiameter, and hence the reduced parallax and ſemidiameter as before.

Find the apparent longitude of the ſtar; from thence, and the Moon's longitude, compute the apparent time at Greenwich of the ecliptic conjunction of the Moon and ſtar, by the laſt problem. Diminiſh the latitude of the place of obſervation by the quantity from table XXXVI.

With the reduced latitude, and the right aſcenſion of the meridian, compute the altitude and longitude of the nonageſimal at the inſtants of obſervation; and find the difference between the longitudes of the nonageſimal and obſerved ſtar, which will therefore be the apparent differences of longitude between the nonageſimal and the Moon's obſerved limb. Compute the parallaxes of the Moon in latitude and longitude, by prob. II. page 170, uſing the apparent latitude of the ſtar in the computation.

Find the Moon's motion in latitude and longitude, during the obſerved interval; and from the change of longitude, ſubtract the difference of the parallaxes in longitude, if the Moon is either eaſt or weſt of the nonageſimal at both obſervations; but if eaſt of the nonageſimal at the time of immerſion, and weſt at that of emerſion; the ſum of the parallaxes in longitude is to be ſubtracted from the change of the Moon's longitude in the above interval. Hence the apparent difference of longitude will be obtained.

If the Moon is approaching the elevated pole of the ecliptic, find the difference between the parallax in latitude at the emerſion, and the ſum of the change of the Moon's latitude in the obſerved interval and the parallax at the immerſion. But if the Moon is receding from the elevated pole, let the difference between the parallax in latitude at the immerſion, and the ſum of the change of the Moon's latitude and the parallax in latitude at the emerſion, be found. Hence the apparent change of the Moon's latitude in the obſerved interval will be known.

From the P. Log. of the apparent difference of longitude, its index being increaſed by 10, ſubtract the P. Log. of the apparent difference of latitude; the remainder will be the log. tangent of the apparent inclination; to the log. co-ſine of which, the P. Log. of the apparent difference of longitude being added, the ſum will be the P. Log. of the apparent motion of the Moon in its relative orbit.

From the P. Log. of the Moon's apparent motion in its proper orbit, ſubtract the P. Log. of the Moon's augmented diameter; the remainder will be the log. ſecant of the central angle.* Find the ſum and difference of this angle and the apparent inclination; whereof the ſum is arc firſt, and the difference arc ſecond, if the Moon is receding from the elevated pole of the ecliptic; but if the Moon is approaching thereto, the difference is arc firſt, and the ſum arc ſecond. In this determination, the ſum of the change of latitude and parallax at immerſion is ſuppoſed greater than the parallax at emerſion, the Moon's diſtance, from the elevated pole of the ecliptic, being increaſing; but when the Moon is approaching the elevated pole, the ſum of the change of latitude and the parallax at emerſion is ſuppoſed greater than the parallax at immerſion, otherwiſe, the contrary rules are to be uſed.

Now to the log. ſecant of arc firſt, add the P. Log. of the Moon's ſemidiameter at the immerſion; the ſum will be the P. L. of arc third; and to the log. ſecant of arc ſecond, add the P. Log. of the Moon's ſemidiameter at the emerſion, the ſum is the P. Log. of arc fourth.

Then, if the Moon is eaſt of the nonageſimal, to arc third add the parallax in longitude at the immerſion, and from arc fourth ſubtract the parallax in longitude at the emerſion; but if the Moon is weſt

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* In ſtrictneſs, this angle ought to be computed both at the immerſion and emerſion, uſing the Moon's apparent diameter at theſe times. This degree of preciſion is however almoſt unneceſſary.

of the nonageſimal, uſe the contrary rules. Now, to the P. Logs. of theſe quantities add the log. coſine of the ſtar's latitude, and the ar-co. of the P. Log. of the Moon's motion in longitude in three hours; the ſums will be the intervals of time between the ecliptic conjunction, and the immerſion and emerſion reſpectively. Hence the apparent time of conjunction of the Moon and ſtar will be known—the difference between which and that inferred from computation by prob. 1. page 183, will be the longitude of the place in time—which is eaſt or weſt, according as the time of conjunction at the place of obſervation is later or earlier than that at Greenwich.

EXAMPLE.

☽ November 26, 1787, in latitude 57° 9′ N. longitude by account 8′ 36″ W. in time, the immerſion of • ♊ was obſerved at 11h. 18′ 8″ apparent time, and the emerſion at 12h. 23′ 12″. Required the true longitude of the place of obſervation?

App. time of immerſ.	11h.	18′	8″	App. time of emerſ.	12h.	23′	12″
Long. by eſtimation		8	36	— — — —		8	36
Reduced time —	11	26	44	— — — —	12	31	48
☽'s hor. par. at imm. - -		61	11	— at emerſion - - -		61	10
Reduction - - -		−	11	— — — — —		−	11
Augmentation - -		+	22	— — — — —		+	22
Reduced parallax - -		61	22	— — — — —		61	21
☽'s ſemidiam. at immerſ.		16	40	— at emerſion - -		16	40
Augmentation - -		+	13	— — — — —		+	14
Inflexion - - -		−	3	— — — — —		−	3
Apparent ſemidiameter -		16	50	— — — — —		16	51

The apparent time at Greenwich of the ecliptic conjunction of the ☽ and • ♊, is found by problem 1. to be 12h. 25′ 5″.

App. time of immer.	11h.	18′	8″	Latitude — —	57°	9′
☉'s R. A. at 11h. 26′ 44″ —	16	10	57	Reduction —	−	14
Right aſcen. meridian	3 6	29	5	Reduced latitude —	56	55
Arch firſt — —	9	29	5			
Supplement —	2	30	55			

Supplement — 2h. 30′ 55″ — — Riſing — 4.32033
Reduced latitude 56 55 — — — Co-ſine — 9.73708
Obliq. ecliptic 23 28 — — — Sine — 9.60012

Difference — 33 27 — Nat. ſine - 55121
4545 — 3.65753

Alt. nonageſimal 53 22 — Nat. co-ſine 59666 - Sine 9.90443
Supplt. arc firſt 2 30 55 — — H. E. T. — 0.21330
Reduced latitude 56 55 — — Secant — 0.26292

Long. nonageſ. 65 24 — — — Secant — 0.38065
Longitude * ♊ 90 29

App. diff. long. 25 5 — — — Co-ſecant — 0.3727
Alt. nonageſim. 53 22 — — Co-ſecant — 0.0956
App. latitude * - - 54′ 48″ — — Co-ſine — 9.9999
Reduced par. ☽ - - 61 22 — — P. L. — — 0.4673

Part 1ſt. par. long. + 20 53 — — P. L. — 0.9355
Part 2d. — 0

Parallax in longitude 20 53

Apparent latitude * 54′ 48″ Secant 0.0001 — Co-ſecant 1.7975
Alt. nonageſimal 53 22 Secant 0.2242 — Co-ſecant 0.0956
Red. parallax of ☽ 61 22 P. L. 0.4673 — — 0.4673

Arc firſt — 36 37 P. L. 0.6916 D.lon. 25° 5′ ſec. 0.0430

Arc ſecond — + 43 — — — P. L. — 2.4034

Part 1ſt. par. in lat. 37 20
Part 2d. — — — 25

Par. in lat. at immer. 36 55

App. time emer. 12 23 12
Sun's R. A. at 12h. 31′ 48″ } 16 11 9

R. A. of meridian 4 34 21
6

Arch = 10 34 21

Arch —	10h. 34′ 21″			
Supplement —	1 25 39	— —	Riſing —	3.83900
Reduced latitude	56 55	— — —	Co-ſine —	9.73708
Obliq. ecliptic	23 28	— — —	Sine —	9.60012
Difference —	33 27	Nat. ſine - 55121		
		1500	—	3.17620
Alt. nonageſ. -	55 31	Nat. co-ſine 56621	Sine —	9.91608
Supplt. arch 1ſt	1 25 39	— —	H. E. T.	0.43761
Reduced latitude	56 55	— — —	Secant —	0.26292
Long. nonageſ. -	76 1	— — —	Secant —	0.61661
Long. η ♊ —	90 29			
Difference long.	14 28	— — —	Co-ſecant —	0.6024
Alt. nonageſimal	55 31	— — —	Co-ſecant —	0.0839
Latitude η ♊ —	54 48	— —	Co-ſine —	9.9999
☽'s reduced parallax	61 21	— —	P. L. —	0.4675
Parallax in longitude	12 38	— —	P. L. —	0.1537
App. latitude η ♊	0° 54′ 48″	Secant 0.0001 —	Co-ſecant	1.7975
Alt. nonageſimal	55 31	Secant 0.2471 —	Co-ſecant	0.0839
☽'s reduced parallax	61 21	P. L. 0.4675	— —	0.4675
Arch firſt — —	34 43	P. L. 0.7147	☽ lon. 14°28′ ſec.	0.0140
Arch ſecond —	+ 47	— —	P. L. —	2.3629
Part 1ſt. par. in lat.	35 30			
Part 2d. tab. XXXIV.	— 25			
Par. in lat. at emer.	35 5			
☽'s mot. in long. in III. h. correct.	1° 52′ 41″	P. L. 2034	Par. in lon. at imm.	20′ 53″
Obſerved interval	1 5 4	P. L. 4419	— — at emer.	12 38
True diff. longitude	40 44	P. L. 6453	Difference —	8 15
Difference of parall.	8 15			
App. diff. of long.	32 29			

Moon's

☽'s mot. in lat. in III. h. correct.	0° 10′ 22″	P. L.	1.2396	Par. in lat. at imm.	36′ 55″
Obſerved interval	1 5 4	P. L.	0.4419	— — at em.	35 5
True diff. of latitude	3 40	P. L.	1,6815	Difference —	1 50
Difference of parallax	1 50				
Apparent diff. lat. -	1 50	P. L.	1.9920		
Apparent diff. long.	32 29	P. L.	10.7436	— — —	0.7436
Appar. inclination	3° 14′	tang.	8,7516	- Co-ſine —	0.9993
☽'s mot. in rel. orbit	32 32	— — —		P. L. —	0.7429
☽'s diameter —	33 41	— — —		P. L. —	9.7278
Central angle -	15 1	— — — —		Secant —	0.0151
Apparent inclinat.	3 14				
Arc firſt —	18 15	— — —		Secant —	0.0224
☽'s ſemid. at immer.	16 50	— — —		P. L. —	1.0291
Arc third — —	15 59	— — —		P. L. —	1.0515
Par. in long. at imm.	20 53				
Sum — —	36 52	— — —		P. L. —	0.6886
Latitude of * —	54 48	— — —		Co-ſine —	9.9999
☽'s mot. in long. in III. hours	1 52 41	— — —		Ar-co-P. L. -	9.7966
Int. bet. imm. & ☌	58 54	— — —		P. L. —	0.4851
App. time of imm.	11 18 8				
App. time of con.	12 17 2				
Ap. t. of ☌ at Gr.	12 25 5*				
Longitude in time	8 3 W.				

Since the time of conjunction, deduced from obſervation, is compared with that found by laſt problem; the longitude of the place is therefore affected by the errors ariſing from thoſe in the longitudes of the Moon and ſtar. If the time of conjunction at Greenwich had been alſo inferred from obſervation, the longitude of the place would have been accurately determined.

---

* The apparent time of conjunction may alſo be inferred from the emerſion, by uſing arc ſecond and the Moon's ſemidiameter at that time.

It often happens, that the immerſion, or the emerſion only, is obſerved. In this caſe, the longitude may be found by the following problem, though not with equal accuracy, as it will now be affected by the errors ariſing from the error in the latitudes of the Moon and ſtar, as well as from thoſe in the longitudes.

PROBLEM III.

To find the Longitude of a Place, by an Occultation of a fixed Star by the Moon, when the Immerſion or Emerſion only is obſerved.

RULE.

Compute the apparent time at Greenwich of the conjunction of the Moon and ſtar, by prob. 1. page 183. Find the reduced latitude and parallax at the reduced time of obſervation, with which compute the altitude and longitude of the nonageſimal, and the parallaxes in latitude and longitude as uſual. Hence the Moon's apparent latitude at that time will be known.

Find the difference between the apparent latitudes of the Moon and ſtar, at the inſtant of obſervation—to the P. Log. of the ſum of which, and the Moon's ſemidiameter, add the P. Log. of their difference; half the ſum will be the P. Log. of arc firſt.

Now, if the Moon is eaſt of the nonageſimal, and the obſervation that of an immerſion, the ſum of the above arc and the parallax in longitude is to be taken; but if an emerſion, take their difference. If the Moon is weſt of the nonageſimal, uſe the contrary rules.—Then, to the P. Log. of this ſum or difference, add the conſtant log. 0.4771, the ar-co. of the P. Log. of the Moon's horary motion in longitude, and the log. co-ſine of the ſtar's latitude; the ſum will be the P. Log. of arc ſecond. Now, in caſe of an immerſion, and the Moon eaſt of the nonageſimal, arc ſecond is to be added to the apparent time; but if the Moon is weſt of the nonageſimal, the above arc is to be applied to the apparent time of obſervation, by addition or ſubtraction, according as arc firſt is greater or leſs than the parallax in longitude. If an emerſion be obſerved, and the Moon at that time eaſt of the nonageſimal, arc ſecond is to be ſubtracted from the apparent time of emerſion; but if the Moon is weſt of the nonageſimal, arc ſecond is additive or ſubtractive, according as the parallax in longitude is leſs or greater than arc firſt. Hence the apparent time of conjunction of the Moon and ſtar will be obtained—the difference between which, and the computed time at Greenwich, will be the longitude in time of the place of obſervation, as before,

EXAM.

EXAMPLE.

♄ October 18, 1788, in latitude 57° 9′ N. longitude by account 8° 36′ W. in time, at 11h. 47′ 17″ apparent time, the emerſion of ι ♉ was obſerved. Required the correct longitude?

```
Mean long. ι ♉ 2ˢ 13° 50′  4″        Mean latitude  -  1° 13′ 23″ S.
 Eq. equinoxes +            17        Aberration    —           0
 Aberration  - +            13                             ────────
               ──────────────         Apparent latitude 1  13  23  S.
App. longitude 2  13  50  34
☽'s lon. at mid. 2 13  48  56 diff. 0°  1′38″   —   —   P. L. 2.0422
 —    at noon 2  20  41  30 diff. 6  52 34÷4=1°43′8½″P.L. 0.2418
                                                             ──────
Apparent time of conjunction - 12   2 51   —   —   P. L. 1.8004

App. time emer. 11h. 47′ 17″  ☽'s hor. par. 58′ 29″  Semidia. 15′ 56″
Long. in time  -       8  36  Reduction  —  11   Augm.   +  10
               ─────────────  Augmentat. +  21   Inflex. —   3
Reduced time  - 11  55  53               ──────           ──────
App. time emer. 11  47  17    Red. parall. 58  39  Red. ſem. 16   3
⊙'s right aſcenſ. 13  37  28
               ─────────────
                 1  24  45
                 6
               ─────────────
Arch firſt   —   7  24  45   —     —     Riſing   —   5.13399
Reduced latitude 56  55  —   —     —     Co-ſine  —   9.73708
Obliq. ecliptic  23  28  —   —     —     Sine     —   9.60012
                ────────                                ──────
Sum    —    —   80  23  —  Nat. ſine - 98595
                                       29593   —      4.47119
                                       ─────
Alt. nonageſimal 46  22  —  Nat. co-ſine 69002 Sine - 9.85960
Supplt. arc firſt - 4 35 15   —    —    H. E. T.  —   0.03040
Reduced latitude 56  55  —    —    —    Secant    —   0.26292
                                                      ───────
Longitude nonag. 45  19  —    —    —    Secant    —   0.15292
Longitude of ſtar 73  51
                ────────
App. diff. long. 28  32  —   —   —   Co-ſecant  —     0.3209
Alt. nonageſimal 46  22  —   —   —   Co-ſecant  —     0.1404
Latitude of ſtar - 1  13  23  —  —   Co-ſine  —   —   9.9999
Reduced parallax     58  39  —  —   P. L.     —       0.4870
                                                      ──────
Part 1ſt par. in lon. + 20  17  —  —  P. L.  —   —    0.9482
Part 2d par. in lon. +       3
                    ──────────
Parallax in longitude  20  20
```

Lati-

Latitude of ſtar	1° 13′ 23″	Secant	0.0001	Co-ſec.	1.6707
Alt. nonageſimal	46 22	Secant	0.1611	Co-ſec.	0.1404
Reduced parallax	58 39	P. L.	0.4870		0.4870
Arc firſt	40 28	P. L.	0.6482	D. long. 28° 32′ } ſecant	0.0562
Arc ſecond	48			P. L.	2.3543
Part 1ſt. par. in lat.	41 16				
Part 2d. par. in lat.	24				
Parallax in latitude	40 52				
Moon's true latitude	28 48				
Moon's app. lat.	1 9 40				
Star's latitude	1 13 23				
Difference	3 43				
Moon's ſemidiameter	16 3				
Sum	19 46			P. L.	0.9593
Difference	13 20			P. L.	1.1303
					2.0896
Arc firſt	16 14			P. L.	1.0448
Parallax in longitude	20 20				
Difference	4 6			P. L.	1.6425
Conſtant log.					0.4771
Star's latitude	1 13 23			Co-ſine	9.9999
☽'s hor. mot. in lon.	34 17			Ar-co-P. L.	9.2798
Interval	7 11			P. L.	1.3993
App. time emer.	11 47 17				
App. time conj.	11 44 28				
Ap. time at Gr.	12 2 51				
Long. in time	8 23 W.				

CHAP.

CHAP. V.

The method of finding the Longitude of a Place,

By

The Eclipſes of the Satellites of Jupiter.

INTRODUCTION.

THE ſatellites of Jupiter were firſt obſerved by Galileo in the year 1610; who is ſaid to have applied them to the diſcovery of the longitude. This laſt diſcovery is however claimed by P. Herigone, in the fifth volume of his *Curſus Mathematici*, printed at Paris in 1644.

The facility of the computation recommends this method, and at land, the obſervations are eaſily made; but the motion of a ſhip at ſea prevents the uſe of a teleſcope proper for this purpoſe. Several attempts have indeed been made in order to render a teleſcope manageable in obſerving the eclipſes of the ſatellites at ſea, ſuch as the Marine Chairs of Irwin and Fyot. The firſt of theſe was fully tried by Dr Maſkelyne, in his voyage to Barbadoes in 1763; but it was not found to anſwer the expectation of the inventor. It however appears, that ſucceſsful experiments have been made at land on the other; and it is to be hoped that theſe chairs are ſtill capable of improvement.

The obſervation neceſſary to find the longitude of a place by this method, is the apparent time of an immerſion or emerſion of either of the ſatellites, but particularly that of the firſt, upon account of its periodic time being ſhorter than that of either of the other ſatellites; and becauſe its elliptic equation, and other inequalities, except that whoſe period is 437 days, are inſenſible. The inſtruments proper for making the above obſervation are, a clock or watch beating ſeconds, an achromatic teleſcope of three or four feet focal length, and a quadrant or ſome other inſtrument proper for determining the error of the watch.

An Immerſion of a ſatellite, is the inſtant of its entrance into the ſhadow of Jupiter, and an Emerſion is that of its re-appearance out of the ſhadow.

When the time of Jupiter's paſſage over the meridian, as expreſſed in the Nautical Almanac, is leſs than XII. hours, the eclipſes of the ſatellites happen on the eaſt ſide of the planet; and when that time exceeds XII. hours, theſe obſervations happen on the weſt ſide of Jupiter. In the firſt caſe, the emerſions of the firſt and ſecond ſatellites are viſible; and in the latter, the immerſions only may be obſerved. Both the immerſion and emerſion of the other two ſatellites are generally viſible, except when Jupiter is too near the oppoſition or conjunction.

In order to obſerve an immerſion or emerſion of any of the ſatellites, the obſerver muſt apply to his teleſcope, a magnifying power ſuitable to the ſtate of the weather; and adjuſt it to diſtinct viſion, previous to the time of obſervation. The watch muſt be contiguous to him, ſo that he may be able to catch the ſecond, as ſoon as the obſervation of the ſatellite is completed. An aſſiſtant clock will however be found more convenient. The obſerver is then to place himſelf at the teleſcope, about three or four minutes before the expected time of an eclipſe of the firſt ſatellite; twice as much before that of the ſecond or third ſatellites; and a quarter of an hour before that of the fourth, the longitude of the place of obſervation being pretty well known. If its ſituation is uncertain, let four times as many minutes as there are degrees of uncertainty, be added to the above times. If the obſervation to be made is that of an immerſion, the obſerver muſt keep his eye directed to the ſatellite, till it diſappears; but if it is an emerſion, he muſt keep his ſight directed to that part of the ſhadow, from which he expects the ſatellite to emerge; and the inſtant of the laſt appearance of the ſatellite, if the obſervation is that of an immerſion, or that of its firſt appearance if an emerſion, muſt be wrote down, and the obſervation is completed.

If an aſtronomical eye glaſs is uſed, it muſt be remembered, that it inverts objects, making that which is the right ſide, apparently the left—*et contra.*

The eclipſes of Jupiter's ſatellites are given in the Nautical Almanac, in page III. of the month. Thoſe viſible at Greenwich are marked with an aſteriſk.

PROBLEM I.

To find, if an Eclipſe of one of Jupiter's Satellites will be viſible at a given Place.

RULE.

To the apparent time of the eclipſe at Greenwich, as given in the Nautical Almanac, apply the eſtimate longitude of the given place, by addition or ſubtraction, according as it is eaſt or weſt, and the apparent time nearly, by the meridian of the place, will be known.

Now if, at the reduced time, the altitude of Jupiter exceeds ſix or eight degrees, and adjacent ſtars of the third magnitude viſible to the naked eye, the eclipſe may be obſerved; but if the ſun is above the horizon, it will be inviſible.

EXAMPLES.

I. Will the immerſion of the firſt ſatellite of Jupiter, of 17th January 1792, be viſible at Eaſter iſland?

Apparent time of immerſion at Greenwich	11h. 48′ 42″
Longitude of Eaſter iſland in time —	7 19 7 W.
Eſtimate time of obſervation — —	4 29 35

This eclipſe will not be viſible, becauſe at the reduced time the Sun is above the horizon.

II. Will the emerſion of the ſecond ſatellite of Jupiter of 25 May 1792, be viſible at Achem, in latitude 5° 22′ N. and longitude 95° 34′ E.?

Apparent time of emerſion at Greenwich —	6h. 2′ 46″
Longitude of Achem in time — —	6 22 16
Apparent time of emerſion at Achem —	12 25 2

The ſun is conſiderably under the horizon at the above time; this eclipſe will therefore be viſible, provided the altitude of Jupiter exceeds 7 or 8 degrees. Now, to the latitude of the place 5° 22′ N. and Jupiter's declination 7° 24′ S. the ſemidiurnal arc is about 5h. 57′. the time of Jupiter's tranſit over the meridian of the place is 9h. 14′. hence the time of the riſing of Jupiter is 3h. 17′, and that of its ſetting 15h. 11′. The altitude of the planet is therefore ſufficient for the above obſervation.

PROBLEM II.

To find the Longitude of a Place, from an Obſervation of an Immerſion or Emerſion of one of the Satellites of Jupiter.

RULE.

To the obſerved time of the eclipſe, apply the error of the watch for apparent time, deduced from an obſervation of the altitude of the Sun, or a fixed ſtar, taken as near the time of the eclipſe as poſſible. Hence the apparent time of obſervation will be known. The difference between this time, and that given in the Nautical Almanac will be the longitude of the place in time: being eaſt or weſt, according as the time by obſervation is later or ealier than the Greenwich time.

EXAMPLE.

I. December 9, 1791, an immerſion of the firſt ſatellite of Jupiter was obſerved at 16h. 58′ 35″ per watch, which was 3′ 58″ ſlow for apparent time. Required the longitude of the place of obſervation?

Time per watch of obſervation —	16h. 58′ 35″
Watch ſlow — — —	3 58
Apparent time of obſervation —	17 2 33
App. time at Greenwich, per N. Alm.	13 42 33
Longitude in time — —	3 20 0 = 50° 0′

Which is eaſt, becauſe the time at the place of obſervation is later than the time at Greenwich.

II. May 27, 1792, in latitude 31° 18′ N. and longitude by account 41° 20′ W. the emerſion of the firſt ſatellite of Jupiter was obſerved at 10h. 25′ 33″ per watch. In order to find the error of the watch, the following altitudes of Regulus were obſerved. Required the longitude of the place of obſevation ?

Time per watch	10h. 28′ 20″	Altitude	19° 46′
	29 30	—	19 31
	30 46	— —	19 17
	32 2	— —	19 0
	33 3	— —	18 46
	3 41	— —	80
Mean —	10 30 44	— —	19 16
		Dip to 12 feet —	3.3
		Refraction — —	2.7
		Corrected altitude	19 10

Latitude —	31° 18′	— — —	Secant -	0.06831
Declination of Regulus	12 58.5	— — —	Secant -	0.01123
Mer. zenith diſtance -	18 19.5	Nat. co-ſine	94928	
Altitude Regulus —	19 10.0	Nat. ſine -	32832	
		Difference -	62096	log. 4.79306
Meridian diſt. Regulus	5 1 5	— — —	Riſing -	4.87260
Right aſcenſ. Regulus	9 57 18			
Right aſcen. meridian	14 58 23			
Sun's R. A. at noon -	4 19 43			
Approximate time —	10 38 40			

Ap-

Approximate time	-	10 38 40
Eq tab. XVIII. to ☉'s R. A. & time of em. at Greenwich — —	—	2 15
Apparent time	—	10 36 25
Time per watch	—	10 30 44
Watch ſlow	— —	5 41
Time per watch of em.		10 25 33
Apparent time	—	10 31 14
App. time at Greenw.		13 20 8

Longitude in time — 2 48 54 = 42° 13½′ W.

III. December 31, 1792, in latitude 39° 35′ S. longitude 48° E. by account, the immerſion of the ſecond ſatellite of Jupiter was obſerved at 15h. 32′ 18″ per watch; and, previous thereto, the following altitudes of Canopus were obſerved, the height of the eye being 8 feet. Required the correct longitude?

Time per watch	14h. 56′ 50″	Altitude		53° 44
	14 58 4	—	—	33
	14 59 36	—	—	19
	15 1 15	—	—	4
	35 45			100
Mean =	14 58 56	—	—	53 25
	Dip	— —	—	2.7
	Refraction	—	—	.7
	Altitude corrected			53 21.6

True altitude	—	53° 21′.6			
Polar diſtance	—	37 24.7	— —	Co-ſecant —	0.21643
Latitude	—	39° 35	— —	Secant —	0.11312
Sum	— —	130 21.3			
Half	—	65 10.7	— —	Co-ſine —	9.62304
Difference	—	11 49.1	— —	Sine — —	9.31135
					19.26394
Arch	—	25 22⅜	—	Sine — —	9.63197

Arch

Arch —		25	$22\frac{3}{8}$
			8
Mer. diſtance Canopus	3	22	59
Right aſcen. Canopus	6	19	22
Right aſcenſion merid.	9	42	21
⊙'s R. A. at Green. time of immerſion	18	48	32
Apparent time —	14	53	49
Time per watch —	14	58	56
Watch faſt —		5	7
Obſerved time of im.	15	32	18
App. time of immer.	15	27	11
App. time p. N. Alm.	12	13	4
Longitude in time -	3	14	7 = 48° 31¾′ E.

Becauſe the time at the place is later than the Greenwich time.

When it is required to ſettle the longitude of a place, as accurately as poſſible by this method, ſeveral immerſions and emerſions of the ſame ſatellite muſt be obſerved. The mean longitude, as given by the immerſions, is to be found, and alſo that deduced from the emerſions. The mean of theſe will be the longitude of the place, unaffected by the errors ariſing from the teleſcope, and is therefore more to be depended on, than that deduced from a ſingle obſervation. If the compariſon be made with correſponding obſervations taken at another place, the difference of the meridians of thoſe places will be accurately found.

DETERMINATION

Of the Longitude of the Obſervatory at Aberdeen, by obſervations of the eclipſes of the ſatellites of Jupiter.

Theſe obſervations were made with one of Dollond's four feet achromatic teleſcopes; and the powers uſed were about 80 and 115, according to the ſtate of the atmoſphere. The time per clock of obſervation was reduced to apparent time, and compared with the Nautical Almanac. Upon this account, it is thought proper to ſubjoin only the reſults, as given by the obſervations of the firſt and ſecond ſatellites.

FIRST SATELLITE.

Year	Month and Day.	Long. by	
		Imm.	Emer.
			′ ″
1786	♂ Jan. 3		8 21
	♃ — 19	′ ″	8 9
	☽ Sept. 18	8 36	
	☿ — 20	8 59	
	♃ Dec. 7		7 47
	♃ — 14		7 23
	♄ — 30		8 9
1787	☽ Jan 8		8 32
	☽ — 15		8 25
	☽ — 22		8 11
	☿ — 31		8 28
	♀ Feb. 23		8 25
	♃ Oct. 25	9 13	
	☿ Dec. 26		8 53
	♀ — 28		8 23
1788	♀ Jan. 4		7 51
	☉ — 27		7 43
	☉ Dec. 14	9 30	
	☉ — 21	9 5	
1789	♃ Jan. 22		8 37
1791	♂ March 1	9 20	
	Sum —	43	197
	Mean —	9 7.1	8 13.1
			9 7.1
			17 20.2
	Mean longitude		8 40.1

SECOND SATELLITE.

Year	Month and Day.	Longitude by	
		Imm.	Emer.
		′ ″	
1786	☽ Sep. 18	8 12	
	♂ Nov. 7	9 43	′ ″
	♄ Dec. 2		8 11
1787	☿ Jan. 3		9 29
	☉ Feb. 4		9 2
	♃ March 8		9 9
	♀ Dec. 21		7 36
	♀ Dec. 28		7 28
1788	♀ Jan. 4		8 25
	☉ May 11		8 52
1789	♃ Jan. 22		7 21
	☽ Dec. 9	8 49	
1791	♄ April 9	8 45	
	Sum -	209	213
	Mean —	8 52.2	8 23.6
			8 52.2
			75.8
	Mean longitude -		8 37.9
	Mean by firſt ſat. -		8 40.1
	Long. of Obſervatory		8 39.0

CHAP.

CHAP. VI.

1. *The Method of finding the Longitude of a Ship at Sea,*

By

A Chronometer or Time Keeper.

INTRODUCTION.

WE are indebted to Gemma Friſius for the diſcovery of this celebrated method of finding the longitude of places. It is alſo mentioned by Adrian Metius, Blundevil, and others. The recommendations of this method are, ſimplicity of principle, and the ſhortneſs of the neceſſary calculations; and were it poſſible to obtain a watch, at a reaſonable price, that would keep an uniform rate in every different poſition and climate, it is more than probable, that no other method for this purpoſe would ever be practiſed at ſea.

About the year 1664, Mr Huygens applied himſelf very diligently to the improvement of timekeepers. In his paper on this ſubject, in the Phil. Tran. No. 47, he recommends two watches, ſo that in caſe the one ſhould meet with any accident, the other may be ſtill ſafe. He then ſhows, how carefully the watches ſhould be kept; the manner of ſetting them to mean ſolar time, under a given meridian, and of finding their daily rate; and the difference of longitude between the ſhip and the place to which the watches were regulated. Laſtly, he gives a ſpecimen of their utility in Major Holmes' voyage to the iſland of St. Thomas.

Soon after the publication of the reward offered by the Britiſh Parliament in 1714, for the diſcovery of the longitude at ſea, Mr Henry Sully applied himſelf to the improvement of watches; but he died at Bourdeaux, at the time he was engaged in making experiments on his time-keeper. The next in France who proſecuted this branch were, M. Julien le Roy and Son, and M. Berthoud. Time-keepers of their conſtruction underwent a trial, by order of the French King, but were not found to anſwer the intended purpoſe.

In England, Mr John Harriſon applied himſelf with indefatigable induſtry, to the improvement of timekeepers—and his ſucceſs therein juſtly merited the attention of the public. In the year 1736, he made trial of his firſt machine in a voyage to Liſbon, and found it to anſwer his expectation. In his return to England, he corrected the dead reckoning about a degree and a half. The following year, the Board of Longitude were pleaſed to give him a gratuity, and deſired him to proſecute his diſcoveries.

In 1739, he completed a ſecond machine more perfect than the firſt; and in 1741, he produced a third, which according to the report of ſome of the members of the Royal Society, was more commodious, and more ſimple than either of the former. In 1761, he completed his fourth timekeeper, for the trial of which his ſon Mr. William Harriſon made a voyage to Jamaica. Before his departure, the error and rate of the timekeeper were determined by Mr. Robertſon, maſter of the academy at Portſmouth. Its error on the 6th of November at noon was 2′ 57″ faſt; on the 7th, it was put back 3′; its error therefore, reduced to the 6th at noon, was 3″ ſlow; and it was found to have loſt 24″ in 9 days, on mean ſolar time.

Mr. Harriſon ſailed from Portſmouth in the Deptford, November 18, 1761, and arrived at Madeira, 9th December following. During the courſe of this part of the voyage, the timekeeper was found of great utility, having corrected the longitude by account, about a degree and a half. In the paſſage from Madeira to Jamaica, the longitude by account was found to differ from that given by the timekeeper, above 3 degrees; and the longitudes of the ſeveral iſlands by which the ſhip paſſed, agreed with the ſame as given by the timekeeper. The Deptford arrived at Jamaica, January 19, 1762; and by equal altitudes, obſerved at Port Royal, the 26th of the ſame month, the time of mean noon by the timekeeper was found to be 4h. 59′ 7″.5; to which (3″) its error, Nov. 6th at noon, and rate ($3'\ 36''.5 = \frac{24''\times 81\text{d}.5\text{h}.}{9\text{d}.}$) being applied, gives 5h. 2′ 47″ the mean time at Portſmouth of noon at Port Royal, and which is therefore the longitude of Port Royal in time, according to the timekeeper; but the longitude of the ſame place, deduced from the tranſit of Mercury obſerved at Kingſton and reduced thereto, is 5h. 2′ 51″, the difference of which from the former is only 4″ of time, which in the parallel of Jamaica is leſs than one nautical mile.

On the 28th of January 1762, Mr Harriſon ſailed from Jamaica in the Merlin, and ſome time after, met with a violent ſtorm, which obliged him to remove the timekeeper to another place where it was more expoſed than before. The Merlin arrived at Portſmouth, March 26, and by equal altitudes obſerved the 2d of April, the time of mean noon was 11h. 51′ 31″.5; to which its former error (3″) and accumulated rate during the interval ($6'\ 32'' = 8''\times 49 = \frac{24''\times 147}{9}$) being applied, gives 11h. 58′ 6″.5, the time of noon. Hence, from Nov. 6, 1761, to April 2, 1762, during which period, the timekeeper had paſſed through a variety of climates, and undergone violent agitations at ſea, its error was no more than 1′ 53″.5, or $28'\frac{1}{3}$ of longitude, which, in the parallel of Portſmouth, does not amount to 18 nautical miles.

Several objections were made to this trial. However, in conſideration of the accuracy of the timekeeper in the above voyage

Mr. Harriſon received 5000l. and was ordered to make a ſecond trial, in a voyage to Barbadoes. He accordingly ſailed from Portſmouth on the 28th of March 1764, in his Majeſty's ſhip the Princeſs Louiſa, having previouſly ſettled the error and rate of the timekeeper. He arrived at Barbadoes the 13th of May, and returned to England the 13th of September following, and in that interval, its accumulated rate was only 54″ of time—in conſequence of which, the Board of Longitude unanimouſly agreed, that Mr. Harriſon's watch had given the ſhip's longitude in theſe trials within the limits preſcribed by act of Parliament. He therefore received 5000l. and was promiſed the remainder of the reward, as ſoon as it was found that a proper timekeeper, conſtructed on the ſame principles by another perſon, was found to anſwer equally well.

The above inſtrument,* together with its explanation, were delivered to the Board of Longitude—which being examined by a committee appointed for that purpoſe, the following report was made—*That Mr Harriſon has taken his time-keeper to pieces, in preſence of us, and explained the principles and conſtruction thereof, and every thing relative thereto, to our entire ſatisfaction, and that he alſo did, to our ſatisfaction, anſwer every queſtion propoſed by us or any of us relative thereto; and that we have compared the drawings of the ſame with the parts, and do find that they perfectly correſpond.*

The Board of Longitude ordered Mr Kendal (one of the committee) to conſtruct a timekeeper upon the ſame principles. This inſtrument was committed to the care of Mr. Wales, in his voyage round the world with Capt. Cook, in the years 1772, 1773, &c. and its ſucceſs was ſuch, that in 1774, Mr Harriſon received the remaining part of the reward offered by Parliament.

Of late, ſeveral improvements have been made in chronometers; particularly that of a new balance ſpring, and an improved thermometer piece, or a compenſation for the effects of heat and cold in the balance, by Mr. John Arnold, for which he claimed one of the new rewards offered, for improvements in diſcovering the longitude. In 1780, he publiſhed an account of the going of a pocket chronometer, kept at the Royal Obſervatory, in which he obſerves, that its greateſt error in the period of thirteen months, during which time it was under trial at the Royal Obſervatory, was 2′ 32″.2, which in the Channel would amount to about 25 geographical miles: and that during the laſt eight months of trial, the extreme difference did not amount to one minute of time, or about 10 miles in the Channel. That, during the whole period of 13 months, there were only four days in which the daily difference amounted to three ſeconds per day; ſo that this machine, in its preſent improved ſtate, is well adapted to ſhew the effect of currents, and to determine with great exactneſs the run of a ſhip, from day to day. He concludes by ob-

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* The author had the pleaſure of ſeeing Harriſon's machines at the Royal Obſervatory in the year 1790.

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obſerving, that the chronometer is better adapted for determining the relative longitude of places, by a compariſon with the longitude of thoſe which have been determined by a ſeries of aſtronomical obſervations on ſhore, than any other method which can be practiſed at ſea.†

The utility of a timekeeper at ſea, is fully exemplified, in the quarto edition of Cook's laſt voyage, vol. I. page 46. vol. III. pages 321. 479. to which the reader is referred.

In order to find the longitude at ſea, by means of a chronometer, its daily rate on mean ſolar (or ſiderial) time, muſt be eſtabliſhed by obſervations made at ſome particular place, and its error aſcertained, for the meridian of that, or any other known place.

An Obſervatory is the moſt proper and convenient place for this purpoſe, as there the rate and error may be both determined, with the utmoſt accuracy, by equal altitudes, or rather tranſits of the Sun or fixed ſtars. But if an obſervatory is not adjacent, the rate and error of the chronometer may be found by altitudes taken daily for ſeveral days, from the horizon of the ſea, or by the method of reflection.

If, by theſe obſervations, the daily rate is found to be nearly the ſame—that is, if the chronometer gains or loſes nearly the ſame portion of abſolute time daily, it may be depended on for finding the longitude; but if its rate is unequal, it muſt be rejected, as the longitude inferred from it cannot be depended on.

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† In the month of July 1788, a Chronometer of Arnold's conſtruction, belonging to his Grace the Duke of Gordon, was put under the care of the author, in order to aſcertain its rate in mean ſolar time; in doing which, the greateſt care imaginable was taken. It was always compared at noon with the Obſervatory clock, whoſe error was generally determined by tranſits of the Sun, which were obſerved with one of Ramſden's Tranſit Inſtruments; and which, previous to every obſervation, was levelled and adjuſted to the meridian mark on the Grampian Hills, about 13900 feet diſtant from the Obſervatory. The mark is ſeen againſt the ſky, and has a circular perforation in its top, of ſuch a ſize, that the middle wire of the tranſit inſtrument covers about a third, and therefore a ſegment of light is ſeen on each ſide.

The obſervations began on the 15th of July, and continued regularly for ſeveral months, in which time it was tried in every different poſition. In what follows, its rate, while in a horizontal poſition only, is given.

Daily rate.	Therm.	No. of days.	Daily rate.	Therm.	No. of days.
— 3″.99	65°	7	— 4″.60	60	7
— 5.60	60½	7	— 4.64	60	7
— 4.89	59¾	7	— 4.80	57¼	5

The price of this inſtrument is 60 guineas. It is in form of an octagon, the radius of the circumſcribing circle is 3.44 inches, and its depth is 3.3 inches—the caſe is mahogany; it has a bow handle for lifting it, and the whole is put into a mahogany box, 10.1 inches ſquare, 6.4 inches deep, having its inſide lined with ſoft elaſtic ſubſtances, to prevent the effects of ſudden ſhocks. Theſe chronometers beat half ſeconds.

PROBLEM.

To find the Longitude of a Ship at Sea by a Chronometer.

RULE.

Let ſeveral altitudes of the Sun, or of any fixed ſtar, be obſerved; and correct the mean altitude as uſual—with which, the ſhip's latitude, and object's declination, compute the apparent time of obſervation, to which apply the equation of time, reduced to the time and place of obſervation, according to its title in the Nautical Almanac, and hence the mean time of obſervation will be known.

To the mean of the times of obſervation, as ſhown by the chronometer, apply its error and accumulated rate. Hence the mean time, under the meridian of the place where the error and rate were eſtabliſhed, will be known; to which apply the difference of longitude in time between the given place and Greenwich—and the mean time of obſervation under the meridian of Greenwich will be obtained. Now the difference between the time at the place of obſervation and that at Greenwich, will be the longitude of the place in time; and which is eaſt or weſt, according as the time by obſervation is later or earlier than the Greenwich time.

EXAMPLES.

I. May 18, 1792, in latitude 42° 15′ N. the following altitudes of the Sun's lower limb were obſerved in the afternoon, and the times of obſervation were given by a chronometer, whoſe error, as ſettled at the Royal Obſervatory at Greenwich, March 15. at noon, was 1′ 18″.4 faſt for mean time, and its daily gain was 7.″83. Required the longitude of the place of obſervation?

Time per chronometer	6h.	58′	40″	Altitude Sun's l. limb		43°	24′
	6	59	36	— — — —		43	33
	7	0	51	— — — —		43	44
	7	2	12	— — — —		43	57
	7	3	21	— — — —		44	7
		4	40				225
Mean — —	7	0	56	Mean — —		43	45
Error — —	—	1	18	Semidiameter —	+		15.8
Accumulated rate - -	—	8	23	Dip — —	—		4.9
				Correction —	—		.9
Mean time at Greenw.	6	51	15				
				Corrected altitude		43	55

To latitude 42° 15′ N. and reduced declination, the number from

Table xxvii. — —	4.1571	Table xxviii. =	3272
Altitude 43° 55′ Sine -	9.8411		
	3.9982	Natural number —	9959
Apparent time —	3 12 8	Table xxix. —	6687
[*]Equation of time -	— 3 53		
Mean time of obſervat.	3 8 15		
Mean time at Greenw.	6 51 15		

Longitude in time — 3 43 9 = 55° 45′ W.

II. October 17, 1791, in latitude 57° 9′ N. the following altitudes of the Sun's lower limb were obſerved, the height of the eye being 13 feet. The time was ſhown by a watch, whoſe error and rate were ſettled at the Obſervatory at Aberdeen; the error, Oct. 1ſt, was 3′ 11″ faſt, and it loſt 4″.6 daily. Required the ſhip's longitude?

Time per watch —	7h. 12′ 8″ A. M.	Alt. ☉'s l. limb	7° 14′
	7 13 0	— — — —	7 20
	7 14 7	— — — —	7 27
	9 15		61
Mean — —	7 13 5	— — — —	7 20.3
Error — —	— 3 11	Semidiameter —	+ 16.1
Accumulated rate —	+ 1 13	Dip and correction	— 10.4
Mean time at Aberdeen	7 11 7	Corrected altitude	7 26
Longitude of Aberdeen	+ 8 36		
Equation of time —	+ 14 33		
Apparent time Green.	7 34 16 A. M.		

To lat. 57° 9′ N. and reduced decl. 9° 20′.5 S. the number from

Table xxvii. =	4.2715	Table xxxviii. —	2547
Altitude 7° 26′ Sine -	9.1118		
	3.3833	Natural number =	2417
Time from noon - - -	4 0 57	Table xxix. —	4964
Apparent time —	7 59 3		
App. time at Greenwich	7 34 16		

Longitude in time — 24 47 = 6° 10¾′ E.

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[*] If the equation of time be applied, with a contrary ſign, to the mean time of obſervation by the chronometer, it will give the ſame reſult.

III. February 2, 1792, being in latitude 15° 48′ N. the mean of ſeveral altitudes of Spica Virginis, eaſt of the meridian, was 53° 24′, and that of the correſponding times 15h. 18′ 22″ per watch, which had been ſet to mean ſolar time at Rio Janeiro, December 4, 1791, and was then gaining 53″.8 daily on mean time. The height of the eye was 16 feet. Required the longitude of the ſhip?

Daily rate — — — — — —	53.″8
No. of days between 4th Dec. 1791, and 2d Feb. 1792	60
Gain in 60 days — — — —	53′ 48″
Now 15h. 18′ − 54′ = 14h. 24′, in which time it gains	32
Accumulated rate — — — — —	54 20
Time per watch of obſervation — —	15 18 22
Mean time of obſervation at Rio Janeiro —	14 24 2
Longitnde of Rio Janeiro in time — —	2 50 55 W.
Mean time at Greenwich — —	17 14 57
Equation of time — — —	− 14 14
Apparent time at Greenwich — —	17 0 43

Mean of ob. alt. =	53° 24′.0	Sun's R. A. at noon	21h. 3′ 42′
Dip and refraction —	4.5	Equation, tab. XVIII. +	2 53
Altitude corrected	53 19.5	Reduced right aſcen.	21 6 35

To lat. 15° 48′ N. and reduced decl. 10° 6′ S. the number from

Table XXVII. =	4.0235	Table XXVIII. =	0504
Altitude 53° 19½′ Sine -	9.9042		
	3.9277	Natural number —	8467
Mer. diſt. Spica Virg.	1h. 44′ 53″	Table XXIX. —	8971
Right aſc. Spica Virg.	13 14 15		
Right aſcen. merid.	11 29 22		
Sun's right aſcenſion	21 6 35		
Apparent time —	14 22 47		
App. time at Greenw.	17 0 43		

Longitude in time - 2 37 56 = 39° 29′ W.

August

IV. Auguſt 15, 1792, in latitude 38° 19′ S. the mean of ſeveral altitudes of Antares, weſt of the meridian, was 14° 28′.4, the height of the eye being 12 feet, and the mean of the times per watch 11h. 41′ 38″ P. M. which had been compared with mean time at the Cape of Good Hope, June 21ſt, and was found to be 1h. 10′ 28″ ſlow, and gained 3″.54 daily. Required the ſhip's longitude?

Daily gain	3″.54
Number of days between 21ſt June and 15th Auguſt	55
Gain in 55 days	3 14.7
The Green. time of obſ. is about 11h. 35′, and corr. gain	1.7
Accumulated rate	3 16
Error, 21ſt June	1 10 28
Watch ſlow at time of obſervation	1 7 12
Time per watch of obſervation	11 41 38
Mean time of obſervation at Cape of Good Hope	12 48 50
Longitnde of Cape of Good Hope	1 13 33 E.
Mean time at Greenwich	11 35 17
Equation of time	− 3 52
Apparent time at Greenwich	11 31 25

Obſerved alt. Antares =	14°	28′.4	Sun's R. A. at noon	9 42 17
Dip and refraction		6.9	Equation, table xviii.	+ 1 44
Corrected altitude	14	21.5	Reduced right aſcen.	9 44 1
Polar diſtance	64	2.7	Co-ſecant	0.04618
Latitude	38	19.0	Secant	0.10535
Sum	116	43.2		
Half	58	21.6	Co-ſine	9.71981
Difference	44	0.1	Sine	9.84178
				19.71312
Arch	45	57	Sine	9.85656
		8		
Mer. diſt. Antares	6	7 36		

Mer. diſt. Antares —	6	7	36
Right aſcen. Antares -	16	16	43
Right aſcen. meridian	22	24	19
Sun's right aſcenſion	9	44	1
Apparent time —	12	40	18
Apparent time at Green.	11	31	25
Longitude in time - -	1	8	43=17° 10¼ E.

In practice, it will be found very convenient to have a table conſtructed, ſhewing the error of the chronometer at the noon of every day for ſeveral weeks, or during the eſtimated time of the run to a place where its error and rate can be again ſettled. To this table a column ſhould be added, containing its hourly rate continued up to 24 hours.

Thus—ſuppoſe the daily rate of a chronometer, deduced from a ſeries of obſervations, was — 4.″72, and its error for mean time, May 9, 1797, at noon, was 3′ 58″.6 ſlow—then

Error of Chronometer at Mean Noon.							*Hourly Rate.*			
♂	May	9.	Error	=	3′	58″.6	1	hour	=	0″.2
☿	—	10	—	—	4	3.3	2	—	—	0.4
♃	—	11	—	—	4	8.0	3	—	—	0.6
♀	—	12	—	—	4	12.8	4	—	—	0.8
♄	—	13	—	—	4	17.5	5	—	—	1.0
☉	—	14	—	—	4	22.2	6	—	—	1.2
☽	—	15	—	—	4	26.9	7	—	—	1.4
♂	—	16	—	—	4	31.6	8	—	—	1.6
☿	—	17	—	—	4	36 4	9	—	—	1.8
♃	—	18	—	—	4	41.1	10	—	—	2.0
♀	—	19	—	—	4	45.8	11	—	—	2.2
♄	—	20	—	—	4	50.5	12	—	—	2.4

DETERMINATION

Of the Longitude of the Obſervatory at Aberdeen, by a Chronometer of Arnold's conſtruction.

The chronometer was ſet to mean ſolar time at Greenwich, June 16, 1788, and loſt 7″.5 in eleven days. It was ſent to Aberdeen by ſea, and being compared with the obſervatory clock, July 15, it was found to be 7′ 26″.6 faſt; and was loſing 6″.4 daily. It is hence probable, that the motion of the ſhip had altered its rate. Now, ſuppoſing this alteration to have commenced when the ſhip left London,

don, which was July 8, its error at that time for the meridian of Greenwich was 15″.0, from which time till July 15, it had loſt 44″.8 (=6″.4 × 7) its rate being ſuppoſed uniform. Hence its error for the meridian of Greenwich, July 15, at noon, was — 59″.8; But its error for the meridian of the obſervatory at Aberdeen, at the ſame time, was + 7′ 26″.6. Hence the longitude of the obſervatory is 8′ 26″.4 in time.

CHAP. VII.

The Method of finding the Longitude at Sea,

By

The Variation Chart.

INTRODUCTION.

THE principle employed in the former methods of determining the longitude of a place, is that of finding the difference of time between the place whoſe longitude is wanted, and the firſt meridian. In this, the variation of the compaſs is uſed for the ſame purpoſe. In order therefore, to diſcover the longitude by this method, charts muſt be conſtructed, in which the variation of the compaſs at all different places of the earth, is repreſented by curve lines, accurately drawn from obſervation. Now the variation being obſerved at the ſhip, and compared with the correſpondent variation on the chart, the longitude anſwering thereto will be obtained.

Meſſ. Nautonier and Linton appear to have been the firſt who propoſed the variation of the compaſs for the diſcovery of the longitude at ſea. In 1676, Mr. Henry Bond publiſhed his *Longitude Found* in 4to. But the want of a ſufficient number of obſervations rendered this method of little or no ſervice at that time.

In 1700, Dr. Halley publiſhed the firſt variation chart, from a great number of obſervations, which for many years he had been collecting for that purpoſe. As the variation is, however, ſubject to an annual change, this chart therefore, in a few years, became almoſt uſeleſs. In the years 1744 and 1756, it was re-publiſhed at London by Meſſ. Mountaine and Dodſon from near one hundred thouſand obſervations. It was alſo publiſhed at Paris in 1765, by M. Bellin. Variation charts, adapted to different years, have been publiſhed by Mr. Samuel Dunn.

Although many of the obſervations, uſed by Dr. Halley, and his followers in the conſtruction of variation charts, were accurate, yet it is not to be expected, that this was the caſe with them all, and therefore it is not probable, that any curve drawn according to theſe obſervations, repreſented the actual variation at all places through which it paſſed, or which is the ſame, it will not be a line of equal variation. It ſhould alſo be remembered that theſe curves are ſubject to a change of place and figure.

This method of finding the longitude, is liable to a very great degree of uncertainty at thoſe places where the variation lines coincide with a parallel of latitude, or where they are nearly in that direction. This happens in the tract between the ſouthern parts of Europe and North America, and in ſome parts of the Pacific and Indian Oceans. But, in places where the variation lines are in a direction perpendicular to the former, a tolerable exact ſolution to this problem may be obtained, provided a good chart, adapted to the time of obſervation, is uſed. This, however, will be difficult to be procured, until theory and obſervation are blended together in the conſtruction of variation charts.

Another and very particular objection againſt the practice of this method, was the want of a proper inſtrument to obſerve the magnetic azimuth with accuracy, in order to determine the variation of the compaſs.* This is however now removed, by the invention of a new azimuth compaſs, by Mr M'Culloch, which is ſo conſtructed, that the centers of motion, of gravity, and of magnetiſm, are brought almoſt all to the ſame point. This inſtruments has alſo many other advantages over the common azimuth compaſſes. An account of its great utility and ſuperiority over them may be ſeen in his Report, printed at London in 1789.

PROBLEM I.

Given the Latitude of a place, and the variation of the compaſs, to find the Longitude of that place.

RULE.

Draw a line to repreſent the parallel of latitude of the given place, and its interſection with the line of the given variation will be the ſituation of the given place; and hence its longitude is known.

When the given variation falls between two of the lines of equal variation on the chart, allowance is to be made as uſual, and, if the given time differs conſiderably from that to which the chart is adaped, allowance is to be made for the change of variation in the interval.

EXAMPLES.

I. Let the ſhip's latitude be 13° 10′ S. and the variation of the compaſs 10° 30′ W. Required the longitude?

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* See alſo Cook's laſt voyage, ſecond edit. 4to. vol. 1. page 50.

By laying a rule over 13° 10′ S. the given latitude, in a direction parallel to the equator, it will be found to interſect the line repreſenting 10° 30′ W. variation, in a certain point, the longitude of which is about 9° 20′ W.

II. June 1, 1793, being in latitude 40° 0′ S. the variation of the compaſs deduced from the mean of ſeveral azimuths of the ſun was 10° E. and by comparing two variation charts, the lateſt of which being adapted to the year 1765, it is found that the annual decreaſe of variation is about 10 minutes. Required the ſhip's longitude?

The decreaſe of variation in the given time is about 4° 40′ (= 10′ × 28 y.); hence the obſerved variation reduced to the time to which the chart is adapted is about 14° 40′ E. Now by drawing the parallel of 40° S. its interſection with the line expreſſing the above variation, is found to be in longitude 56° 30′ W. which is therefore that required.

Although the variation chart, in many caſes, is not to be depended on, for the purpoſe of determining the longitude at ſea; yet its uſe in finding the variation of the compaſs anſwering to a given latitude and longitude by inſpection only, is evident. Upon this account, it demands the particular attention of the practical navigator.

Beſides the above chart, another expreſſing the lines of equal *DIP*, was publiſhed in the year 1768, by M. Wilcke of Sweden. By a chart of this conſtruction Mr. W. Whiſton propoſed to find the latitude, the longitude being given; or the longitude, if the latitude be known. If both methods be combined, the latitude and longitude may be found, the Declination and Inclination of the needle being given. For this purpoſe, a chart expreſſing the lines of equal variation and dip is neceſſary.

B O O K V.

CONTAINING

The Demonſtrations of the preceding Rules and Formulæ.

Demonstration

Of Dr. Maſkelyne's Rules for computing the Longitude and Latitude of a fixed Star or Planet, from its known right Aſcenſion and Declination, and for computing its right Aſcenſion and Declination; its Longitude and Latitude being given, Pages 22. 23.

LET the primitive circle, (fig. 2.) repreſent the ſolſtitial colure, EQ the equator, ♋ ♑ the ecliptic, and S the poſition of a ſtar or planet, through which let circles of latitude and declination be drawn. Then is ♈ A the right aſcenſion, AS the declination; ♈ B the longitude, and BS the latitude of the ſtar: and the angle A ♈ B the obliquity of the ecliptic. Now in the right angled ſpherical triangle ♈ AS, are given A ♈ and AS, to find the angle A ♈ S.

Sine A ♈ : R : : tangent AS : tangent A ♈ S.

Hence, tangent A ♈ S $= \dfrac{\text{tangent AS,}}{\text{Sine A ♈}}$ to radius unity;

And A ♈ S ∓ A ♈ B = B ♈ S.

Again, in the right angled ſpherical triangles A ♈ S, B ♈ S, we have

R : co-ſine A ♈ S : : tan. ♈ S : tan. A ♈
R : co-ſine B ♈ S : : tan. ♈ S : tan. B ♈
Hence co-ſine A ♈ S : co-ſine B ♈ S : : tan. A ♈ : tan. B ♈
And tan. B ♈ = ar. co. co-ſine A ♈ S. co-ſine B ♈ S. tan. A ♈, or
= tan. A ♈ S. ar. co. [illegible]ſ. A ♈ S. co-ſ. B ♈ S. tan. A ♈,

becauſe the co-ſine of an arch is equal to the rectangle under the co-tangent and ſine of that arch, to radius unity.

Now R : ſine B ♈ : : tan. B ♈ S : tan. BS.
Hence tan. BS = ſine B ♈. tan. B ♈ S, or
= tan. B ♈. co-ſine B ♈. tan. B ♈ S. *Q. E. D.*

If the poſitions of the ecliptic and equator be ſuppoſed to be mutually changed, the demonſtration of the rule for computing the right aſcenſion and declination will be the ſame as the above.

Theory

THEORY *of* HADLEY'S QUADRANT, *of the* SEXTANT, *and* CIRCULAR INSTRUMENT.

LEMMA.

The angle contained between a ray of light falling on any reflecting ſurface, and a line perpendicular to that ſurface at the point of incidence, is equal to the angle contained between the ſame perpendicular and the reflected ray.

THEOREM I.

In the Fore Obſervation, the Altitude of an Object, or the angular Diſtance between two Objects, is equal to double the Arch on the Limb paſſed over by the Index.

Let ABC (fig. 27.) repreſent a quadrant, D the index glaſs, F the fore horizon glaſs, and O, G, two objects, whoſe angular diſtance is the arch OG, or the angle GEO, the poſition of the eye being E. Now let the index be ſo placed, that the reflected image of the object O, may coincide with the object G, ſeen directly. Then is the angle GEO equal to twice the angle CDH; that is, equal to twice the arch paſſed over by the index.

DEMONSTRATION.

Let the ray OD incident on the ſpeculum at D, be reflected to the fore horizon glaſs, and from thence to the eye at E; alſo, let DL be perpendicular to the index glaſs at the point of incidence D: and FM perpendicular to the fore horizon glaſs at the point of incidence F. Then, by the above lemma, the angle LDF = LDO, and MFE = MFD. Let OD and GF produced meet in E, and FED will be the angular diſtance of the objects.

Now ſince LD is parallel to EG, and perpendicular to the ſpeculum, the angle EFD = FDL = LDO = DEF; and EFD is biſected by the perpendicular FM, therefore the angle DEF = 2MFE. But becauſe the right angled triangles MFK, MFI, have the angle M common, the remaining angles MFI, MKF, are therefore equal: hence the angle DEF = 2MKF; but MKF = KDC, therefore DEF = 2KDC. Now DEF is the angular diſtance of the objects, and KDC is meaſured by the arch CH. The angular diſtance between any two objects, is therefore double the arch on the limb paſſed over by the index. Q. E. D.

This alſo holds good with reſpect to the Sextant, and Circular Inſtrument.

Theorem

THEOREM II.

In the back Obſervation, the Supplement of the Angle contained between any two Objects, is equal to double the Exceſs of the Inclination of the Mirrors above a right Angle; and therefore, equal to Twice the Arch paſſed over by the Index.

Let D (fig. 28.) be the ſpeculum, N the back horizon glaſs, whoſe planes produced meet in M: Hence the angle MNI is equal to the exceſs of the inclination of theſe planes above a right angle; alſo, let O, G, be any two diſtant objects, then PEO, the ſupplement of the angular diſtance GEO, is equal to twice the angle MNI = twice CDH = twice the arch CH.

DEMONSTRATION.

Let the index be ſo placed, that the reflected image of the object O may coincide with the direct object G, the eye being at P. Alſo let DL be perpendicular to the ſpeculum, and LN perpendicular to the back horizon glaſs. Then, is the angle LDN = LDO, and LNP = LND; hence, MND = RNP = MNI, therefore DNI = 2 MNI: and becauſe the triangles MNI, MKN, are ſimilar, and DC parallel to NK, therefore the angle MNI = (MKN = KDC =) the arch CK; hence DEN = twice the arch CH. Therefore, in the back obſervation, the ſupplement of the angular diſtance between two diſtant objects, is equal to double the arch on the limb paſſed over by the index. Q. E. D.

DEMONSTRATION

Of the Rule for computing the Length of the Scale, or of any Portion of the Scale on the Limb of Hadley's Quadrant, Page 37.

Let ABC (fig. 26.) repreſent an octant, and therefore the angle BAC = 45°. Join BC, and draw AS perpendicular thereto. Now in the triangle ASC, are given AC, and the angle CAS, to find CS.

R : ſine CAS : : AC : CS.

Hence CS = AC. ſine CAS, and BC = 2AC, ſine CAS.

If AC = 1, then BC = 2 ſine CAS. *Q. E. D.*

DEMONSTRATION

Of the Formula for computing the greateſt Augmentation of the Moon's Semidiameter, Page 56.

Let ML (fig. 29.) repreſent the moon's ſemidiameter, E the earth's center, and P a point on its ſurface: Hence the angles MEL, MPL, will be the moon's ſemidiameter, as ſeen from thoſe points reſpectively;

ſpectively; and their difference PLE is therefore the augmentation. Now, ſince theſe angles are ſmall, the meaſure of each may be ſubſtituted for its ſine. Let therefore, the moon's horizontal ſemidiameter MEL $= s$, the augmentation PLE $= x$, hence MPL $= s + x$. Alſo let ME $= d$, and PE $=$ ME $-$ MP $= 1$, hence MP $= d - 1$.

Then $d - 1 : a :: s : s + x$
Convert°. $d - 1 : 1 :: s : x$

Hence $x = \dfrac{s}{d - 1}$

If the Moon's horizontal parallax be expreſſed by b, then d becomes $= \dfrac{57^\circ\ 17'\ 44.''8}{b}$, and $d - 1 = \dfrac{57^\circ\ 17'\ 44''.8 - b}{b}$. But the ratio of the horizontal parallax to the ſemidiameter of the Moon is as $1 : ,27249$ nearly: hence $d - 1 = \dfrac{57^\circ\ 17'\ 44''.8 - b}{b}\ .27249$ $= \dfrac{15^\circ\ 46'\ 45'' - s}{s}$; which being ſubſtituted for $d - 1$ in the former equation, gives $x = \dfrac{s^2}{15^\circ\ 46'\ 45'' - s}$.

Let LP be produced to F, and EF being drawn parallel to ML, will therefore be the greateſt augmentation to the radius EM. Let the moon be in any other point A of its diurnal path; join AE, AF; make AG $=$ AE; and EG, which is the meaſure of the angle EAF, to the radius AE $=$ LE will be the augmentation correſponding to that altitude.

Now in the right angled rectilineal triangle EFG, given the angle EFG $=$ complement of AEM $=$ ☽'s altitude, and EF the greateſt augmentation, to find EG.

R : ſine EFG : : EF : EG $=$ EF. ſine EFG, to radius 1. *Q. E. D.*

It hence obviouſly follows, that if to the true altitude of the moon's limb, the horizontal ſemidiameter of the moon as given in the ephemeris, be applied, the true altitude of the moon's center will be obtained: and therefore in this caſe, the augmentation is unneceſſary. Hence alſo, the change of parallax at any given altitude, anſwering to a change of altitude equal to the moon's ſemidiameter, will be the augmentation anſwering to that altitude.

DEMONSTRATION

Of the Rule for computing the Dip of the Horizon, Page 58.

In fig. 21ſt, BE repreſents the height of the obſerver, and ECH $=$ FEI the correſponding dip. Now in the right angled rectilineal triangle CHE, are given CH the ſemidiameter of the earth, and BE the height of the eye, to find the angle HCE.

Let CH $= s$, and BE $= h$, hence CE $= s + h$.

Now $\overline{HE}^2 = \overline{2s+h} \times h$. per 36. III Euclid.

Hence HE $= \sqrt{\overline{2s+h} \times h} = \sqrt{2sh}$. becauſe at any probable elevation, $2s + h$, is not ſenſibly different from $2s$h.

And $s : \sqrt{2sh} :: r :$ tan. HCE $= \frac{r\sqrt{2sh}}{s}$.

Since, in the preſent caſe, the arch may be ſubſtituted for its tangent, the radius therefore becomes 57° 17′ 44″.8.

Hence the dip $= \frac{r\sqrt{2sh}}{s} = \sqrt{\frac{2r^2 h}{s}} = \sqrt{\frac{2r^2}{s} \times h}$. *Q. E. D.*

DEMONSTRATION

Of the Rule for computing the Parallax in Altitude, Page 62.

In the rectilineal triangles DAC, HAC, (fig. 23,) the angle ADC is the horizontal parallax, AHC the parallax anſwering to the apparent altitude DAH.

Now DC : AC : : R : ſine ADC.

And DC (= HC) : AC : : ſine ZAH : ſine AHC.

Ex. eq. R : ſine ADC : : ſine ZAH : ſine AHC = ſine ADC. ſine ZAH, to radius 1.

Since in angles not exceeding one, or one and a fourth degrees, the arches are nearly proportional to their ſines.

Therefore AHC = ADC. ſine ZAH.

That is, Par. in alt. = hor. par. × coſine of apparent altitude.

Or, by the nature of Proportional Logarithms,

P. log. Par. in alt. = P. L. hor. par. + log. ſecant app alt. *Q. E. D.*

DEMONSTRATION

Of the Rule for converting Degrees into Time, p. 72.

Let a be the given degrees, and x the correſpondent time; then ſince the ſun is in the plane of any given meridian once in 24 hours,

Therefore $360^\circ : a^\circ :: 24\text{h.} : x\text{ h.}$

Hence $x = \frac{24\text{ h. } a}{360^\circ} = \frac{1440' a}{360} = 4a$. *Q. E. D.*

DEMONSTRATION

Of the Rule for reducing Time into Degrees, page 72.

Let a be the given time, and x the degrees anſwering thereto.

Now $24\text{h.} : a\text{h.} :: 360^\circ : x^\circ$

Hence $x = \frac{360}{24} \times a = 15a = 10a + \frac{10a}{2}$. *Q. E. D.*

Demonstration

Of the Rule for reducing the Time under a given Meridian, to the Time at Greenwich, and converſely, page 73.

The rotation of the earth being from weſt to eaſt, it is therefore evident, that the ſun, or any celeſtial object, will be ſooner on the meridian of a place to the eaſtward, and later on that of a place to the weſtward, than on the meridian of Greenwich, by a quantity proportional to the longitude of the place from Greenwich. And converſely. Q. E. D.

Demonstration

Of the Rule, to find the Correction of Noon ariſing from the Ship's Run between the Obſervations of the equal Altitudes, pages 86, 89.

Let $a =$ interval of time between the obſervations, and $b =$ hourly rate of ſailing. Then, according to the principles of plane ſailing,

$$\left.\begin{array}{l} \text{The diff. latitude} = \dfrac{ab.\text{coſine courſe}}{\text{1 hour}} \\ \text{And departure} = \dfrac{ab.\text{ſine courſe}}{\text{1 hour}} \end{array}\right\} \text{to radius unity.}$$

Now the change in the horary angle anſwering to a given change of latitude is $= \text{D. Lat.}\ \dfrac{\text{tan. azimuth}}{\text{coſine lat.}}$ [*] $= \dfrac{ab.\ \text{cos. courſe. tan. azim.}}{\text{coſine latitude.}}$ to radius 1. and therefore

$$\text{The equation of latitude in time} = \frac{2ab.\text{coſine courſe. tan. azimuth}}{\text{coſine of latitude.}}.$$

Or if proportional logarithms be uſed, and the hours reckoned as minutes, then the P. Log. of eq. of lat. $=$ P. L. $a +$ P. L. $b +$ L. ſecant courſe $+$ L. tan. azim. $+$ L. cos. lat. $+$ L. 2 $-$ P. L. 1, minute.

$$\text{And the equation of long.} = \frac{2\,ab.\ \text{ſine courſe.}}{\text{1 hour cos.lat.}}$$

Hence, P. L. eq. of long. $=$ P. L. $a +$ P. L. $b +$ L. co-ſecant courſe $+$ L. co-ſine lat. $+$ L. 2 $-$ P. L. 1, minute.

Demonstration

Of the Formula for computing the Altitude of a circumpolar Star, when its Motion in Altitude is a Maximum, page 90.

Let P, (fig. 30.) repreſent the pole, Z the zenith of the place of obſervation, DS*d* the parallel of the declination of the ſtar, ZSA a vertical circle touching the parallel of declination in the point S, which is therefore the place of the ſtar, when its motion in altitude is quickeſt.

Now in the right angled ſpherical triangle ZSP, ZP and SP are given to find ZS.

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[*] Simpſon's Fluxions, page 282, line 2, from bottom.

Co-ſine SP : R : : co-ſine ZP : co-ſine ZS
Co-ſine ZS = coſine ZP. ſecant SP, to radius unity:
That is, ſine altitude = ſine latitude. co-ſecant declination.

Q. E. D.

Demonstration

Of the Rule for computing the apparent Time, from the Altitude of a celeſtial Object.

Method First, *page* 91.

In any Spherical Triangle ZPS. (fig. 31.)

$$\text{Sine } \tfrac{1}{2} P = \sqrt{\frac{\text{ſine } \tfrac{1}{2}(ZS+PZ-PS).\text{ſine } \tfrac{1}{2}(ZS+PS-PZ)}{\text{ſine } PZ.\ \text{ſine } PS}}.\ \text{per Spher.}$$

Wherein ZP may repreſent the co-latitude of the place of obſervation, ZS the zenith diſtance, and PS the polar diſtance of the obſerved object: and the angle ZPS the horary diſtance of the object from the meridian.

Now, let the altitude $= a$, latitude $= m$, and polar diſtance $= p$.

Then $\tfrac{1}{2}(ZS + PZ - PS) = \tfrac{1}{2}(90 - a + 90 - m - p) = \tfrac{1}{2}(180 - a - m - p)$
$= 90 - \tfrac{1}{2}(a + m + p)$.

And $\tfrac{1}{2}(ZS + PS - PZ) = \tfrac{1}{2}(90 - a - \overline{90 - m} + p) = \tfrac{1}{2}(-a + m + p)$
$= \tfrac{1}{2}(a + m + p) - a$.

Hence by ſubſtitution we have

$$\text{Sine } \tfrac{1}{2} P = \sqrt{\frac{\text{co-ſ. } \tfrac{1}{2}(a+m+p).\ \text{ſine } \overline{\tfrac{1}{2}(a+m+p)-a}}{\text{co-ſ. } m.\ \text{ſine } p.}} =$$

$$= \sqrt{\text{co-ſ. } \tfrac{1}{2}(a+m+p).\ \text{ſine } \tfrac{1}{2}\overline{(a+m+p)}-a.\ \text{ſecant } m.\ \text{co-ſe. } p.}$$

Method Second, *page* 93.

In the Spherical Triangle ZPS. (fig. 31.)

$$\text{Sine } PS.\ \text{ſine } PZ : R^2 : : \text{v. ſine } ZS. - \text{v. ſine } \overline{PS - PZ} : \text{v. ſine } P.$$

per Spherics.

But the difference of the verſed ſines is equal to the difference of the co-ſines; therefore,

$$\text{Sine } PS.\ \text{ſine } PZ : R^2 : : \underline{\text{co-ſ. } \overline{PS - PZ} - \text{co-ſ. } ZS} : \text{v. ſine } P.$$

$$\text{Hence v. ſine } P = \frac{\text{co-ſ. } \overline{PS - PZ} - \text{co-ſ. } ZS.\ R^2}{\text{ſine } PS.\ \text{ſine } PZ}$$

$$= \frac{\text{co-ſec. } PS.\ \text{co-ſec. } PZ.\ \overline{\text{co-ſ. } \overline{PS - PZ} - \text{co-ſ. } ZS.}}{R^2}$$

Whence log. v. ſine P = log. co-ſec. PS — log. co-ſec. PZ
+ log. co-ſ. $\overline{PS - PZ}$ — co-ſ. ZS — 2 log. rad.

But PS — PZ = the meridian zenith diſtance = latitude ± declination,

nation, according as they are of different, or of the ſame name. And column of riſing is a table of log. verſed ſines, adapted to time. Therefore,

$$\text{Log. ſecant lat.} + \text{log. ſec. Dec.} + \text{log.}\,\overline{\text{coſ. mer. zen. diſt.} - \text{ſine ob. alt.}} - 2\ \text{log. rad.} = \text{log. riſing of apparent meridian diſtance of object.}$$

INVESTIGATION

Of that Poſition in which an Object ſhould be, that the Time deduced from its Altitude may be as little as poſſible affected, by unavoidable Errors in the Obſervations and Latitude, page 102.

When the latitude and declination are of the ſame name, the change of altitude, in a given time, is moſt rapid; firſt, when the ſtar is in the prime vertical, the declination being leſs than the latitude; ſecond, when the object is in that part of its diurnal path, which is in contact with an azimuth circle—in this caſe, the declination will exceed the latitude of the place of obſervation; and thirdly, when the object is in the zenith, the latitude and declination being equal.

When the latitude and declination are of contrary names, the change of altitude is quickeſt when the object is in the horizon: which, therefore, is the moſt proper poſition for obſervation, abſtracting from refraction. But becauſe of the irregularity to which the horizontal refraction is ſubject, it is therefore improper to uſe altitudes obſerved near the horizon, for the purpoſe of determining the apparent time.

Let P. Z. S. (fig. 32.) repreſent the pole, zenith, and ſtar, reſpectively: Let MPS be a given change in the hour angle: And AM being a portion of a parallel of altitude, SA will be the correſpondent change of altitude.

Let AS $= a$, and MPS $= h$.

Now R : ſine PS : : h : SM $= h$. ſine PS. to radius unity.

And R : ſine AMS : : h.ſine PS : $a = h$. ſine PS. ſine AMS

$= h$. ſine PS. ſine ZSP. to rad. 1.

Now PS being conſtant, the change of altitude AS anſwering to a given change in the hour angle MPS, will be a maximum, when the ſine of the angle ZSP is the greateſt poſſible; that is, when ZSP is a right angle. Now ZSP is a right angle when the vertical ZS touches the parallel of declination of the ſtar. And if ZSP be conſtant, the change of altitude will be quickeſt when PS $= 90°$; that is, when the object is in the equator.

Again, in the triangle ZSP.

Sine ZP : ſine PS : : ſine ZSP : ſine PZS.

Therefore, ſine ZP. ſine PZS = ſine PS. ſine ZSP.

Hence, ſine ZP. ſine PZS, being ſubſtituted for its equal in the former equation, we have $a = b.$ ſine ZP. ſine PZS.*

Now ZP being conſtant, the change of altitude will be a maximum, when PZS is a right angle; that is, when PS is the prime vertical.† And if PZS be conſtant, the change of altitude, anſwering to a given change in the horary angle, will increaſe with the ſine of PZ, and is therefore greateſt when the latitude is leaſt; that is, when the place is ſituated on the equator.

Since, $\text{ſine ZSP} = \dfrac{\text{ſine PZ. ſine ZPS}}{\text{ſine ZS}}$; which, ſubſtituted in the general equation, gives $a = b. \dfrac{\text{ſine PS. ſine PZ. ſine ZPS}}{\text{ſine ZS}}$. Hence, the latitude and declination being inſtant, the change of altitude will increaſe with the altitude and hour angle.

DEMONSTRATION

Of the Rules for computing the Correction of apparent Time, anſwering to given Errors in the Altitude, Declination, and Latitude.—Prob. x. xi. xii. *Pages* 103, 104.

The error in altitude being given, the correſpondent error of the hour angle may be eaſily deduced from the preceding formulæ.

For ſince $a = b.$ ſine PZ. ſine PZS.

$$\text{Therefore } b = \frac{a}{\text{ſine PZ. ſine PZS. } 15}. \text{ in time:}$$

And P. log. $b =$ P. log. $a +$ log. ſine PZ + log. ſine PZS + log. 15.

Whence, in any ſpherical triangle, the change in any of its angles occaſioned by a change in the oppoſite ſide, is equal to this change, multiplied by the co-ſecant of one of the other ſides, and the co-ſecant of the intervening angle—and converſely.

The Error in Declination being given, to find the reſulting Error in the Hour Angle.

Let SB, SM, (fig. 32.) be portions of the parallels of altitude and declination of the given object, and BM the given change of declination : Hence, MPS will be the correſpondent change of the hour angle.

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* If proportional logarithms be uſed, the equation becomes

P log. $a =$ P. log. b + co ſecant ZP + l. co-ſecant PZS.

This is the demonſtration of the rule in page 55, for computing the change of altitude, anſwering to a given interval.

† If the latitude and declination are of contrary names, the parallel of declination will not interſect the prime vertical above the horizon; therefore, the change of altitude is quickeſt when the object is in the horizon.

Now in the right angled triangle BMS, which may be conſidered as rectilineal, are given MB, and the angle BSM = ZSP, to find MS.

R : co-tan. ZSP : : MB : MS = MB. co-tan. ZSP, to radius unity.
And, ſine PM : R : : MB. co-tan. ZSP : MPS = MB. co. tan. ZSP. co-ſecant PM.*

Or by uſing P. logarithms, the change of the hour angle expreſſed in time is = P. log. BM + log. tangent ZSP + log. ſine PM + log. 15.

Hence, in any ſpherical triangle, the change in any of its angles, ariſing from a given change in one of the adjacent ſides, is equal to the above change multiplied by the co-tangent of one of the other angles, and the co-ſecant of the ſide contained between theſe angles ; and converſely.

If *l* be the change of latitude, and *h* the correſponding change of the hour angle, then, by the preceding, we have

$$h = l.\ \text{co-tangent PZS. co-ſecant PZ} \times \tfrac{1}{15}.$$

And P. log. *h* = P. log. *l*. + log. tan. PZS + log. ſine PZ + log. 15.

Or, ſince co-tangent Z = co-ſecant P. co-tangent PS. ſine PZ − co-tangent P. co-ſine PZ. Hence by ſubſtitution, we have

$$h = l.\ \overline{\text{co-ſec. P. co-tang. PS} - \text{co-tang. P. co-tang. PZ.}}\quad \mathcal{Q}.\ E.\ \mathcal{R}.$$

DEMONSTRATION

Of the Rule for computing the Apparent Time from an Obſervation of two Stars in the ſame Vertical, page 105.

Let P repreſent the elevated pole, Z the zenith, and S, A, two ſtars in the ſame azimuth circle ZSAN, to which let PB be perpendicular. Now, in the triangle APS are given, AP, SP, the polar diſtances of the ſtars, and the angle APS the difference of right aſcenſion, to find the angle ASP.

$$\text{Now tang. S} = \frac{\text{ſine APS}}{\text{ſine PS. co-tan. AP} - \text{co-ſine PS. co-ſine P}}\ \text{per ſpherics.}$$

And in the right angled ſpherical triangle PBS

$$\text{R : coſ. PS : : tang. S : co-tang. BPS} = \text{coſ. PS. tang. S} =$$

$$= \frac{\text{ſine APS}}{\text{tang. PS. co-tang. AP} - \text{coſ. P}}$$

Hence,

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* Or, ſince co-tan. ZSP = co-ſec. P. co-tan. PZ. ſine PS − co-tan. P. co-ſ. PS, which being ſubſtituted in the former equation, we have

$$\text{MPS} = \text{MB.}\ \overline{\text{co-ſec. P. co-tan. PZ} - \text{co-tan. P. co-tan. PS.}}$$

$$\text{Hence tangent BPS} = \frac{\text{tangent PS. co-tangent AP} - \text{co-ſine P.}}{\text{ſine APS}} =$$

$$= \text{tang. PS. co-tan. AP. co-ſec. P} - \text{co-tan. P.}$$

Again, in the triangles PSB, PZB,

$$\text{co-t.PS} : \text{co-tan.PZ} :: \text{coſ.BPS} : \text{coſ.BPZ} = \text{tan.PS. co-t.PZ. coſ.BPS}$$

$$\text{And BPS} \mp \text{BPZ} = \text{ZPS.} \qquad \textit{Q. E. D.}$$

DEMONSTRATION

Of the Rule for computing the Hour Angle, from the Interval of Time between the Riſing of two known Stars, page 106.

Let P (fig. 34) repreſent the pole, S the firſt obſerved ſtar when in the horizon, and A the ſituation of the other at the ſame inſtant. The angle APS is the difference of right aſcenſion. Let *a* be the poſition of the ſecond ſtar at the time of riſing. Then will the angle AP*a* be the interval of time between the inſtants when the ſtars were obſerved in the horizon; and the angle *a*PS will be the difference between the above interval, and the difference of right aſcenſion.

In the ſpherical triangle *a*PS, are given P*a*, PS, and the angle *a*PS, to find PS*a*. Now, per ſpherics,

$$\text{Tangent PS}a = \frac{\text{ſine } a\text{PS}}{\text{ſine PS. co-tan. P}a - \text{coſ. PS. coſ. } a\text{PS}}$$

And, in the right angled ſpherical triangle SPO,

$$\text{R} : \text{coſ. PS} :: \text{tan. PS}a : \text{co-tan. SPO} = \text{coſ. PS. tan. PS}a =$$

$$= \frac{\text{ſine } a\text{PS}}{\text{tan. PS. co-t. P}a - \text{coſ. } a\text{PS}}$$

$$\text{And tang. SPO} = \text{tan. PS. co-tan. P}a\text{. co-ſec. } a\text{PS} - \text{co-tangent } a\text{PS.}$$

Q. E. D.

DEMONSTRATION

Of the Rule for computing the Apparent Time, from three Altitudes of the ſame Object, with the Intervals of Time between the Obſervations, page 107.

Let A, B, C, (fig. 35) be the poſitions of the object at the times of obſervation; and AD, BF, CG, the ſines of the altitudes. Deſcribe the ſemicircle KMR, and draw AN, BM, CL, perpendicular to KR, which will therefore be the ſines of the times of obſervation from noon, to the radius VK. Join LM, LN, MN, and let MS, LU, be drawn perpendicular to LN and BM reſpectively. Now the angle KVL = VLU = time from noon when the greateſt altitude was obſerved; the angle LVN = interval of time between the extreme obſervations, and conſequently VLN = complement of half that interval. And becauſe the triangles MTS, LTU, are ſimilar, therefore the angle TMS = TLU.

In the right angled triangle LSM, are given, the angle MLS = half the arch MN, and LM = twice the ſine of half the arch LM, to find MS and LS.

$$\text{R} : \text{ſine MLS} :: \text{LM} : \text{MS} :: \tfrac{1}{2}\text{LM} : \tfrac{1}{2}\text{MS}.$$

Hence $\frac{1}{2}$ MS = ſine $\frac{1}{2}$ arc MN. ſine $\frac{1}{2}$ arc LM.

$\text{R} : \text{coſ. MLS} : \frac{1}{2}\text{LM} : \frac{1}{2}\text{LS} = \text{coſ.}\ \frac{1}{2}\ \text{arc MN. ſine}\ \frac{1}{2}\ \text{arc LM}.$

And becauſe the lines CX, CA, and LN, are cut proportionally in the points I, B, and T.

Therefore $\text{CX} : \text{CI} :: \text{LN} : \text{LT} :: \frac{1}{2}\text{LN} : \frac{1}{2}\text{LT}.$

$$\text{Hence } \tfrac{1}{2}\,\text{LT} = \frac{\text{CI. ſine } \frac{1}{2} \text{ arc LMN}}{\text{CX}}$$

$$\text{And } \tfrac{1}{2}\,\text{LS} - \tfrac{1}{2}\,\text{LT} = \tfrac{1}{2}\,\text{TS}$$

Now in the right angled triangle TSM,

$$\tfrac{1}{2}\,\text{TS} : \tfrac{1}{2}\,\text{MS} :: \text{R} : \text{co-tangent TMS} = \text{TLU}.$$

$$\text{Therefore, co-tangent TLU} = \frac{\frac{1}{2}\text{MS}}{\frac{1}{2}\text{TS}} = \frac{\text{ſine } \frac{1}{2} \text{ arc MN. ſine } \frac{1}{2} \text{ arc LN}}{\frac{1}{2}\,\text{TS}}$$

$$\text{And TLV} - \text{TLU} = \text{VLU} = \text{KVL},$$

$$\text{Or MTS} - \tfrac{1}{2}\text{LVN} = \text{KVL}. \quad \mathcal{Q}.\ E.\ D.$$

Demonstration

Of the Method of reducing the Apparent to the True Diſtance.

Method First (page 112).

Given the apparent diſtance *ms* (fig. 24) the apparent zenith diſtances Z*m*, Z*s*, and the true zenith diſtances ZM, ZS, to find the true diſtance MS.

$$\left.\begin{array}{l}\text{ſine Z}s\text{. ſine Z}m : \text{R}^2 :: \text{v. ſine } ms - \text{v. ſine } \overline{\text{Z}s - \text{Z}m} : \text{v. ſine P} \\ \text{ſine ZS. ſine ZM} : \text{R}^2 :: \text{v. ſine MS} - \text{v. ſine } \overline{\text{ZS} - \text{ZM}} : \text{v. ſine P}\end{array}\right\} \begin{array}{l}\text{per}\\ \text{ſph.}\end{array}$$

$$\text{ſine Z}s\text{. ſine Z}m : \text{ſine ZS. ſine ZM} :: \text{v. ſine } ms - \text{v. ſine } \overline{\text{Z}s - \text{Z}m} : \\ : \text{v. ſine MS} - \text{v. ſine } \overline{\text{ZS} - \text{ZM}}$$

$$\text{v. ſine MS} - \text{v. ſine } \overline{\text{ZS} - \text{ZM}} = \overline{\text{v. ſine } ms - \text{v. ſine } \overline{\text{Z}s - \text{Z}m}} \cdot \frac{\text{ſine ZS. ſine ZM}}{\text{ſine Z}s\text{. ſine Z}m}$$

$$\text{v. ſine MS} = \overline{\text{v. ſine } ms - \text{v. ſine } \overline{\text{Z}s - \text{Z}m}} \cdot \frac{\text{ſine ZS. ſine ZM}}{\text{ſine Z}s\text{. ſine ZM}} + \\ + \text{v. ſine } \overline{\text{ZS} - \text{ZM}}$$

But table of log. diff. = $\log.\dfrac{\text{ſine ZS. ſine ZM}}{\text{ſine Z}s\text{. ſine Z}m}$, which call *d*, and let *a* be the number anſwering to $\log.\ \overline{\text{v. ſine } ms - \overline{\text{Z}s - \text{Z}m}} + d$

Then $\text{v. ſine MS} = \text{v. ſine } \overline{\text{ZS} - \text{ZM}} + a.$ $\mathcal{Q}.\ E.\ D.$

Me-

METHOD SECOND, *page* 113.

$$\left.\begin{aligned}\overline{\text{Co-ſine}}|^2\tfrac{1}{2}Z &= \frac{\text{ſine}\,\tfrac{1}{2}(ZM+ZS+MS).\text{ſine}\,\tfrac{1}{2}(ZM+ZS-MS}{\text{ſine }ZM.\ \text{ſine }ZS}\\ &= \frac{\text{Sine}\,\tfrac{1}{2}\,Zm+Zs+ms).\text{ſine}\,\tfrac{1}{2}(Zm+Zs-ms)}{\text{ſine }Zm.\ \text{ſine }Zs}\end{aligned}\right\}\text{p. ſph.}$$

Now let A, *a*, B, and *b*, be the complements of ZM, Z*m*, ZS, and Z*s*, reſpectively: alſo, let C and *c* repreſent MS and *ms*.

$$\begin{aligned}\text{Then }\ \tfrac{1}{2}(ZM+ZS+MS)&=\tfrac{1}{2}(90-A+90-B+C)=90-\tfrac{1}{2}(A+B-C)\\ \tfrac{1}{2}(ZM+ZS-MS)&=\tfrac{1}{2}(90-A+90-B-C)=90-\ (A+B+C)\\ \tfrac{1}{2}(Zm+Zs+ms)&=\tfrac{1}{2}(90-a+90-b+c)=90-\tfrac{1}{2}(a+b-c)\\ \tfrac{1}{2}(Zm+Zs-ms)&=\tfrac{1}{2}(90-a+90-b-c)=90-\tfrac{1}{2}(a+b+c)\end{aligned}$$

Hence, by ſubſtitution,

$$\frac{\text{Co-ſine}\,\tfrac{1}{2}\,(A+B-C).\ \text{co-ſine}\,\tfrac{1}{2}(A+B+C)}{\text{ſine }ZM.\ \text{ſine }ZS.}$$

$$=\frac{\text{Co-ſine}\,\tfrac{1}{2}\,(a+b-c).\ \text{co-ſine}\,\tfrac{1}{2}(a+b+c).}{\text{ſine }Zm.\ \text{ſine }Zs.}$$

$$\text{Co-ſ.}\,\frac{A+B-C}{2}.\text{co-ſ.}\frac{A+B+C}{2}$$

$$=\text{co-ſ.}\,\frac{a+b-c}{2}\ \text{co-ſ.}\frac{a+b+c}{2}.\ \ \frac{\text{ſine }ZM.\ \text{ſine }ZS,}{\text{ſine }Zm.\ \text{ſine }Zs.}$$

Now, log. of $\frac{\text{ſine }ZM.\ \text{ſine }ZS}{\text{ſine }Zm.\ \text{ſine }ZS.}$ is contain. in tab. XLII. which call *d*.

Alſo let $\frac{a+b-c}{2}=D$, $\frac{a+b+c}{2}=E$, and $A+B=F$,

Then, $\text{co-ſine}\,\frac{F-C}{2}.\ \text{co-ſine}\,\frac{F+C}{2}=\text{co-ſine }D.\ \text{co-ſine }E.\ d.$

But $\text{co-ſ.}\,\frac{F-C}{2}.\text{co-ſ.}\frac{F+C}{2}=\tfrac{1}{2}\times\overline{\text{co-ſ. }F+\text{co ſ. }C}$, to radius 1.

Hence $\tfrac{1}{2}\cdot\overline{\text{co-ſine }F+\text{co-ſine }C}=\text{co-ſine }D.\ \text{co-ſine }E.\ d.$

Let $\text{co-ſine }D.\ \text{co-ſine }E.\ d=\overline{\text{ſine}}|^2\ \tfrac{1}{2}\,n$;

Then, $\tfrac{1}{2}\times\overline{\text{co-ſine }F+\text{coſine }C}=\overline{\text{ſine}}|^2\ \tfrac{1}{2}\,n.$

And $\text{co-ſine }F+\text{co-ſine }C=2\,\overline{\text{ſine}}|^2\ \tfrac{1}{2}\,n=1-\text{co-ſine }n.$

But $\text{co-ſine }C=1-2\,\overline{\text{ſine}}|^2\ \tfrac{1}{2}\,C$:

Hence, $1-2\,\overline{\text{ſine}}|^2\ \tfrac{1}{2}\,C=1-\text{co-ſine }n-\text{co-ſine }F$;

Therefore, $\overline{\text{ſine}}|^2\ \tfrac{1}{2}\,C=\tfrac{1}{2}+\overline{\text{co-ſine }n+\text{co-ſine }F}$,

$$=\text{co-ſine}\,\frac{n+F}{2}.\ \text{co-ſine}\frac{n-F.}{2}$$

Q. E. D.

ME-

METHOD THIRD, *page* 115.

Let *sm* (fig. 25.) be the apparent diſtance, S*n* the diſtance cleared of refraction, and SM the true diſtance. From Z draw ZD perpendicular to the diſtance, which let be biſected in E.

Then, tan. $\frac{sm}{2}$: tan. $\frac{Zs+Zm}{2}$: : tan. $\frac{Zs-Zm}{2}$: tang. $\frac{DE}{2}$ per ſpherics.

Hence, tan. arch firſt (= tan. $\frac{DE}{2}$) = co-tan. half ſum alt. tan. half diff. alt. co-tan. half diſt. to rad. 1.

And, half diſtance ± arch firſt = *s* D and D*m* reſpectively = arches ſecond and third.

Now, in the right angled triangles, *s*DZ, S*as*.

Tangent Z*s* : tan. D*s* : : R : co-ſine *s*.

And R : co-ſ. *s* : : S*s* : S*a*

Hence, Ex. eq. pertur. tan. Z*s* : tan. D*s* : : S*s* : S*a* $= \frac{Ss.\ \text{tan.}\ Ds.}{\text{tan.}\ Zs.}$

But, S*s* = 57″. tang. Z*s*, nearly;

Hence, S*a* = 57″. tang. D*s* :

And hence alſo, *nb* = 57″. tang. D*m*.

S*a* and *nb* may therefore be taken from a table of refraction.

Again, in the triangles *m* D Z, M *c n*;

R : co-tan. Z*m* : : tan. D*m* : co-ſ. *m* = co-t. Z*m*. tan. D*m*,

R : co-tan. Z*m* .tan. D*m* : : M*n* : *nc* = M*n*. co-t. Z*m*. tan. D*m*.

But M*n* = hor. par. ſine Z*n*, to radius 1. And as Z*m* may be aſſumed = Z *n*, therefore *nc* = hor. par. tan. D*n*. co-ſine Z*n*.

Hence, P. log. Par. in diſtance = P. log. hor. par. + log. co-tan. D*n* + log. co-ſecant altitude.

The laſt correction of parallax is found by means of table XXII. It is the difference between S*c* and SM, and may be computed by various methods. The reaſon for entering the table twice will be obvlous, from a due conſideration of the triangles S *c* M, M *c n*.

Q. E. D.

DEMONSTRATION

Of the Rule for computing the true Altitude of the Sun, Moon, or a fixed Star, page 132.

In the ſpherical triangle ZPS, (fig. 31.) are given PZ the co.latitude, PS the polar diſtance, and ZPS the horary angle, to find ZS the co-altitnde. Then per ſpherics,

Sine PS. ſine PZ : R^2 : : v-ſine ZS − v-ſine $\overline{PS-PZ}$: v-ſine P.

Hence, V-Sine ZS = v-ſine $\overline{PS-PZ}$ + v-ſine P. ſine PS. ſine PZ.

Now, $\overline{PS-PZ}$ = mer. zenith diſtance, and ZS = co-altitnde.

Hence co. v-ſine alt. = v-ſine, mer. zen. diſt. + v-ſine P. co-ſine
decli. co-ſine latitude.

Or, becauſe the difference of the verſed ſines is equal to that of the co-ſines,

Therefore, ſine alt. = co-ſine mer. zen. diſt. — v-ſine P. co-ſine dec.
co-ſine latitude.

And column of riſing is a table of log. verſed ſines : Hence, the rule is obvious. *Q. E. D.*

DEMONSTRATION

Of the Method of computing the Latitude, Longitude, and Time, from the ſame Set of Obſervations, p. 137.

Let P. (fig. 36.) repreſent the elevated pole, Z the zenith, A, B, *a*, *b*, the true and apparent places of the objects: Hence, ZA, ZB, Z*a*, Z*b*, the true and apparent zenith diſtances: AB, *ab*, the true and apparent diſtances, and PA, PB, their reſpective polar diſtances.

The demonſtration of the method of reducing the apparent to the true diſtance, is given in page 223.

In the ſpherical triangle BAZ, given ZA, ZB, the true zenith diſtances, and AB the true diſtance of the objects. Required the angle ZAB.

Sine ZA. ſine AB : R^2 : : v-ſine BZ—v-ſine $\overline{AZ-AB}$: v-ſine ZAB.
Or, by uſing a table of natural ſines,

R^2 : co-ſec. ZA. co-ſec. AB : : co-ſine $\overline{AZ-AB}$—co-ſine BZ : v-ſine ZAB.

Log. co-ſecant ZA + log. co-ſec. AB + log. $\overline{\text{co-ſine } \overline{AZ-AB} - \text{co-ſ.}BZ}$
— 2 log. R = log. v-ſine ZAB.

Now, the co-ſine of the difference of any two arches, is equal to the ſine of the ſum of the leſs, and complement of the greater; and column of riſing is a table of log. verſed ſines. Wherefore, log. ſecant alt. object fartheſt from mer. + log. co-ſecant diſtance + log. ſine alt. ob. fartheſt from mer. + diſt. —ſine other objects alt. —2 log.
R = (log. verſed ſine angle ZAB =) log. riſing of arch firſt.

In the ſpherical triangle PAB, are given PA, PB, the polar diſtances, and AB the true diſtance. Sought the angle PAB.

Sine AB. ſine AP : R^2 : : v-ſine BP — v-ſine $\overline{AB-AP}$: v-ſine
PAB, per Spherics.
Or by uſing natural ſines,

R^2 : co-ſec. AB. co-ſec. AP : : co-ſine $\overline{AB-AP}$ $\mp$ co-ſine BP : v-ſine PAB.

R^2 : co-ſec. diſt. ſec. dec. ob. fartheſt from mer. : : ſine $\overline{\text{diſt.} + \text{dec.}}$
$\mp$ ſine other ob. dec. : v-ſine PAB.

Log. co-ſec. diſt. + log. ſec. dec. ob. fartheſt from mer. + log. ſine $\overline{\text{diſt.} + \text{dec.}}$ $\mp$ ſine other objects dec. − 2 log R = log. riſing arch ſecond.

In the ſpherical triangle PZA, given AZ, AP, the zenith and polar diſtances of the object fartheſt from the meridian, and the angle ZAP = BAP − BAZ, to find ZP the co-latitude, and the angle ZPA, the horary diſtance of the object fartheſt from the meridian.

Sine AZ. ſine AP : R^2 : : v-ſine ZP − v-ſine $\overline{AZ - AP}$: v-ſine ZAP.

Hence, the v-ſine ZP is eaſily obtained. Or by ſubſtituting natural ſines,

R^2 : ſine AZ. ſine AP : : v-ſine ZAP : co-ſ. $\overline{AZ - AP}$ − co-ſ. ZP.

$$\frac{\text{Co-ſ.alt . co-ſ.dec. v-ſine ZAP}}{R^2} = \text{ſine } \overline{\text{alt.} \mp \text{dec}} - \text{ſine latitude}$$

$$\text{Sine lat.} = \text{ſine } \overline{\text{alt.} \mp \text{dec.}} - \frac{\text{co-ſ. alt.co-ſ. dec.v-ſine ZAP.}}{R^2}$$

Let natural number of log. co-ſine alt. + log. co-ſine dec. + log. co-ſine ZAP − 2 log. R = *a* ;

Then, nat. ſine latitude = nat. ſine $\overline{\text{alt.} \mp \text{dec.}}$ − *a* :

Again, Sine ZP : ſine ZA : : ſine ZAP : ſine ZPA,
Co-ſ. lat. : co-ſ. alt. : : ſine ZAP : ſine ZPA,
Sec. lat. : ſec. alt. : : co-ſec. ZAP : co-ſec. ZPA ;

Log. co-ſec. ZPA = log. co-ſine lat. + log. ſec. alt. + log. co-ſecant ZAP − 2 log. R.

But column of half elapſed time is a table of log. co-ſecants − R. Whence the operation is manifeſt. *Q. E. D.*

DEMONSTRATION

Of the Rule for computing the Difference of Longitude between the Moon and a fixed Star, or Planet.—Page 146.

Let P (fig. 37.) repreſent the pole of the ecliptic, PM the complement of the moon's latitude, PS the complement of the ſtar's latitude, MS the true diſtance between the moon and ſtar, and the angle MPS their difference of longitude, which is required.

Now, the polar diſtances being of the ſame affection,

$$\text{Co-ſine } \tfrac{1}{2} P = \sqrt{\frac{\text{ſine } \frac{1}{2}(PM + PS + MS).\ \text{ſine } \frac{1}{2}(PM + PS - MS.)}{\text{ſine PM. ſine PS.}}}$$

Let the moon's latitude = m, the latitude of the ſtar = s, and the diſtance = d.

Then, $\tfrac{1}{2}(PM + PS + MS) = \tfrac{1}{2}(90 - m + 90 - s + d) = 90 - \tfrac{1}{2}(m + s - d)$
$= 90 - \tfrac{1}{2}(m + s + d) - d.$

And, $\tfrac{1}{2}(PM+PS-MS)=\tfrac{1}{2}(90-m+90-s-d)=90-\tfrac{1}{2}(m+s+d)$.

Hence,

$$\text{Co-ſ.}\ \tfrac{1}{2}P=\sqrt{\text{co-ſ.}\ \tfrac{1}{2}(m+s+d).\ \text{co-ſ.}\ \overline{\tfrac{1}{2}(m+s+d)-d}.\ \text{ſec.}\ m.\ \text{ſec.}\ s.}$$

Again, let the polar diſtances be of a different affection,

$$\text{Sine}\ \tfrac{1}{2}P=\sqrt{\frac{\text{ſine}\ \tfrac{1}{2}(MS+MP-PS).\ \text{ſine}\ \tfrac{1}{2}(MS+PS-MS.)}{\text{ſine}\ MP,\ \text{ſine}\ PS.}}$$

Now, the ſame notation being uſed, we have

$\tfrac{1}{2}(MS+MP-PS)=\tfrac{1}{2}(d+90-m-90+s)=\tfrac{1}{2}(d+s-m)=\tfrac{1}{2}(d+s+m)-m$

And $\tfrac{1}{2}(MS+PS-PM)=\tfrac{1}{2}(d+90+s-90+m)=\tfrac{1}{2}(d+s+m)$

Hence, $\text{ſine}\ \tfrac{1}{2}P=\sqrt{\text{ſine}\ \tfrac{1}{2}(d+m+s).\ \text{ſine}\ \overline{\tfrac{1}{2}(d+m+s)-m}.\ \text{ſec.}\ m.\ \text{ſec.}\ s.}$

Q. E. D.

DEMONSTRATION

Of the Rule for computing the Longitude, from an Obſervation of the Moon's Altitude. Page 154.

Let the ſun's right aſcenſion $=s$, the moon's $=m$, the moon's relative motion in right aſcenſion in 12 hours $=d$, moon's horary angle $=h$, and the apparent time $=x$.

$$\text{Now,}\ 12\ \text{h.} : x\ \text{h} :: d : \frac{dx}{12}$$

$$\text{And,}\ \overline{x+s}-m+\frac{dx}{12}=h$$

$$x+s=h+m+\frac{dx}{12}$$

$$x-\frac{dx}{12}=h+m-s$$

$$12x-dx=\overline{h+m-s}\,.\,12$$

$$x=\frac{h+m-s}{12-d}.\ 12=\frac{h+m-s}{180-d}.\ 180.$$

Q. E. D.

DEMONSTRATION

Of the Rule for computing the Altitude and Longitude of the Nonageſimal.

Let the circle PEBQ (fig. 38) repreſent the ſolſtitial colure, EQ the equator, ♋♑ the ecliptic, P,p their reſpective poles, ♈A the right aſcenſion of the meridian PZAB, which paſſes through Z the zenith of the place, whoſe latitude is AZ; hence PZ is the co-latitude, Pp the diſtance of the poles, or the obliquity of the ecliptic; ZPp the right aſcenſion of the meridian from ♑. Zp♋ the longitnde of the nonageſimal from ♋; N the nonageſimal—hence ZN is the zenith diſtance, and Zp the altitude of the nonageſimal.

METHOD FIRST (page 167)

In the right angled ſpherical triangle PMZ, are given PZ the co-latitude of the place of obſervation, and the angle ZPM, the difference between the right aſcenſion of the meridian and ♋ (or ♑) to find PM.

$$R : \text{co-ſine } ZPM :: \text{tan. } ZP : \text{tan. } PM = \frac{\text{ſine } ZP♈ . \text{co-tan. latitude}}{R}$$

And $PM \pm Pp = Mp$. The uppermoſt ſign is to be uſed, when the right aſcenſion of the meridian is between o and XII. hours, and the other ſign, when the right aſcenſion is between XII. and XXIV. hours.

Again, in the right angled ſpherical triangles MPZ, MpZ,

$$\text{ſine } MP : \text{ſine } Mp :: \text{co-tan. } MPZ : \text{co-tan. } MpZ.$$

$$\text{Hence tan. long. nonageſimal} = \frac{\text{tang. } ♈PZ. \text{ coſecant } MP. \text{ ſine } Mp}{R^2}$$

$$R : \text{ſecant } MpZ :: \text{tan. } Mp : \text{tan. } pZ,$$

$$\text{Whence tan. alt. nonageſ.} = \frac{\text{co-ſec. } ♈pZ. \text{ tan. } Mp}{R}$$ *Q. E. D.*

METHOD SECOND (page 168)

In the ſpherical triangle ZpP are given, ZP the co-latitude, Pp the obliquity of the ecliptic, and the angle pPZ the right aſcenſion of the meridian from ♑, to find Zp the altitude, and the angle ♈pN the longitude of the nonageſimal.

$$R^2 : \text{ſine } PZ. \text{ ſine } Pp :: \text{v-ſine } ZPp : \text{v-ſine } Zp - \text{v-ſine } \overline{PZ - Pp}$$
$$:: \text{v-ſine } ZPp : \text{co-ſine } \overline{PZ - Pp} - \text{coſ. } Zp$$

Now the complement of the ſum of two arcs is equal to the ſum of the leaſt arc and complement of the greater; therefore, as the latitude is aſſumed leſs than 66½ degrees.

$$R^2 : \text{ſine } PZ. \text{ ſine } Pp :: \text{v-ſine } ZPp : \text{ſine } \overline{Pp + \text{comp. } PZ} - \text{coſ. } ZP$$

$$R^2 . \overline{\text{ſine } \overline{Pp + \text{comp. } PZ} - \text{co-ſine } ZP} = \text{ſine } PZ. \text{ ſine } Pp. \text{ v-ſine } ZPp$$

$$\text{ſine } \overline{AZ + Pp} - \text{co-ſine } ZP = \frac{\text{co-ſine } AZ. \text{ ſine } Pp. \text{ v-ſine } ZPp}{R^2}$$

$$\text{co-ſine } ZP = \text{ſine } \overline{AZ + Pp} - \frac{\text{co-ſine } AZ. \text{ ſine } Pp. \text{ v-ſine } ZPp}{R^2}$$

Now, column of riſing is a table of log. verſed ſines.

Let the nat. N°. to log. coſine AZ + log. ſine Pp + log. riſing ZPp
$- 2 \log. R = a,$

Then, coſine alt. nonageſimal = ſine $\overline{\text{AZ} + \text{Pp}} - a$.
Again, ſine Zp : ſine ZP :: ſine ZPp : ſine ZpP,
co-ſec. Zp : coſ. ZP :: coſ. ZPp : ſec. ♈pA,
Hence ſec. ♈pA = co-ſec. ZPp. co-ſec. ZP. ſine Zp to radius 1.
But column of half elapſed time is a table of log. co-ſecants.
Therefore, log. ſecant long. nonag. = log. H,E,T,ZPp + log. co-ſec. ZP + log. ſine Zp. to radius 1.
Q. E. D.

DEMONSTRATION

Of the Rule for computing the Parallax in Latitude and Longitude, (page 170.)

Let ZNHEP (fig. 39) repreſent the meridian paſſing thro' the nonageſimal N—NE the ecliptic, P its pole, Z the zenith, L the place of an object unaffected by parallax, and M its apparent place, both being in the ſame vertical ZLM*n*. Thro' L, M, draw the circles of latitude PLp, PMp, and thro' M draw the parallel of latitude MA; then will the angle APM be the parallax in longitude, and AL the parallax in latitude. Alſo ZP = NH = altitude of the nonageſimal; MPZ the apparent diſtance of the planet from the nonageſimal, and MP its apparent polar diſtance.

Now, in the right angled triangle ALM, which may be conſidered as rectilineal,

R : co-ſine AML :: ML : AM = ML. co-ſine AML.

$$\text{But co-ſine AML} = \text{ſine ZMP} = \frac{\text{ſine P. ſine ZP}}{\text{ſine ZM.}}$$

And ML = hor. par. ſine ZM.
Hence AM = hor. par. ſine P. ſine ZP.

$$\text{And angle APM} = \frac{\text{hor. par. ſine P. ſine ZP}}{\text{ſine MP}}$$

That is, par. in lon. = hor. par. ſine diſt. from non. ſine alt. non. ſec. lat.
Again, R : co-tang. ZMP :: AM : AL = AM. co-tangent ZMP.
But AM = hor. par. ſine P. ſine ZP.
Hence AL = hor. par. ſine P. ſine ZP. co-tangent ZMP.

$$\text{But co-tangent ZMP} = \frac{\text{co-tan. ZP. ſine MP.} - \text{coſine P. coſine MP}}{\text{ſine P.}}$$

Whence by ſubſtitution we obtain

AL = hor. par. ſine ZP. co-tan. ZP. ſine MP — hor. par. ſine ZP. co-ſine P. co-ſine MP.
= hor. par. co-ſine ZP. ſine MP — hor. par. ſine ZP. co-ſine P. co-ſine MP.

That is, par. in lat. = hor. par. co-ſine alt. non. co-ſine lat. — hor. par. ſine alt. non. co-ſine diſt. of object from non. ſine lat. object.

Q. E. D.

DE-

DEMONSTRATION

Of the Rule for computing the Longitude of a Place, by an Obſervation of a Solar Eclipſe, or an Occultation of a Fixed Star by the Moon, pages 173, 184.

Let CE (fig. 40) repreſent a ſmall portion of the ecliptic, S a ſtar (or the ſun's center*) M the apparent place of the moon at the immerſion, and L its apparent place at the emerſion. Now the parallaxes in longitude at the immerſion and emerſion being applied to the moon's relative motion in longitude in the interval between the obſervations, will give the apparent difference of longitude EC, which, multiplied by the co-ſine of the moon's apparent latitude, gives MN. The apparent difference of latitude LN is found, by applying the parallaxes in latitude to the computed change of latitude in the above interval.

In the right angled triangle LNM, which may be conſidered as rectilineal, are given, the ſides MN, NL, to find the apparent inclination NML, and the apparent motion of the moon in its relative orbit ML.

$$\text{MN} : \text{NL} :: \text{R} : \text{tangent LMN} = \frac{\text{NL}}{\text{MN}}$$

$$\text{R} : \text{ſecant LMN} :: \text{MN} : \text{ML} = \text{MN. ſecant LMN.}$$

Again, in the triangle MSL, the augmented ſemidiameters of the moon (or ſum of ſun and moon's ſemidiameters) MS, LS, and the Moon's relative motion ML being given, to find the angles SML, SLM—to which the angle PLM=LMN being applied, the angles DSM, BSL will be obtained.

Laſtly, in the right angled triangle MDS are given, SM, and the angle DSM, to find DS; and hence AC, which being reduced to time, will be the interval between the immerſion and conjunction. The interval between the emerſion and conjunction may, in like manner, be inferred from the triangle SBL; and if no error be committed, the times of conjunction will agree. Now, the time of conjunction thus found, being compared with that deduced from the Nautical Almanac, will give the longitude of the place of obſervation. *Q. E. D.*

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* But in this caſe BD would be a portion of the ecliptic.

BOOK VI.

APPENDIX.

CONTAINING

Various Methods of finding the Latitude, and Variation of the Compaſs.

CHAP. I.

Of finding the Latitude of a Place.

INTRODUCTION.

THE ſituation of a place, with reſpect to the equator, was anciently determined, by aſcertaining the length of the longeſt day, and by the comparative length of the ſhadow of a *Gnomon* at that time. This inſtrument was afterwards uſed in many aſtronomical obſervations, ſuch as, for determining the obliquity of the ecliptic, the times of the tropics and equinoxes, the length of the year, and the ſeaſons, &c. The method of reckoning the latitude in *degrees* and *minutes* being introduced, inſtruments for obſerving altitudes were divided accordingly.— The *Aſtrolabe*, (a circular ring, having a moveable index and ſights,) was applied to obſerve altitudes at ſea. It was, however, ſupplanted by the *Croſs Staff*, and that again by the *Quadrants* of Davis and Hadley, in ſucceſſion.

The moſt ſimple, and at the ſame the moſt accurate method of determining the latitude of a place, is by an obſervation of the meridian altitude of the Sun, or of any other of the celeſtial bodies—and, therefore, ſuch obſervations ſhould not be neglected, when poſſible to be obſerved. It however frequently happens, that the meridian altitude cannot be obſerved, by reaſon of clouds, fog, &c. or that the declination of the object is unknown. In theſe caſes, therefore, recourſe muſt be had to other methods.

PROB.

PROBLEM I.

Given the Sun's Meridian Altitude, to find the Latitude of the Place of Obſervation.

RULE.

Find the true altitude of the Sun's center, by prob. x. page 78. Call it S or N, according as the Sun is ſouth or north at the time of obſervation, which, ſubtracted from 90°, will give the zenith diſtance of a contrary denomination.

Reduce the Sun's declination to the meridian of the place of obſervation, by prob. v. page 74. Then, the ſum or difference of the zenith diſtance and declination, according as they are of the ſame, or of a contrary denomination, will be the latitude of the place of obſervation of the ſame name with the greater.*

EXAMPLES.

I. October 17, 1792, in longitude 32° E. the meridian altitude of the Sun's lower limb was 48° 53′ S. height of the eye 18 feet.—Required the latitude?

Obſ. alt. ⊙'s low. limb	48° 53′ S.	⊙'s dec. 17 Oct. noon	9° 37′ S.
Semidiameter —	+ 16	Equation, table XIII.	— 2
Dip and refraction	— 5		
		Reduced declination -	9 35 S.
True alt. Sun's center	49 4 S.	Zenith diſtance —	40 56 N.
		Latitude — —	31 21 N.

II. November 6, 1792, in longitude 158° W. the meridian altitude of the Sun's lower limb was 87° 37′ N. height of the eye 12 feet. Required the latitude?

Mer. alt. Sun's l. limb	87° 37′ N.	⊙'s dec. p. N. Alm.	16° 17′.8 S.
Semidiameter —	+ 16.3	Equat. tab. XIII.	+ 7.7
Dip and refraction	— 3.3		
		Reduced declination	16 25.5 S.
True altitude —	87 50.0 N.	Zenith diſtance - -	2 10.0 S.
		Latitude — —	18 35.5 S.

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* A method very often practiſed at ſea, eſpecially by coaſters, is to correct the obſerved altitude, by adding 12′; and from thence and the declination, the latitude is to be found. Or, ſubtract the declination from 89° 48′; the remainder will be the latitude. Theſe methods may no doubt give the latitude tolerably exact in ſome caſes; but in others, the mariner will be deceived above half a degree; and in any caſe they are ſeldom free of error.

Remark. The dip of the horizon, in table III. anſwers to a free or unobſtructed horizon; but if the land intervenes, and the ſhip at no great diſtance therefrom, the dip will be conſiderably greater, and that in proportion to the nearneſs of the ſhip to the land. This dip may be found as follows:

Let two perſons obſerve the Sun's altitude at the ſame inſtant, the one being as near the maſt head as poſſible, and the other on deck immediately under. Then, to the log. of the ſum of the heights of the obſervers above the ſea, add the ar-co. of the log. of their difference, and the log. ſine of the difference of altitude; the ſum will be the log. ſine of an arch. Now, half the ſum of this arch, and the difference of altitude, will be the dip anſwering to the greateſt height; and half their difference will be that correſponding to the height of the loweſt obſerver.

This remark, in a ſimilar caſe, is to be applied to the following problems.

Example.

Being cloſe in with the land, September 28, 1793, a perſon on deck, 16 feet above the water, obſerved the meridian altitude of the Sun's lower limb to be 29° 55′ S. and another obſerver on the croſs trees, 68 feet above the ſea, found the altitude at the ſame inſtant to be 30° 8′ S. Required the latitude?

Height	68 feet.		
—	16		
Sum	84	log.	1.92428
Difference	52	ar-co-log.	8.28400
Differ. of alt.	0° 13′	ſine	7.57767
Arch	0 21	ſine	7.78595
Sum	0 34	Half = 17′ = dip to greateſt altitude.	
Difference	0 8	Half = 4 = dip to leaſt altitude.	

Altitude	30° 8′ S.
Sun's ſemidiameter	+ 16
Dip	− 17
Refraction	− 2
True altitude	30 5 S.
Zenith diſtance	59 55 N.
Declination	2 17 S.
Latitude	57 38 N.

Prob.

PROBLEM II.

Given the Meridian Altitude of a Fixed Star, to find the Latitude of the Place of Obſervation.

RULE.

Reduce the obſerved to the true altitude, by prob. IX. page 77, and find the ſtar's zenith diſtance. Take the declination of the ſtar from table XLVI, and reduce it to the time of obſervation. Now, the ſum or difference of the zenith diſtance and declination, according as they are of the ſame, or of a contrary name, will be the latitude of the place of obſervation.

EXAMPLE.

December 1, 1793, the meridian altitude of Sirius was 59° 50′ S. height of the eye 14 feet. Required the latitude?

Obſerved altitude Sirius	59° 50′S.
Dip and refraction	— 4
True altitude	59 46 S.
Zenith diſtance	30 14 N.
Declination of Sirius	16 27 S.
Latitude	13 47N.

PROBLEM III.

Given the Meridian Altitude of a Planet, to find the Latitude of the Place of Obſervation.

RULE.

Compute the true altitude of the planet, as directed in laſt problem.* Take its declination from the Nautical Almanac, and reduce it to the time of obſervation. Then the ſum or difference of the zenith diſtance and declination of the planet will be the latitude as before.

EXAMPLE.

December 10, 1792, the meridian altitude of Saturn was 68° 42′ N. and height of the eye 15 feet. Required the latitude?

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* Being ſufficiently accurate for correcting altitudes obſerved at ſea.

Obſerved altitude of Saturn —	68° 42′ N.
Dip and refraction —	— 4
True altitude — —	68 38 N.
Zenith diſtance — —	21 22 S.
Declination — — —	7 26 N.
Latitude — — —	13 56 S.

PROBLEM IV.

Given the Meridian Altitude of the Moon, to find the Latitude of the Place of Obſervation.

RULE.

Take the number from table XX. anſwering to the given longitude, and daily variation of the moon's paſſing the meridian; which, being applied to the time of tranſit at Greenwich, by addition or ſubſtraction, according as the longitude is weſt or eaſt, will give the time of paſſage over the meridian of the ſhip.

Reduce this time to the meridian of Greenwich, and find the moon's declination anſwering thereto, by prob. VII. page 76, and the horizontal parallax and ſemidiameter, by prob. VIII.

Correct the obſerved altitude of the moon's limb, by prob. XI. page 79. Hence, the moon's zenith diſtance will be known; the ſum or difference of which, and the declination, will be the latitude of the place, as before.

EXAMPLE.

December 24, 1792, the meridian altitude of the moon's lower limb was 81° 15′ N. height of the eye 16 feet. Required the latitude?

T. paſ. over mer. Green. =	9h. 19′
Equation, table XX.	+ 4
T. paſ. over mer. ſhip —	9 23
Longitude in time —	2
Reduced time — —	11 23

☽'s dec. at midn. =	14° 53′ N.
Eq. to time from mid.	— 4
Reduced dec. —	14 49 N.
Moon's parallax —	55′ 25″
Moon's ſemidiameter	15 6
Augmentation —	+ 14
Aug. ſemidiameter	15 20

Obſerved

Obſerved alt. moon's lower limb =	81° 15'.0 N.
Augmented ſemidiameter	+ 15.3
Dip — —	— 3.8
Apparent alt. moon's center —	81 26.5 N.
Correction, table IX. — —	+ 8.1
True altitude — — —	81 34.6 N.
Zenith diſtance — —	8 25.4 S.
Declination — — —	14 49.0 N.
Latitude — — —	6 23.6 N.

Remark. If the object, at the time of obſervation, is in the oppoſite meridian, then the ſum of the true altitude, and the complement of the declination, will be the latitude. If the altitude is negative, it muſt be ſubſtracted from the polar diſtance, in order to obtain the latitude.

EXAMPLES.

I. July 1, 1792, in longitude 15° W. the meridian altitude of the ſun's lower limb was 8° 58', height of the eye 18 feet. Required the latitude?

Sun's dec. at noon =	23° 4'.3 N.	Ob. alt. ☉'s l. lim. =	8° 58'.0
Eq. table XIII. to 15° —	.2	Semidiameter —	+ 15.8
— — to 12h.	— 2.2	Dip and refraction —	9.8
Reduced dec. —	23 1.9	True alt. — —	9 4.0
Polar diſtance —	66 58.1	— — — —	66 58.1
		Latitude — —	76 2.1 N.

II. May 18, 1795, in longitude 20° E. at midnight, the ſun's lower limb was obſerved to be in contact with the horizon, height of the eye 12 feet. Required the latitude?

Sun's dec. at noon =	19°36'	Obſerved alt. —	0° 0'
Eq. table XIII. to 20° E. —	1	Semidiameter — —	+ 16
— — to 12h.	+ 6	Dip and refraction	— 36
Reduced declination	19 41	Depreſſion —	— 20
Polar diſtance — —	70 19	— — —	70 19
		Latitude — —	69 59 N.

PROBLEM V.

Given equal Altitudes of the Sun obſerved the ſame Day, with the Interval of Time between the Obſervations, to find the Latitude ?

METHOD FIRST,

Without a ſuppoſed Latitude.

RULE.

To the co-tangent of the declination, add the co-ſine of half the interval in degrees, the ſum will be the co-tangent of *arch firſt.*

To the co-ſecant of the declination, add the ſine of the altitude, and the ſine of arch firſt, the ſum will be the co-ſine of *arch ſecond.*

If the latitude and declination are of the ſame name, the ſum of arches firſt and ſecond will be the latitude; but if of contrary names, their difference will be the latitude.

EXAMPLE.

In north latitude, at 11h. 10′, and at 12h. 40′ per watch, the altitude of the ſun's limb was the ſame; which being corrected, was 26° 55′, and the ſun's declination 5° 17′ S. Required the latitnde ?

Declination - -	5° 17′	co-tan. 1.03398	—	co-ſecant	1.03583
Halfinter. 45′=	11 15	co-ſine 9.99157	alt. 26° 55′.	ſine	9.65580
Arch firſt -	5 23	co-tan. 1.02555	— —	ſine	8.97229
Arch ſecond	62 32	— — —	coſine	— —	9.66392
Latitude -	57 9 N.				

METHOD SECOND.

The Latitude by Account being given.

RULE.

Take the log. anſwering to half the obſerved interval from col. of riſing, to which add the log. co-ſines of the declination, and latitude by account. Find the natural number anſwering to the above ſum, which being added to the natural ſine of the corrected altitude, will give the natural co-ſine of the meridian zenith diſtance; and hence the latitude is to be found as formerly.

EXAMPLE.

July 1, 1794, in latitude 42° 50′ N. by account, at 11h. 30 A. M.

A. M. per watch, the altitude of the Sun's lower limb was 69° 3′, and at 12h. 31′, the Sun had the ſame altitude; height of the eye 24 feet. Required the latitude?

Time per watch —	11h.	30′	Obſerved altitude —	69° 3′
— — —	12	31	Semid. — dip and refr. - +	11
Elapſed time —	1	1	True altitude —	69 14
Half elapſed time —	0	30 30″	— riſing —	2.94656
Declination — —	23	7	— — co-ſine —	9.96365
Latitude by account —	42	50	— — co-ſine —	9.86530
Altitude — —	69	14	Nat. ſine = 93503	
			Nat. num. 596 —	2.77551
Meridian zenith diſtance	19	47	Nat. coſine 94099	
Declination —	23	7		
Latitude —	42	54	N.	

If the ſhip makes any conſiderable run in the interval between the obſervations, the change of altitude in conſequence of that run is to be eſtimated, and the index of the quadrant altered accordingly. The obſerved interval is alſo to be corrected by the change of longitude. See prob. VIII. The half interval ſhould alſo be corrected by the equation of equal altitudes, page 81.

PROBLEM VI.

Given two Altitudes of the Sun, and the Apparent Times of Obſervation, to find the Latitude and Declination.

RULE.

To the ar-co. of the log. of the difference of the natural verſed ſines of the times from noon reduced to degrees, add the log. of the verſed ſine of the greateſt interval, and the log. of the difference of the natural ſines of the corrected altitudes, the ſum will be the log. of arch firſt.

To the conſtant log. 5.30103, add the two laſt mentioned logs. the ſum will be the log. of arch ſecond.

The ſum of arch firſt, and the natural ſine of the leaſt altitude, will be the natural co-ſine of arch third ; and this natural co-ſine being ſubtracted from arch ſecond, leaves the natural co-ſine of arch fourth.

Now, half the ſum and half the difference of arches third and fourth, will be the latitude and declination reſpectively.

Remark. The apparent time of each obſervation may be found, by taking equal altitudes, and correcting the half interval, by the equation, anſwering thereto.

Example.

At 9h. 23′ 20″, A. M. apparent time, the true altitude of the ſun's center was 34° 29′: and at 11h. 9′ 32″ the altitude was 42° 19′. Required the latitude ?

T.fr.n.	39° 10′	N.V.S.	22469	log. =	4.35158	con. log.	5.30103
—	12 37	—	02415				
		Differ.	20054	ar.co.log.	5.69780	- -	5.69780
Alt. -	42 19	N. ſine	67323				
—	34 29	—	56617				
		Differ.	10706	log. -	4.02963	- - -	4.02963
Arc 1ſt	—	—	11995	— —	4.07901	ar. 2d. 106772	5.02846
Leaſt alt.	34 29	N. ſine	56617				
Arc.3d	46 40	N.co-ſ.	68612	—	— —	68612	
Arc.4th	67 34	Natural coſine		—	—	38160	

Sum 114 14 half = 57° 7′ N. = latitude.
Dif. 20 54 half = 10 27 N. = declination.

Suppoſe the above obſervation to have been made ſome time between June and December, then the time, anſwering to this declination, is Auguſt 25th.

Problem VII.

Given the Latitude by Account, the Declination, and two obſerved Altitudes of the Sun, and the Time elapſed between them; to find the true Latitude.

Rule.

To the log. ſecant of the latitude by account, add the log. ſecant of the declination; the ſum, rejecting 20 from the index, is the *log. ratio.* To this add the log. of the difference of the natural ſines of the two altitudes, and the logarithm of the half elapſed time from its proper column.

Find this ſum in column of middle time, and take out the correſponding time, the difference between which and the half elapſed time will be the time from noon, when the greateſt altitude was obſerved.*

Take the log. anſwering to this time from column of riſing, from which ſubtract the log. ratio; the remainder is the logarithm of a natural number—which being added to the natural ſine of the greater altitude, the ſum is the natural co-ſine of the meridian zenith diſtance; from which, and the Sun's declination, the latitude is obtained as formerly.†

If the latitude thus found differs conſiderably from that by account, the operation is to be repeated, uſing the computed latitude in place of that by account, until the latitude laſt found agrees nearly with that uſed in the computation.

EXAMPLES.

I. June 4, 1795, in latitude by account 37° N. at 10h. 29′ forenoon per watch, the corrected altitude of the Sun was 65° 24′, and at 12h. 31′, the altitude was 74° 8′. Required the true latitude?

T. p. watch	Alt. ⊙'s cent.	Nat. ſine	Lat. by ac. 37° 0′ ſec.	0.09765
10h. 29′	65° 24′	90924	Declinat. 22 28 ſec.	0.03428
12 31	74 8	96190		
			— Logarithm ratio —	0.13193
2 2	Difference	5266	— log. —	3.72148
1 1	— —	Half elapſed time	—	0.57999
31 10	—	Middle time	— —	4.43340
29 50	— —	Riſing	— —	2.92740
			Log. ratio —	0.13193
Natural number	— —	624	—	2.79547
Greateſt altitude	74 8	N. ſine 96190		
Mer. zenith diſt.	14 30	N. co-ſ. 96814		
Declination —	22 28			
Latitude —	36 58 N.			

II.

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* The ſum of the half elapſed time and middle time may be taken. In this caſe, the natural number is to be added to the natural ſine of the leaſt altitude, to find the natural co-ſine of the meridian zenith diſtance.

† This method is only an approximation, and requires to be uſed under certain reſtrictions namely:

The obſervations muſt be taken between nine o'clock in the forenoon, and three in the afternoon. If both obſervations be in the forenoon, or both in the afternoon, the interval muſt not be leſs than the diſtance of the obſervation of the greateſt altitude from noon. If one obſervation be in the forenoon, and the other in the afternoon, the

II. October 17, 1792, in latitude 49° 48′ N. by account, at 0h. 32′ per watch, the altitude of the Sun's lower limb was 28° 32′, and at 2h. 41′ it was 19° 25′, height of the eye 12 feet. Required the true latitude?

Firſt obſerved altitude	28° 32′	Second altitude - -	19° 25′
Semidiameter — —	+ 16	Semidiameter - -	+ 16
Dip and refraction —	— 5	Dip and refraction -	— 6
True altitude — —	28 43	True altitude - -	19 35

T.p.watch	Alt. ⊙'s cent.	Nat. ſine	Lat. by ac. 49° 48′ ſec.	0.19013
0h. 32′ -	28° 43′ -	48048	Declination 9 37 ſec.	0.00615
2 41 -	19 35 -	33518		
			Log. ratio —	0.19628
2 9 —	Diff.	14530	Log. — —	4.16227
1 4 30 —			Half elapſed time —	0.55637
1 37 0 —			Middle time — —	4.91492
32 30 —			Riſing — —	3.00164
			Log. ratio -	0.19628
Natural number — —			639 — —	2.80536

Greateſt altitude	38 43	Nat. ſine	48048
Mer. zenith diſt.	60 52	Nat. co-ſ.	48687
Declination	9 37		
Latitude —	51 15		

As the latitude, by computation, differs 1° 27′ from that by account, the operation muſt be repeated.

Computed latitude	51° 15′	ſecant	0.20348
Declination —	9 37	ſecant	0.00615
Logarithm ratio — — —			0.20963

Log.

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the interval muſt not exceed four hours and a half; and in all caſes, the nearer the greater altitude is to noon the better.

If the Sun's meridian zenith diſtance be leſs than the latitude, the limitations are ſtill more contracted. If the latitude be double the meridian zenith diſtance, the obſervations muſt be taken between half paſt nine in the forenoon, and half paſt two in the afternoon, and the interval muſt not exceed three hours and a half. The obſervations muſt be taken ſtill nearer to noon, if the latitude exceeds the meridian zenith diſtance in a greater proportion. See Req. Tab. 2d ed. and Brit. Mar. Guide.

Logarithm ratio	—	—	—	0.20963
Difference nat. ſines	—	14530	log.	4.16227
Half elapſed time		1h. 4′ 30″	log.	0.55637
Middle time	—	1 40 20	log.	4.92827
Riſing	— —	35 50	log.	3.08630
Logarithm ratio	—	—	—	0.20963
Natural number	—	753	—	2.87667

Greateſt alt. 28° 43 N. ſine 48048

Mer. z. diſt. 60 47 N. coſ. 48801
Declination 9 37

Latitude - 51 10 N.

As this latitude differs only 5′ from that uſed in the computation, it may therefore be depended on as the true latitude.

Problem VIII.

Given the Latitude by account, the Sun's Declination, two obſerved Altitudes, the elapſed Time, and the Courſe and Diſtance run between the Obſervations, to find the Ship's Latitude at the Time of Obſervation of the greater Altitude.

Rule.

Find the angle contained between the ſhip's courſe and the Sun's bearing, at the time of obſervation of the leaſt altitude, with which enter a traverſe table; and the difference of latitude, anſwering to the diſtance made good, will be the *reduction* of altitude.

Now, if the leaſt altitude be obſerved in the forenoon, the reduction of altitude is to be applied thereto by addition or ſubtraction, according as the angle between the ſhip's courſe and Sun's bearing is leſs or more than eight points. If the leaſt altitude be obſerved in the afternoon, the contrary rule is to be applied.

The difference of longitude in time between the obſervations is alſo to be applied to the elapſed time, by addition or ſubtraction, according as it eaſt or weſt. This is, however, in many caſes ſo inconſiderable as to be neglected.

With the corrected altitudes and interval, the lat. by D. R. and declination at the time of obſervation of the greateſt altitude, the computation is to be performed by the laſt problem.

Examples.

I. July 6, 1793, in latitude 58° 14′ N. by D. R. at 10h. 54′ A.

M. per watch, the altitude of the Sun's lower limb was 53° 17′; and at 1h. 17′, the altitude was 52° 51′, and bearing per compaſs S. W. by W. The ſhip's courſe during the elapſed time was S. by W. ½ W, and hourly rate of ſailing 8 knots, the height of the eye 16 feet. Required the true latitude at the time of obſervation of the greater altitude.

☉'s bearing at 2d obſ. S. W. by W.	Interval betw. obſerv. 2h. 23′
Ship's courſe — S. by W. ½ W.	Diſtance run = 2 23 × 8 = 19 m.
Contained angle — 3½ points.	

Now, to courſe 3½ points and diſtance 19 miles, the difference of latitude is 14.7 or 15 miles.

Firſt obſerved alt. =	53° 17′		Second obſer. altitude	52° 51′	
Semidiameter —	+ 16		Semidiameter —	+ 16	
Dip and refraction - —	4		Dip and refraction - —	4	
			Reduction —	— 15	
True altitude —	53 29				
			Reduced altitude —	52 48	

Time per watch | Altitude | Nat. ſine | Lat. by ac. 58° 14′ ſec. 0.27863
10h. 54′ — 53° 29′ - 80368 Declinat. 22 40 ſec. 0.03491
1 17 — 52 48 - 79653
Logarithm ratio — 0.31354
2 23* Difference - 715 — — log. — 2.85431
1 11 30 — — Half elapſed time — 0.51294

5 30 — — Middle time — — 3.68079

1 6 0 — — Riſing — — 3.61469
Log. ratio — — 0.31354

Natural number — — — 2001 — 3.30115
Greateſt altitude — 53 29 Nat. ſine 80368

Mer. zenith diſtance 34 33 N: co-ſine 82369
Declination — 22 40

Latitude — 57 13

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* The correction of elapſed time is here neglected. It however may be found thus. The variation inferred from the obſervations is about 2¼ points W. Hence the true courſe is S ¾ E, to which and diſtance 19 m. the departure is 2′.8, and to latitude 57°, and 2.′8 in a lat. column, the diſtance is 5′, which reduced to time is 20″ additive, becauſe the D. Long. is E.

Since the computed latitude differs ſo much from that by account, it will be neceſſary to repeat the operation.

Computed latitude	57° 13′	ſecant	0.26643
Declination	22 40	ſecant	0.03491
Logarithm ratio			0.30134
Difference natural ſines	715	log.	2.85431
Half elapſed time	1 11 30	log.	0.51294
Middle time	5 20	log.	3.66859
Riſing	1 6 10	log.	3.61686
Logarithm ratio			0.30134
Natural number	2068		3.31552
Greateſt altitude	53 29	N. ſine 80368	
Mer. zenith diſt.	34 29	N. coſ. 82436	
Declination	22 40		
Latitude	57 9		

As this latitude differs only 4 miles from that uſed in the computation, it may therefore be depended on as the true latitude.

II. September 13, 1793, in latitude 38° 12′ N. by account, and longitude 27° W. at 9h. 28′ A. M. per watch, the altitude of the Sun's lower limb was 40° 42′, and azimuth per compaſs S.E.¼S. at 11h. 16′ A. M. the altitude was 53° 11′. The ſhip's courſe during the elapſed time was W.½N. at the rate of 9 knots per hour, and the height of the eye 12 feet. Required the ſhip's true latitude at the time of the ſecond obſervation?

☉'s bearing at 1ſt obſ.	S.E.¼E.	Reduced declination	3° 34′ N.
Ship's courſe	W.½N.	Elapſed time	1h. 48′
		Diſtance run = 1h. 48′×9 = 16 m.	
Contained angle	11¾ points.		
Supplement	4¼ points.		

To courſe 4¼ points and diſtance run 16 miles, the difference of latitude is 10′.7, or 11 miles.

Firſt obſerved altitude	40° 42′	Second obſerved altitude	53° 11′
Sun's ſemidiameter	+ 16	Semidiameter	+ 16
Dip and refraction	− 4	Dip and refraction	− 4
Reduction of altitude	− 11		
		Corrected altitude =	53 23
Reduced altitude =	40 43		

Time per watch | Altitude | Nat. ſine | Lat. by ac. 38° 12′ ſec. 0.10466
9h. 28′ — 40° 43′ - 65232 Declination 3 34 ſec. 0.00084
11 16 — 53 23 - 80264
Logarithm ratio — 0.10550
1 48 Difference - 15032 — log. — 4.17702
0 54 — — Half elapſed time — 0.63181

1 37 — — Middle time — — 4.91433

43 — — Riſing — — 3.24427

Natural number — 1376 — 3.13877

Mer. zen. diſt. 35 16 N. coſ. 81649
Declination - 3 34 — — ſecant — 0.00084

Latitnde — 38 50 — — ſecant — 0.10848

Logarithm ratio — — — — — 0.10932
Difference natural ſines — 15032 log. — 4.17702
Half elapſed time — — 0 54 0 log. — 0.63181

Middle time — — 1 37 50 log. — 4.91815

Riſing — — — 0 43 10 log. — 3.26089

Natural number — — 1418 — 3.15157

Mer. zen. diſt. 35 14 Nat. co-ſine 81682
Declination - 3 34

Latitude — 38 48 N.

PROBLEM IX.

To find the Latitude by double Altitudes, and the elapſed Time, by means of a Table of Log. Sines and Secants only.

RULE.

To the log. co-ſine of half the ſum of the true altitudes, add the log. ſine of half their difference, the log. ſecant of the declination, the log. ſecant of the latitude by account, and the log. co-ſecant of half the interval of time between the obſervations reduced to degrees; the ſum will be the log. ſine of arc firſt.

To the log. co-ſines of the declination and latitude, add the conſtant log. 0.30103, twice the log. ſine of half the difference between the half interval and arc firſt, and the log. ſecant of the eſtimate meridian altitude; the ſum will be the log. ſine of arc ſecond, which being added to the greateſt altitude, will give the meridian altitude, and hence the latitude is found as uſual.

EXAMPLE.

In latitude 57° N. by account, at 10h. 51′ A. M. the corrected altitude of the Sun's center was 25° 18′, and at 1h. 0′ P. M. the altitude was 26° 55′, the Sun's declination being 5° 17′ S. Required the latitude?

Greateſt alt. 26° 55′ Inter. $= 2\text{h}.\,9' \,\&\, \frac{2\text{h}.\ 9'}{8} = 16^\circ\, 7\tfrac{1}{2}' =$ half inter. in degrees.

Leaſt alt. - 25 18

Sum —	52 13	half = 26 6½	co-ſine	9.95326		
Difference -	1 37	half = 48½	ſine -	8.14945		
Declination	5 17	—	ſecant —	0.00185	co-ſine	9.99815
Latitude -	57 0	—	ſecant —	0.26389	co-ſine	9.73611
Half interv.	16 7½	—	co-ſecant -	0.55637	con. log.	0.30103
Arc firſt -	4 49½	—	ſine —	8.92482		
Difference	11 18	—	Eſt. mer. alt. 27° 43′		ſecant -	0.05293
Half —	5 39	—	twice ſine = 8.99322 × 2		=	8.98644

Arc ſecond — 0° 41′ — ſine — 8.07466
Greateſt altitude 26 55

Meridian alt. - 27 36
Zenith diſtance 62 24
Declination - - 5 17

Latitude — 57 7 N.

PROBLEM X.

To find the Latitude by Obſervations taken near the Meridian and Prime Vertical.

RULE.

When the Sun is near the eaſt or weſt points, let its altitude be obſerved, from which, the eſtimated latitude and declination, compute the Sun's diſtance from the meridian, by prob. VI. page 91; which, being applied to the time of obſervation per watch, will give the time per watch when the Sun is in the meridian.

Now obſerve the Sun's altitude when near the meridian; and time the difference between the time per watch of obſervation, and the time of noon before found, will be the Sun's diſtance from the meridian at the time of obſervation—with which, and the altitude and declination, compute the meridian altitude by prob. v. and hence the true latitude will be known.

Allowance is to be made for the change of longitude in the interval between the obſervations.

EXAMPLE.

April 28, 1794, in latitude 68° N. by account, and longitude 15° E. at 6h. 45′ 34″ the correct altitude of the Sun's center was 17° 58′, and at 11h. 42′ 18″, the true altitude was 36° 2′. Required the true latitude?

Declination at time of 1ſt obſ.= 14° 12′ N. at time of 2d = 14° 16′ N.

Altitude —	17° 58′		
Polar diſtance —	75 48	— — co-ſecant -	0.01348
Latitude — —	68 0	— — ſecant —	0.42642
Sum — —	161 46		
Half — —	80 53	— — co-ſine - -	9.20768
Difference — —	62 55	— — ſine —	9.94956
			19.59714
	38 58	— — ſine —	9.79857
	8		
Time from noon —	5 11 44		
Time p. watch of obſer.	6 45 34		
Time p. watch of noon	11 57 18	Lat. 68° 0′ co-ſine	9.57358
Time of ſecond obſer.	11 42 18	Dec. 14 16 co-ſine	9.98640
Difference —	15 0	— — riſing -	2.33063
Natural number —	—	— 78 —	1.89061
Greateſt altitude —	36 3	Nat. ſine 58826	
Mer. zenith diſtance -	53.55	N. co-ſine 58904	
Declination —	14 16		
Latitude —	68 11 N.		

If the latitude by account be aſſumed 69°, the latitude by computation will be 68° 12′, differing only one mile from the former computation.

PROB.

PROBLEM XI.

Given three Altitudes of the Sun, with the Intervals of Time between them, to find the Latitude of the Place of Obſervation, and the Sun's Declination.

RULE.

Find the apparent time of each obſervation, by prob. xv. p. 107 Then, with any two of the altitudes and the correſponding times from noon, compute the latitude and declination, by prob. vi. p. 239.

EXAMPLE.

Let the three altitudes be 57° 23′, 55° 0′, and 49° 55′; the interval of time between the two firſt 40′, and that between the two laſt 50′. The latitude and declination are required; and ſuppoſe theſe obſervations to have been taken in 1794, ſome time between June and December, the day of obſervation is ſought?

Firſt interval 40′ = 10° half = 5°. 0′ = arc 1ſt } Sum = 11° 15′ = arc 3d.
Second inter. 50 = 12 30 h. = 6 15 = arc 2d }
Altitude — 57° 23′ Nat. ſine 84230
55 0 — 81915 diff. 2315 = arc 5th.
49 55 — 76511 - 7719 = arc 4th.
Arc iv. = 7719 ar-co-log. 6.11244 Arc i. = 5° 0′ ſine - 8.94030 ſine - 8.94030
Arc v. = 23 15 - log. - 3.36455 Arc ii. = 6 15 - coſ. - 9.99741 ſine - 9.03690

Arc iii. = 11 15 - ſine - 9.29024 Arc vii. .08664 - log. - 8.93771

Arc vi. — — 8.76723 — .05851

Difference - - - - - - - - .02813 - - - ar-co-log. - 1.55083

Arc viii. - - - - - - - - - 18° 38′ - - - tangent - 9.52803
Arc iii. - - - - - - - - - 11 15

Time from noon when greateſt altitude was obſerved - - - - } 7 23
Hence time of obſerv. of middle alt. - - 17 23

Time of obſerv. 17 23 N. V. S. - 04567 - - - log. - 3.65966 - conſt. log. - 5.30103
7 23 - - - 00829

Difference - - - - - - - - 03738 ar-co-log. - 6.42736 - - - - 6.42736
Diff. natural ſines altitudes - - 02315 - log. - 3.36455 - - - - 3.36455

2829 - - - 3.45157 - 1.23863 - 5.09294
Leaſt altitude - 55 0 N. ſine - 81915

32 4 N. co-ſine 84744 - - - - - - - 84744

66 58 - - - - N. co-ſine - - - - - 39119

Sum - - - 99 2 - half = 49 31 = latitude N.
Difference - - 34 54 - half = 17 27 = declination N. which anſwers to Aug. 3d.

PROBLEM XII.

Given the Sun's Declination and Semidiameter, and the Interval of Time between the Inſtants when the upper and lower Limbs of the Sun were in the Horizon—to find the Latitude of the Place of Obſervation.

RULE.

To the log. of the Sun's ſemidiameter expreſſed in ſeconds, add the ar-co-log. of the interval of time in ſeconds, and the conſtant log. 9.12494; the ſum will be the log. co-ſine of an arc.

To the log. ſine of the ſum of this arc and the Sun's declination, add the log. ſine of their difference; half the ſum will be the log. ſine of the latitude.

EXAMPLES.

I. Let the Sun's declination be 18° 24′, ſemidiameter 15′ 52″, and time required to riſe 4′ 48″. Required the latitude?

Sun's ſemidiameter	= 15′ 52″ = 952″	log.	2.97864
Interval of time	4 48 = 288	ar-co-log.	7.54061
Conſtant log.			9.12494
Arc	63°51′	co-ſine	9.64419
Sun's declination	18 24		
Sum	82 15	ſine	9.99601
Difference	45 27	ſine	9.85287
			19.84888
Latitude	57 10	ſine	9.92444

II. June 21, 1796, the interval of time between the ſetting of the lower and upper limbs of the Sun was 3′ 56″. Required the latitude?

Sun's ſemidiameter	= 15′ 47″ = 947″	log.	2.97635
Interval of time	3 56 = 236	ar-co-log.	7.62709
Conſtant log.			9.12494
Arch	57°39′	co-ſine	9.72838
Sun's declination	23 28		
Sum	81 7	ſine	9.99476
Difference	34 11	ſine	9.74961
			19.74437
Latitude	48 10	ſine	9.87218

CHAP.

CHAP. II.

Of finding the Variation of the Compaſs.

INTRODUCTION.

THE attractive power of the magnet was known in Europe, about 600 years before the Chriſtian æra; and by the Chineſe records it is ſaid, that its directive property was known in that country at leaſt 1000 years earlier. The invention of the compaſs is by ſome aſcribed to John or Flavia Goya of Amalphi in Naples, about the year 1302. It, however, appears from ſome verſes,* in a French tract, intitled, *La Bible de Guyot de Provins*,† to have been known in that country previous to the year 1180.

Until the time of Columbus, the direction of the needle was ſuppoſed to be exactly in the plane of the meridian, and therefore the other points were ſuppoſed to agree with the correſpondent points of the horizon. However, in the month of September 1492, Columbus firſt diſcovered the variation of the needle. This diſcovery is alſo ſaid to have been made by Sebaſtian Cabot in the year 1500.

Soon after, the variation was found to be different at different places. It was however affirmed to be conſtant at the ſame place; but obſervation alſo overturned this hypotheſis. In 1635, Mr Henry Gellibrand publiſhed his diſcovery of the change of the variation, from a compariſon of his own obſervations with thoſe of his predeceſſors.

The magnetic needle is ſubject both to an annual, and to a diurnal vibratory motion. In the firſt of theſe, the motion of the north end of the needle is in general towards the eaſt, from the time of vernal equinox to the ſummer ſolſtice; and during the other nine months, its motion is in general towards the weſt. In the ſecond, the needle is ſtationary from noon till about 3h. P. M. and from thence till 8 in the evening, it ſlowly approaches the eaſt. It again continues ſtationary till about 8 in the morning; and from that time till noon, it gradually approaches the weſt.

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* Icelle leſtoille ne ſe muet,
Un arts ſont qui mentir ne puet,
Par la vertue de la manete,
Un pierre laide et brunette,
Ou li fers volontiers ſe jointe,'
Ont, regardant l'or droit pointe,
Puez l'une aguile l'ont touchie,
Et en un feſtu l'ont fichie,
En langue la mette ſens plus,
Et li feſtue la tient deſus;
Puis ſe torne la pointe toute,
Contre l'eſtoille, ſans doute;
Quant li nuis eſt tenebre et brune,
C'on ne voit eſtoille ni lune,
L'or ſont a l'aguille alumer,
Puez ne puent ils aſſarrer
Contre leſtoille, vers la pointe;
Per ce ſont le marinier cointe
De la droite voir tenir;
C'eſt un art qui ne peut mentir.

Catalogue de Duc de la Valliere, N°. 2707.

The magnetic needle has another property, called the Inclination or Dip. Thus, let a needle before it is rendered magnetical be well balanced; then give it magnetiſm, and the needle being ſuſpended on ſame point as before, will now be found to have loſt its equilibrium; and the angle contained between the direction of the needle, and an horizontal line is called the Dip. This diſcovery was made by chance, in the year 1576, by Mr Robert Norman, a compaſs maker at Wapping, London.

Definitions.

The Variation of the Compaſs, is the deviation of the points of the mariner's compaſs, from the correſponding points of the horizon, and is denominated Eaſt or Weſt Variation.

East Variation is when the north point of the compaſs is to the eaſt of the true north point of the horizon.

West Variation is when the north point of the compaſs is to the weſt of the true north point of the horizon.

The True Amplitude[*] of any celeſtial object, is an arch of the horizon, contained between the north or ſouth points thereof, and the object's center at the time of its riſing or ſetting.

The Magnetic Amplitude, is the arch contained between the object's center when in the horizon, and the magnetic meridian—or, in other words, it is the bearing of the object per compaſs, when in the horizon.

The True Azimuth of any object, is the angle contained between the true meridian, and the vertical paſſing through the object's center.

The Magnetic Azimuth, is the angle contained between the magnetic meridian, and the azimuth circle paſſing through the center of the object.

Use of the Azimuth Compass.[†]

I. *To obſerve the Sun's Amplitude.*

Turn round the compaſs, until the horizontal bar be directed to the Sun. Apply the eye to the coloured glaſs in the ſight vane, and direct the ſight through the magnifying glaſs in the other vane; now move the box, and either glaſs, till the Sun is ſeen thro' both; then ſtop the card by the ſlide; read off the diviſions ſhown by the vernier, applying the eye to the top of the neareſt vane, and the Sun's magnetic amplitude will be obtained.

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[*] The amplitude is commonly reckoned from the eaſt or weſt points. It, however, ſeems more proper to reckon it from the meridian.

[†] For a deſcription of this compaſs, ſee M'Culloch's Report.

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II. *To obſerve the Sun's Azimuth.*

If the Sun is near the horizon, the azimuth is obſerved in the ſame manner as the amplitude; but if the Sun be conſiderably elevated, turn round the compaſs box till the ſhadow of the line joining the tops of the vanes coincides with the middle line on the horizontal bar; the card is then to be ſtopped by the ſlide, and the diviſion ſhown by the vernier will be the Sun's magnetic azimuth.

PROBLEM I.

Given the Latitude of a Place, and the Sun's Magnetic Amplitude, to find the Variation of the Compaſs.

RULE.

To the log. ſecant of the latitude, add the log. ſine of the Sun's declination; the ſum will be the log. co-ſine of the true amplitude, to be reckoned from the north or ſouth, according as the declination is north or ſouth.

The difference between the true and obſerved amplitudes, reckoned from the ſame point, and when of the ſame name, is the variation; but if of a different denomination, their ſum is the variation.

If the obſervation be made in the eaſtern hemiſphere, the variation will be eaſt or weſt, according as the obſerved amplitude is nearer to, or more remote from the north than the true amplitnde. The contrary rule holds good in obſervations taken in the weſtern hemiſphere.

Or, let the obſerver be ſuppoſed to look directly towards the point repreſenting the true amplitude, then, if the obſerved amplitude is to the left of the true amplitude, the variation is eaſterly—but if to the right, it is weſterly.

EXAMPLES.

I. May 15, 1794, in latitude 33° 10′ N. and longitude 18° W. about 5h. A. M. the Sun was obſerved to riſe E. by N. Required the variation?

Sun's decl. May 15. at noon —	18°58′ N.		
Eq. from tab. XIII. to 7h. from noon —	4		
— — to 18° W. —	+ 1		
Reduced declination —	18 55.	— ſine -	9.51080
Latitude — — —	33 10	— ſecant -	0.07723
True amplitude — —	N. 67 13 E.	— co-ſine -	9.58803
Obſerved amplitude —	N. 78 45 E.		
Variation — —	11 32	Which is weſt, becauſe the	

the magnetic amplitude is more diſtant from the north than the true amplitude, the obſervation being made in the eaſtern hemiſphere.

II. December 20, 1793, in latitude 31° 38′ S. and longitude 83° W. the Sun was obſerved to ſet S. W. Required the variation?

The Sun's declination at the time of obſervation is nearly the ſame as at Greenwich at the noon of the given day, namely 23° 28′ S.

Latitude —	31° 38′	— —	ſecant 0.06985
Declination —	23 28	— —	ſine - 9.60012
True amplitude -	S. 62 7 W.	—	coſine 9.66997
Obſer. amplitude	S. 45 0 W.		
Variation —	17 7		

Which is eaſt, becauſe the obſerved amplitude is to the left of the true amplitude.

If the Sun's amplitude be obſerved at the inſtant its center is in the viſible horizon, an allowance depending on the horizontal refraction, the horizontal parallax, the height of the eye, and the latitude of the place is to be applied to the obſerved amplitude, in order to obtain the amplitude of the object, when in the true horizon. This however may be avoided, by obſerving the amplitude when the altitude of the Sun's lower limb is equal to the ſum of 15′ and the dip of the horizon. Thus, if an obſerver be elevated 17 feet, the amplitude ſhould be obſerved, when the altitude of the Sun's lower limb is 19′.

PROBLEM II.

Given the Magnetic Azimuth, the Altitude, and Declination of the Sun, together with the Latitude of the Place of Obſervation; to find the Variation of the Compaſs.

RULE.

Reduce the Sun's declination to the time and place of obſervation, and compute the true altitude of the Sun's center.

Find the ſum of the Sun's polar diſtance and altitude, and the lat. of the ſhip. Take the difference between this ſum and the polar diſtance.

To the log. ſecant of the altitude, add the log. ſecant of the latitude, the log. co-ſine of the half ſum, and the log. coſine of the difference; half the ſum of theſe will be the log. ſine of the Sun's true azimuth, to be reckoned from the ſouth in north latitude, but from the north in ſouth latitude.

The difference between the true and obſerved azimuths will be the variation as formerly.

EXAMPLES.

I. November 18, 1793, in latitude 50° 22′ N. longitude 24° 30′ W. about three quarters paſt eight A. M. the altitude of the Sun's lower limb was 8° 10′, and bearing per compaſs S. 23° 18′ E. height of the eye 20 feet. Required the variation?

⊙'s dec. 18 Nov. at noon	19° 25′S.	Obſerved altitude l. limb =	8° 10′
Eq. tab. XIII. to 3¼ h. —	2	Semidiameter —	+ 16
— to 24° 30′W. +	1	Dip and refraction -	− 10
Reduced declination - -	19 24	True altitude —	8 16

Polar diſtance —	109 24		
Altitude — —	8 16	— — ſecant —	0.00454
Latitude — —	50 22	— — ſecant —	0.19527
Sum — —	168 2		
Half — —	84 1	— — co-ſine —	9.01803
Difference —	25 23	— — co-ſine —	9.95591
			19.17375
	22 43	— — ſine —	9.58687
	2		
True azimuth —	S. 45 26 E.		
Obſerved azimuth -	S. 23 18 E.		
Variation —	22 8		

Which is weſterly, becauſe the obſerved azimuth is to the right of the true azimuth.

II. January 3, 1794, in latitude 33° 52′ N. longitude 53° 15′ E. about half paſt three, the following alt. and azimuths were obſerved, the height of the eye being 20 feet. Required the variation?

Ob. alt. ⊙'s l. l.	41° 58′	Azim. S.	49° 44′W.	⊙'s dec. at noon	21° 24′S.
— —	41 17	—	50 28	Eq. to t. fm. noon —	2
— —	40 39	—	51 3	Eq. to longitude +	2
	54		75	Reduced decl.	21 24 S.
Mean —	41 18	—	50 25	Polar diſtance	111 24
Semidiameter +	16				
Dip and refr. —	6				
True altitude -	41 28				

Polar

Polar diſtance	111 24		
Altitude	41 28	ſecant	0.12532
Latitude	33 52	ſecant	0.08075
Sum	186 44		
Half	93 22	co-ſine	8.76883
Difference	18 2	co-ſine	9.97558
			18.95048
	17 23	ſine	9.47524
	2		
True azimuth	S. 34 46 W.		
Magnetic azimuth	S. 50 25 W.		
Variation	15 39		

Being weſt, becauſe the obſerved azimuth is to the right of the true azimuth.

It is common for one perſon to obſerve the Sun's altitude at the ſame inſtant that another is taking the azimuth. Theſe obſervations may however be made by the ſame perſon, by taking ſeveral azimuths and altitudes alternately, remembering to end with an obſervation of the ſame kind that was firſt taken.

III. Auguſt 13, 1788, in latitude 57° 7′ N. longitude 2° 7′ W. about half paſt eight in the morning, the following obſervations were taken, the height of the eye being 14 feet. Each quadrant of the card of the compaſs with which the azimuths were obſerved, was divided into 96 equal parts. Required the variation?

Azimuth Sun's center	S. 43°.5 E.	Alt. Sun's low. limb	31°38′
	43.3	— — —	31 47
	43.0	— — —	31 55
	42.5	— — —	32 3
	42.3		
	14.6		203
Mean	42.92	Mean	31 50¾
		Semidiameter	+ 16
Now $\frac{42.92\times 90}{96}=40.237=40°\,14'$		Dip and refraction	− 5¾
		True altitude	32 1

Sun's

Sun's decl. 13th Aug.	14° 26′ N.		
Eq. to reduced time -	+ 3		
Reduced declination	14 29 N.		
Polar diſtance —	75 31		
Altitude —	32 1	— — ſecant —	0.07166
Latitude —	57 7	— — ſecant —	0.26526
Sum —	164 39		
Half — —	82 19½	— — co-ſine —	9.12565
Difference —	6 48½	— — co-ſine —	9.99693
			19.45950
	32 28	— — ſine —	9.72975
	2		
True azimuth —	S. 64 56 E.		
Obſerved azimuth -	S. 40 14 E.		
Variation —	24 42 W.		

Problem III.

To find the Variation of the Compaſs by equal Altitudes of the Sun.

Rule.

Let the Sun's altitude be obſerved in the eaſtern hemiſphere, when its meridian diſtance exceeds two hours, and at the ſame inſtant let another obſerver take the azimuth. Then, when the Sun comes to the ſame altitude in the afternoon, let the bearing of its center be again obſerved. Now, half the difference between the eaſtern and weſtern azimuths will be the variation—which, when the obſervations are made in the ſouthern hemiſphere, will be eaſt, or weſt, according as the eaſtern or weſtern azimuth is greateſt.—The contrary rule is to be applied, when the azimuths are obſerved in the northern ſemicircle.

Remark. The weſtern azimuth will be affected by the change of declination in the interval between the obſervations. This correction may be found thus:

To the log. co-ſine of the latitude, add the log. ſine of the horary angle, and the log. of the change of declination in the elapſed time; the ſum will be the log. of the correction of azimuth—which is additive to the weſtern azimuth, when the object is approaching the elevated pole, but ſubtractive, if receding therefrom.

EXAMPLE.

In the forenoon, the bearing of the Sun's center per compaſs was S. S. E. ¾ E. the altitude of its lower limb being 40° 18′, and in the afternoon, when the Sun had attained the ſame altitude, the azimuth was S. W. by W. ¼ W. Required the variation?

Eaſtern azimuth = S. S. E. ¾ E. = S. 2 ¾ E.
Weſtern azimuth = S. W. by W. ¼ W. = S. 5 ¼ W.

Difference — — — 2 ½
Variation — — — — 1 ¼ W.

PROBLEM IV.

To find the Variation of the Compaſs, by Obſervations of equal Altitudes taken at different Places, the Courſe and hourly Rate of Sailing being given.

RULE.

Find the diſtance run in the elapſed time, and hence the difference of latitude. Correct the obſerved interval by the difference of longitude.

To the log. ſecant of the latitude at noon, add the log. co-tangent of the half elapſed time, and the log. of the difference of latitude; the ſum will be the log. of the correction of azimuth—which is to be applied to the weſtern azimuth, by addition or ſubtraction, according as the ſhip's latitude is decreaſing or increaſing. Apply alſo the change of azimuth ariſing from the change of declination—hence the reduced azimuth will be obtained. Then, half the difference between the eaſtern and weſtern azimuths will be the variation, as formerly.

EXAMPLE.

April 23, 1795, at 8h. 44′ A. M. per watch, the Sun's azimuth per compaſs was S. 59° 4′ E. and at 3h. 20′ P. M. the Sun's azimuth was S. 81° 17′ W. the altitude being the ſame as in the forenoon. The latitude of the ſhip at noon was 41° 48′ N. and courſe per compaſs W. S. W. at the rate of 7 knots an hour. Required the variation?

The interval of time is 6h. 36′, which multiplied by 7 knots, gives 46 miles nearly, the diſtance run. By comparing the obſerved azimuths, it appears the variation is about one point weſt—hence the corrected courſe is S. W. by W. to which, and diſtance 46 miles, the difference of latitude is 25.6, and departure 38.2. Now to latitude 41° 48′ or 42°, and 19′.1 half the departure, the difference of longitude is 26 miles—which ſubtracted from 49° 30′, half the elapſed time in degrees, the remainder 49° 4′ is the reduced half interval. The change of declination in the elapſed time is about 6 minutes.

Latitude -	41°48′	ſecant -	0.12757	— —	coſine	9.87243
Half interval	49 4	co-tang.	9.93814	— —	ſine -	9.87822
Diff. of latitude -	25.6	log. -	1.40824	ch. of dec. = 6′	log.	0.77815
Correction —	29.8	log. -	1.47395	Correction 3.4		0.52880

Weſtern azimuth — —	S. 81° 17′ W.
Firſt correction — — —	+ 29.8
Second correction — —	+ 3.4
Weſtern azimuth corrected —	S. 81 50 W.
Eaſtern azimuth — —	S. 59 4 E.
Difference — — —	22 46
Variation — — — —	11 23 W.

Problem V.

Given the Latitude of a Place, the Sun's Declination and Magnetic Azimuth, and the Apparent Time of Obſervation—to find the Variation of the Compaſs.

Rule.

To the log. co-tangent of the Sun's declination, add the log. coſine of its diſtance from the meridian in degrees; the ſum will be the log. co-tangent of arc firſt—which being ſubtracted from, or added to the latitude of the place of obſervation, according as they are of the ſame, or of a different name, the difference or ſum will be arc ſecond.

Now, to the log. co-tangent of the Sun's diſtance from the meridian in degrees, add the log. ſecant of arc firſt, and the log. ſine of arc ſecond; the ſum will be the log. co-tangent of the true azimuth, from the ſouth in north latitude, otherwiſe from the north. The difference between the true and obſerved azimuths will be the variation.

Example.

Auguſt 18, 1792, in latitude 41° 46′ N. longitude 144° 20′ W. at 9h. 13′ 44″ A. M. apparent time, the magnetic azimuth of the Sun's center was S. 80° 20′ E. Required the variation?

App. time obſ. 9h. 13′ 44″ A. M. ☉'s dec. at noon at Gr. 12° 51′ N.
Equation to 2h. 46′ - + 2
Time fm. noon 2 46 16=41° 34′ — to 144° 20′ W. — 8

Reduced declination - 12 45 N.

Declination — 12° 45′ - co-tang. - 0.64536
Sun's mer. diſt. 41 34 - co-ſine - 9.87401 - co-tangent 0.05217

Arc firſt — 16 50 - co-tang. - 0.51937 - ſecant - - 0.01902
Latitude — 41 46 N.

Difference — 24 56 — — — ſine — 9.62486

True azimuth — — S. 63 38 E. - co-tangent 9.69505
Obſerved azimuth — — S. 80 20 E.

Variation — — — 16 42 E.

The obſerved azimuth being to the left of the true.

PROBLEM VI.

Given the Latitude of a Place, the Sun's Declination and Magnetic Azimuth, and the Interval between that Obſervation and the Time of Sun Riſing or Setting—to find the Variation of the Compaſs.

RULE.

To the log. tangent of the latitude, add the log. tangent of the declination; the ſum will be the log. ſine of an arc, which, when the latitude and declination are of the ſame name, is to be added to 90°, but when of different names, to be ſubtracted therefrom.

From this arc take the obſerved interval of time in degrees; the remainder will be the Sun's diſtance from the meridian—with which, the declination and latitude, the true azimuth is to be found by laſt problem—and hence the variation of the compaſs will be known.

EXAMPLE.

February 16, 1793, in latitude 36° 18′ N. long. 33° 24′ W. the Sun's magnetic azimuth was S. 59° 40′ W. at 3h. 12′ P. M. and at 5h. 26′ the Sun was obſerved to be in the true horizon. Required the variation?

Time of obſ. 3h. 12 Sun's dec. 16, at noon at Green. 12° 4′
5 26 Equation to middle interval - — 3
— to long. 33° 24′ W. — 2
Interval — 2 14=33° 30′

Reduced declination — 11 59

Lati-

Latitude	36°18′			tangent	9.86603
Declination	11 59			tangent	9.32685
	8 58			ſine	9.19288
Subtract from 90					
Remainder	81 2				
Interval	33 30				
Diſt. fm. mer.	47 32	co-ſine	9.82941	co-tang.	9.96155
Declination	11 59	co-tang.	0.67315		
Arc firſt	17 27	co-tang.	0.50256	ſecant	0.02046
Latitude	36 18 N.				
Arc ſecond	53 45			ſine	9.90639
True azimuth	S. 52° 17′W.			co-tang.	9.88840
Magnetic azimuth	S. 59 40 W.				
Variation	7 23 W.				

PROBLEM VII.

Given the Declination and Altitude of the Sun, its Magnetic Amplitude, and the Interval of Time between the Obſervations—to find the Latitude, and Variation of the Compaſs.

RULE.

To the log. ſine of half the elapſed time, add the log. co-ſine of the declination; the ſum will be the log. ſine of the half of arc firſt; and the ſum of the log. ſines of half the elapſed time and declination will be the log. co-tangent of arc ſecond.

To the log. co-ſecant of arc firſt, add the log. ſine of the altitude; the ſum will be the log. ſine of arc third. Subtract the ſum of arcs ſecond and third from 180°, the remainder will be arc fourth—to the log. ſine of which, add the log. co-ſine of the declination; the ſum will be the log. ſine of the latitude. And to the log. coſine of arc fourth, add the log. co-tangent of the declination; the ſum will be the log. tang. of the true amplitude, to be reckoned from the north, when the declination is north, but from the ſouth when the declination is ſouth—and hence the variation will be obtained.

EXAMPLE.

Auguſt 1, 1795, the Sun was obſerved to riſe at 3h. 58′ per watch, and amplitude then was N. 80° 0′ E. and at 8h. 58′, the true altitude of the Sun's center was 38° 34′. Required the latitude of the place of obſervation, and the variation of the compaſs?

Interval 5h. 0′							
Half 2 30=	37°30′		ſine -	9.78445	—	tangent	9.88498
Declination -	18	0	co-ſine	9.97821	—	ſine —	9.48998
	35	23	ſine -	9.76266	arc 2d 76°40′	co-t.	9.37496
Arc firſt —	70	46	co-ſec.	0.02494			
Altitude —	38	34	ſine -	9.79478			
Arc third —	41	19	ſine	9.81972			
Arc ſecond -	76	40					
Sum —	117	59					
Subtract from	180	0					
Arc fourth -	62	1	ſine	9.94600	—	co-ſine - -	9.67137
Declination -	18	0	co-ſine	9.97821	—	co-tang.	0.48822
Latitude —	57	7	ſine -	9.92421	am. N. 55 18 E.	tan.	0.15959
			Obſerved amplitude -		N. 80 0 E.		
			Variation —		—		24 42 W.

Problem VIII.

*Given the Declination, and two obſerved Altitudes and Azimuths of the Sun—to find the Latitude of the Place of Obſervation, and the Variation of the Compaſs.**

Rule.

To the log. verſed ſine of the difference of the obſerved azimuths, add the log. co-ſines of the corrected altitudes; find the natural number anſwering thereto, which being added to the natural verſed ſine of the difference of altitude, the ſum will be the natural verſed ſine of arc firſt.

To

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* If the true azimuths be given, the latitude may be found independent of the Sun's declination by the following formula, in which a repreſents the leaſt altitude, b the greater altitude, c the leaſt azimuth, and d the greateſt azimuth.

$$\text{Tangent latitude} = \frac{\text{coſ. } a. \text{ coſ. } d - \text{coſ. } b. \text{ coſ. } c}{\text{ſine } b - \text{ſine } a}$$

To the log. ſine of arc firſt, add the log. ſine of the difference of azimuth, and the log. co-ſine of the greateſt altitude; the ſum will be the log. ſine of arc ſecond.

To the log. tangent of the Sun's declination, add the log. tangent of the half of arc firſt; the ſum will be the log. co-ſine of arc third. The difference between arcs ſecond and third will be arc fourth, if the latitude and declination are of the ſame name; but if of a contrary name, ſubtract arc ſecond from the ſupplement of arc third, and the remainder will be arc fourth.

Now, to the log. verſed ſine of arc fourth, add the log. co-ſine of the leaſt altitude, and the log. co-ſine of the declination; the nat. number anſwering to this ſum being added to the nat. verſed ſine of the difference or ſum of the leaſt altitude and declination, according as the latitude and declination are of the ſame, or of contrary names; the ſum will be the nat. co-verſed ſine of the latitude.

Then, to the log. ſine of arc fourth, add the log. co-ſine of declination, and the log. ſecant of the latitude; the ſum will be the log. ſine of the true azimuth, when the greateſt altitude was obſerved. Hence the variation will be known.

EXAMPLE.

Auguſt 25, 1788, in the forenoon the Sun's magnetic azimuth was S. 27° 41′ E. and corrected central altitude 33° 14′. Afterwards the magnetic azimuth was S. 9° 53′ W. and true altitude 42° 36′. Required the latitude and variation?

Firſt obſ. azim. S.	27° 41′ E.				
Second obſ. az. S.	9 53 W.				
Difference	37 34	Nat. ver. ſine	20736	log.	4.31672
Firſt altitude	33 14			co-ſine	9.92244
Second altitude	42 36			co-ſine	9.80694
Difference	9 22	Nat. ver. ſine	01333		
			12767		4.10610
Arc firſt	30 48	Nat. ver. ſine	14100	co-ſecant	0.29069
Half arc firſt	15 24	tangent	9.44004	Diff. azim. 37° 34′ ſine	9.78510
Declination	10 30	tangent	9.26797	Greateſt alt. 42 36 co-ſine	9.86694
Arc third	87 4	co-ſine	8.70801	Arc ſecond 61 13 ſine	9.94273
Arc ſecond	61 13				
Arc fourth	25 51	log. verſed ſine	2.00027	ſine	9.63950
Leaſt altitude	33 14	co-ſine	9.92244		
Declination	10 30	co-ſine	9.99267	co-ſine	9.99267
Difference	22 44	N. V. S.	07769		
			08230 - 3.91538		
Latitude	57 8½	N. co-V. S.	15999	ſecant	0.26555
	True azimuth	S. 52 12 E.		ſine	9.89772
	Obſerved azimuth	S. 27 41 E.			
	Variation	24 31 W.			

Upon account of the variation of the compaſs, the true courſe between any two places is different from the courſe per compaſs. The true courſe is found by the common rules of navigation; and if the variation of the compaſs be known, the magnetic courſe may be obtained. Again, in order to determine a ſhip's place from the courſe ſteered and diſtance run, the courſe muſt be previouſly corrected by variation. The method of applying the variation in theſe caſes, is as follows:

PROBLEM IX.

Given the True Courſe between any two Places, and the Variation of the Compaſs—to find the Courſe per Compaſs.

RULE.

The variation being allowed to the right or left of the true courſe, according as it weſterly or eaſterly, will give the courſe per compaſs.

EXAMPLE.

Required the courſe per compaſs from the Lizard to St Mary's, the true courſe being S.W.¼W. and the variation at the Lizard 2¼ points weſt?

Two and a fourth points being allowed to the right of S.W.¼W. gives W.S.W.½W. the magnetic courſe.

PROBLEM X.

Given the Courſe per Compaſs, and the Variation—to find the true Courſe.

RULE.

The variation if weſterly being allowed to the left of the courſe ſteered, or to the right if eaſterly, will give the true courſe.

EXAMPLE.

Let the courſe ſteered be S.E.½E. and the variation 1½ points W. Sought the true courſe?

One and a half points allowed to the left of S.E.½E. gives E.S.E. the true courſe.

END OF THE FIRST VOLUME.

606921
SBN

PLATE. I.

Fig 1.

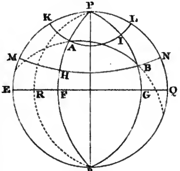
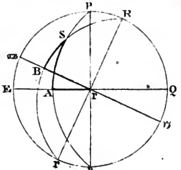


Fig 2.



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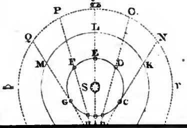

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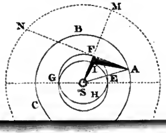

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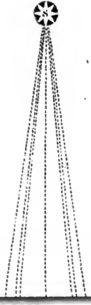

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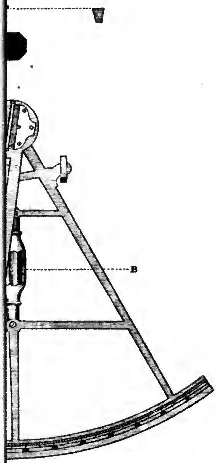
PLATE. II.

1.



ig. 9.

Fig 10.

Carpenter sculpt London.

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PLATE. III.

Fig 12.

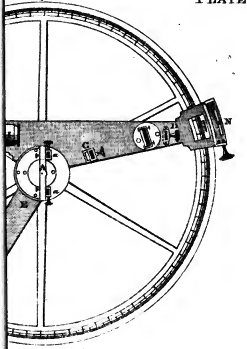

Fig. 13.

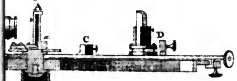

Fig 17.

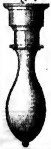

Fig 18.

Fig 19.

Fig 20.

Carpenter sculp^t London.

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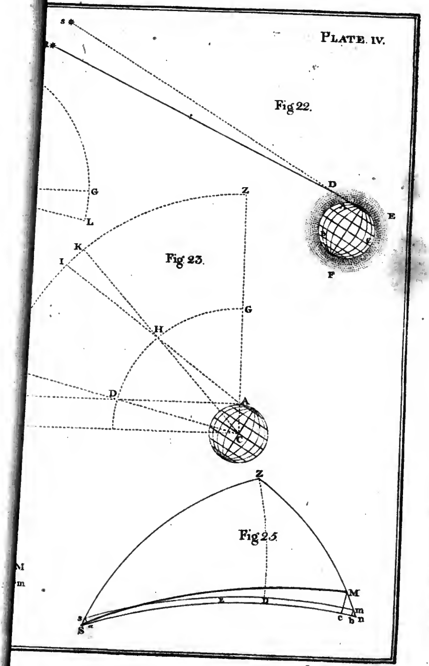
PLATE. IV.



Carpenter Sculpt London.

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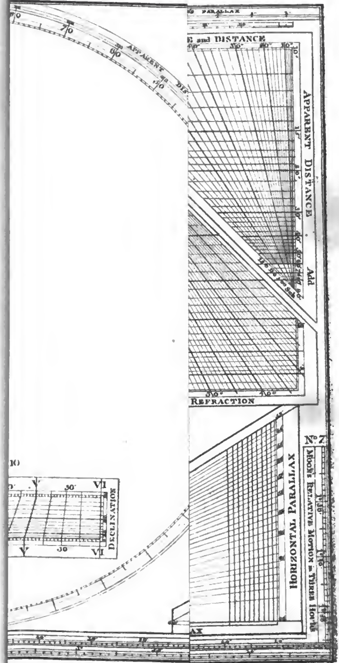

Carpenter sculpt London.

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PLATE. VI.

Fig 28.

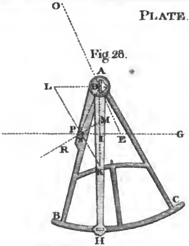

Fig.30.

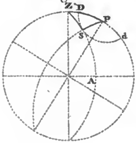

Fig 31.

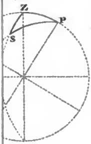

Fig 32.

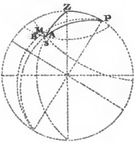

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PLATE. VII.

Fig. 34.

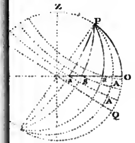
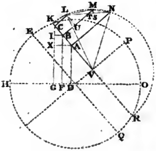


Fig. 35



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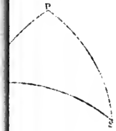


Fig. 38

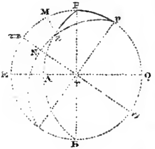
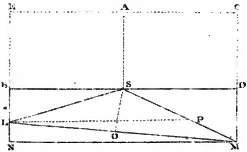


Fig. 40



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